Introduction to Perturbative Quantum-Chromodynamics

CTEQ School 2000 Lake Geneva, WI

Concepts/Vocabulary* (These will appear on the final exam!**)

- **Matter – quarks & leptons, quark model of states and resonances**
- **Parton Model – parton distribution functions (pdf's), fragmentation functions**
- **Symmetries – global and local, gauged – e.g., SU(3) of QCD (unbroken)**
  U(1) of E&M (unbroken)
  $SU(2)_L$ of Weak, SU(2) (to SU(6)) of Flavor (broken-approximate)
- **Interactions – mediated by gauge bosons**
  Strong – gluons (massless)
  Electromagnetic – photons (massless)
  Weak – Z^0, W^+, W^- (massive)
- **Quantum Field Theories – local non-Abelian gauge symmetry, renormalization (scale and scheme, e.g., $\overline{MS}$), factorization (scale), UV, IR & collinear singularities, running couplings – UV freedom & IR slavery, perturbative expansions, power corrections, log resummation**
- **Experimental quantities – exclusive cross section, inclusive cross sections, IR safe quantities, jets**
- **Experimental processes – $e^+e^- \to$ hadrons, $e(\nu)p \to e$+hadrons, $pp \to$hadrons, $pp \to \gamma$+X, $pp \to \mu^+\mu^-$+X**

* Of course, in 4 hours we won't really cover everything, nor will you really be able to do NNLO calculations at the end!
** Just kidding – NO exam!

The big picture:

Interaction	Observed	Strength	Range	Carrier	
Strong	nuclear forces	~1	$\sim 10^{-15}$ m	Gluon	Standard Model
EM	Atomic systems	$\sim 10^{-2}$	$1/r^2$	Photon	Standard Model
Weak	Decays	$\sim 10^{-5}$	$\sim 10^{-18}$ m	$W^{\pm}$,Z	Standard Model
Gravity	Astronomy	$\sim 10^{-39}$	$1/r^2$	Graviton	"SUSY Strings"

References:

- **The previous lectures at CTEQ Schools by George Sterman, Wu-Ki Tung and Dave Soper.**
- **QCD Summary on the Web at the Particle Data Group site: http://pdg.lbl.gov/1999/contents_sports.html#stanmodeletc**
- **The CTEQ Handbook in Rev. Mod. Phys. Volume 67, Number 1, January 1995, (pp. 157-248) and on the Web: http://www.phys.psu.edu/~cteq/#Handbook**
- **"The Pink Book" – *QCD and Collider Physics*, R.K. Ellis, W.J. Stirling and B.R. Webber, Cambridge University Press, 1996**

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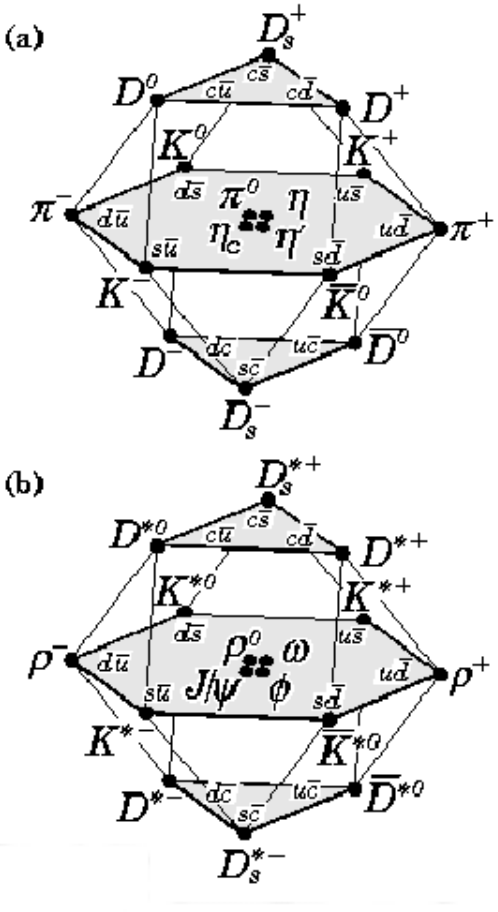
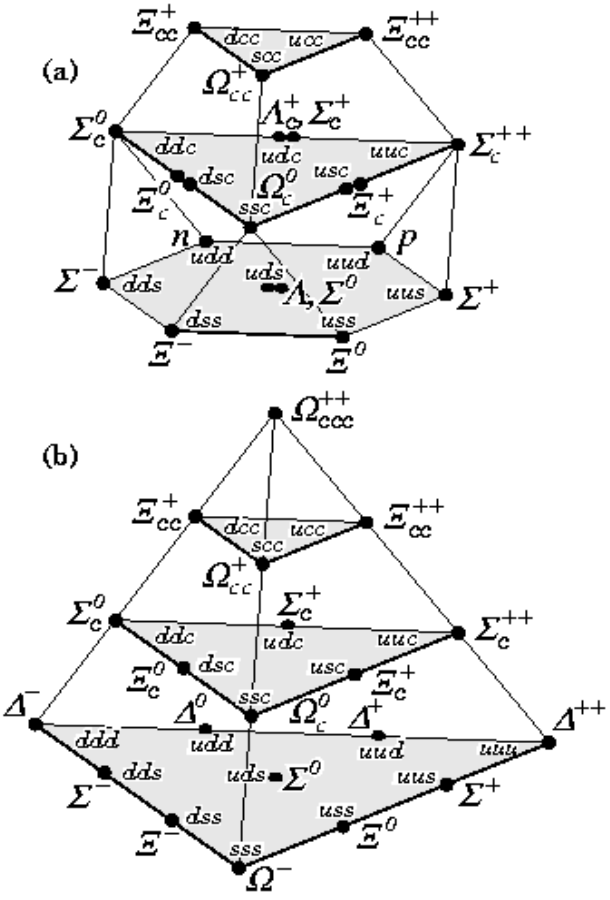
First a Bit of History

PreQCD I (< ~1968) Quarks "algebraic" with no dynamics (of course, we had dynamics in the form of Regge theory, the bootstrap picture and dual models but …). Hadron quantum numbers correct if

$q\bar{q} \rightarrow$ mesons $\quad qqq \rightarrow$ baryons – approximate flavor SU(3)

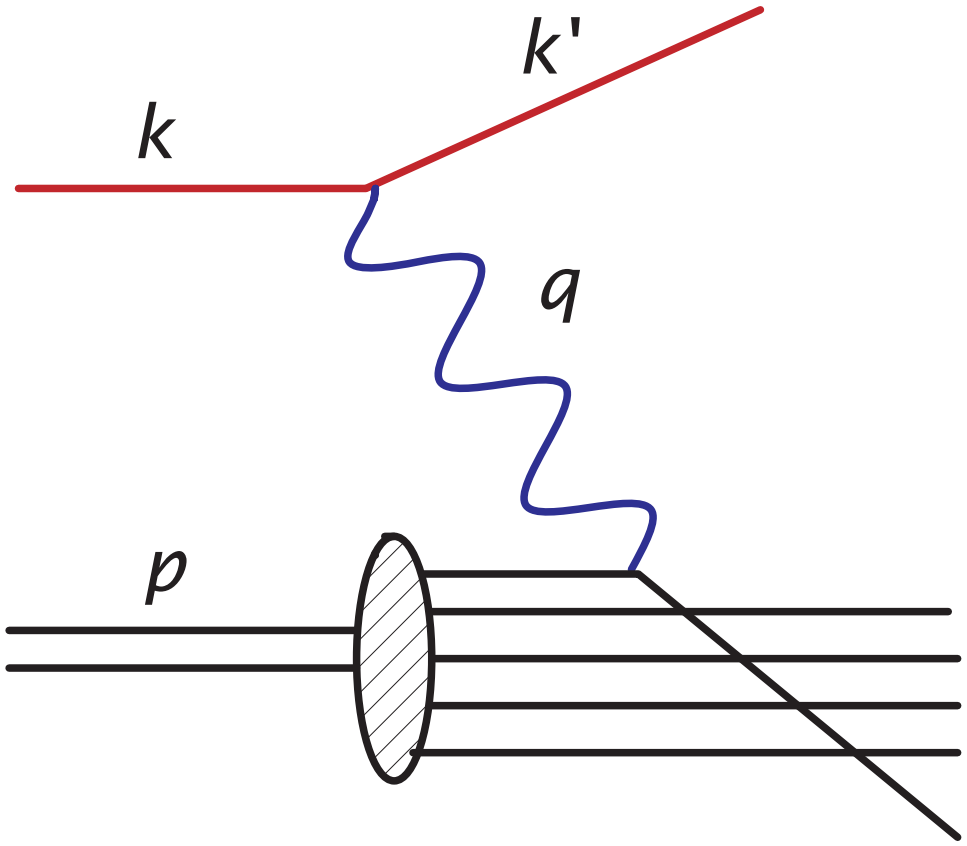
Property/Quark	u	d	s
Electric Charge	+2/3	-1/3	-1/3
Isospin, I_3	+1/2	-1/2	0
Strangeness	0	0	1





PreQCD II (<~1972) Parton Model –
quarks are the building blocks of hadrons *and* play a role in the dynamics (even if we don't understand it!).

Consider the inclusive deeply inelastic scattering of electrons from protons – DIS, (i.e., SLAC).



In the proton rest frame the kinematics look like:

$$p^{\nu} = \left(m, \vec{0}\right), k^{\mu} = \left(E, \vec{k}\right), k'^{\mu} = \left(E', \vec{k}'\right)$$

$$\nu = p \cdot q = m\left(E - E'\right)$$

$$x = \frac{-q^2}{2\nu} = \frac{Q^2}{2m\left(E - E'\right)}$$

$$y = \frac{q \cdot p}{k \cdot p} = 1 - \frac{E'}{E}$$

Invoking E&M and relativity ⇒ 2 dimensionless functions

$$\frac{d\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\begin{array}{l}\left[1+(1-y)^2\right]F_1(x,Q^2) \\ +\left[\frac{1-y}{x}\right]\left(F_2(x,Q^2) - 2xF_1(x,Q^2)\right)\end{array}\right]$$

with $m << E$. (You should confirm this!)

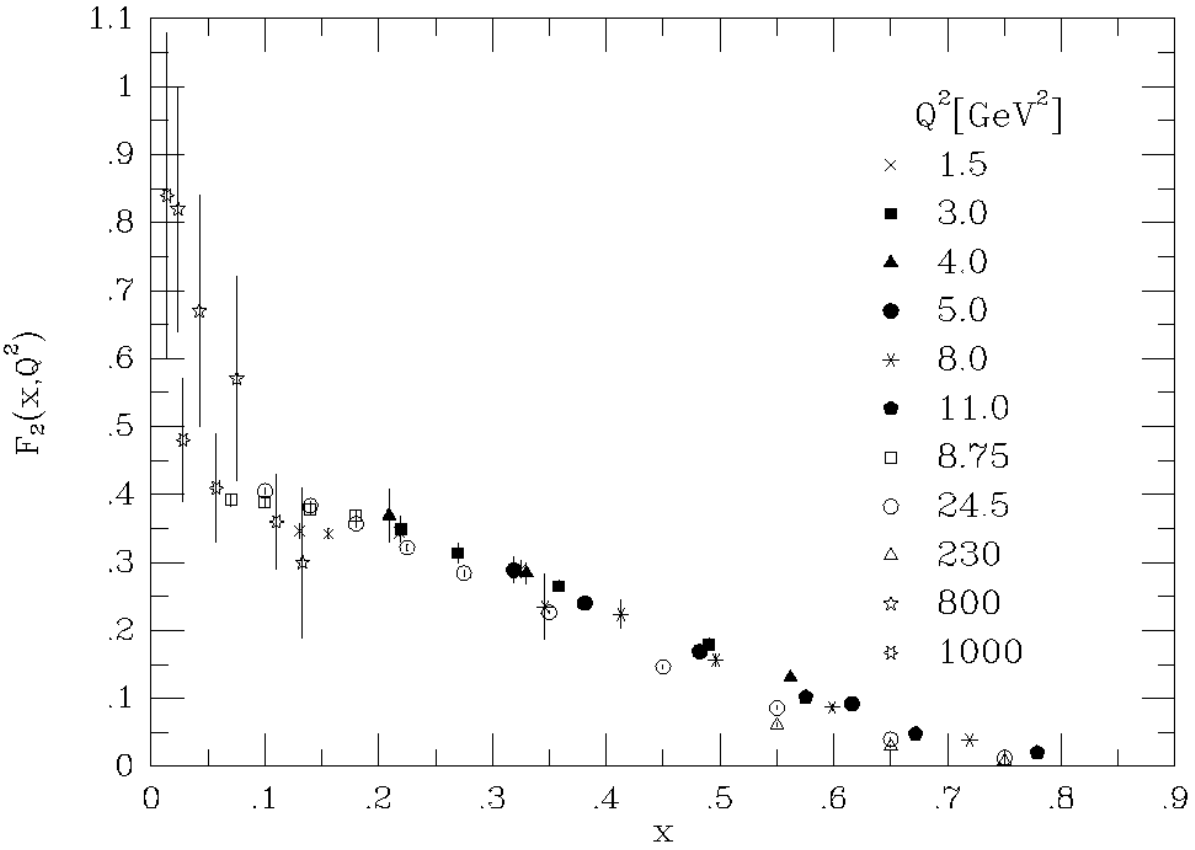
- F_1 **⇒ absorption of transversely polarized photons**
- $F_2 - 2xF_1$ **⇒ longitudinally polarized photons (in the high energy limit).**

Note: for an *elementary* fermion (requiring $(p+q)^2 = p^2$**), electric charge** e_f

$$\frac{d\sigma}{dxdQ^2}\Big|_f = \frac{4\pi\alpha^2}{Q^4}\left[1+(1-y)^2\right]\frac{e_f^2}{2}\delta(1-x)$$

For *elastic* e-p scattering, same form but with the EM form factor squared of the proton [~1/Q⁴] - this specific channel thus makes a negligible contribution at large Q. It is very difficult for a proton to stay a proton when you hit it hard because it is NOT elementary!

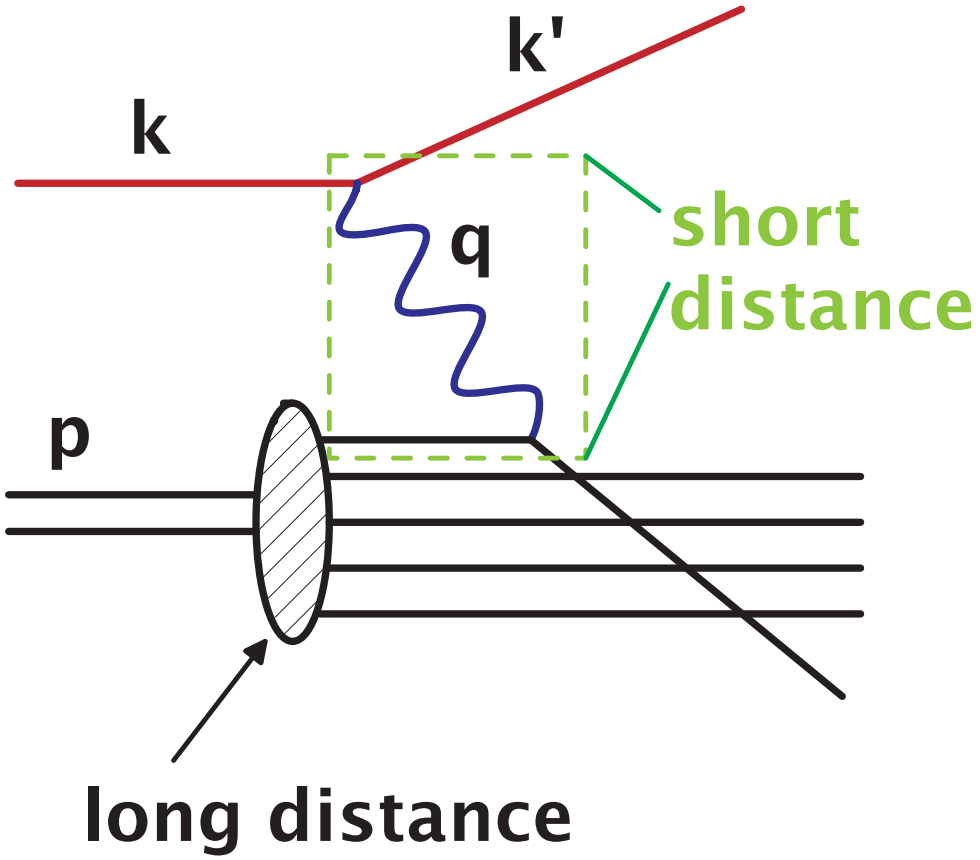
Limit $Q^2, \nu \to \infty; x$ fixed (the "bj" or scaling limit), if there is no large hadronic scale (the hadronic physics is soft or "slow"), we naively expect $F_i\left(x, Q^2\right) \to F_i\left(x\right)$, i.e., scaling.



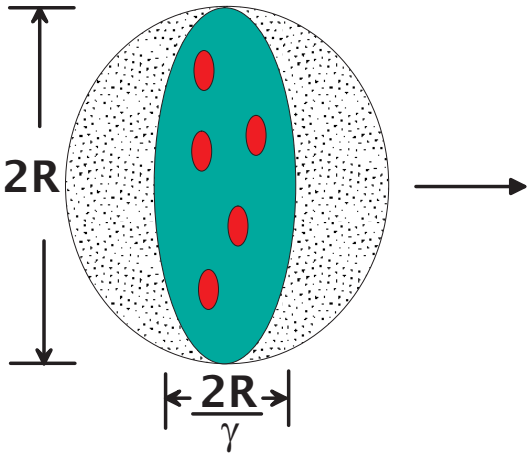
Interpretation – the proton is composed of essentially free, point-like quarks – what could be simpler!

***x* is the fraction of the proton's moment carried by the scattered quark.**

Impact scenario – electron and quark interaction on short time scale (~1/Q); quarks interact on long time scale (~1/m).



Aside Also useful to consider the "infinite momentum frame" where P→infinity, the proton is "mostly" contracted (except wee partons ~dx/x), internal interactions are "frozen" (dilated)



Each quark contributes

$$\hat{F}_2(x) = 2x\hat{F}_1 = e_q^2 x\delta(x-\xi)$$

ξ **= the momentum fraction carried by the quark.**

For the proton we have

$$F_2(x) = 2xF_1(x) = \sum_{q,\bar{q}} e_q^2 x F_{q/p}(x)$$

$F_{q/p}(x) \equiv q(x)$ $\Rightarrow$ **distribution of quarks within the proton.**

$2xF_1(x) = F_2(x)$ $\Rightarrow$ **Callan-Gross relation.**

Experimentally true - evidence that partons are quarks (or at least fermions).

Define distributions for each flavor, with valence quarks and a flavor neutral sea:

$$F_{u/p}(x) \equiv u(x) = u_V(x) + S(x); F_{d/p}(x) \equiv d(x) = d_V(x) + S(x)$$
$$S(x) \equiv \bar{u}(x) = \bar{d}(x) = s(x) = \bar{s}(x)$$

So that

$$F_2^{\gamma}(x) = x\left[\frac{4}{9}(u(x)+\bar{u}(x)) + \frac{1}{9}(d(x)+\bar{d}(x)+s(x)+\bar{s}(x))\right]$$

with (experimentally correct)

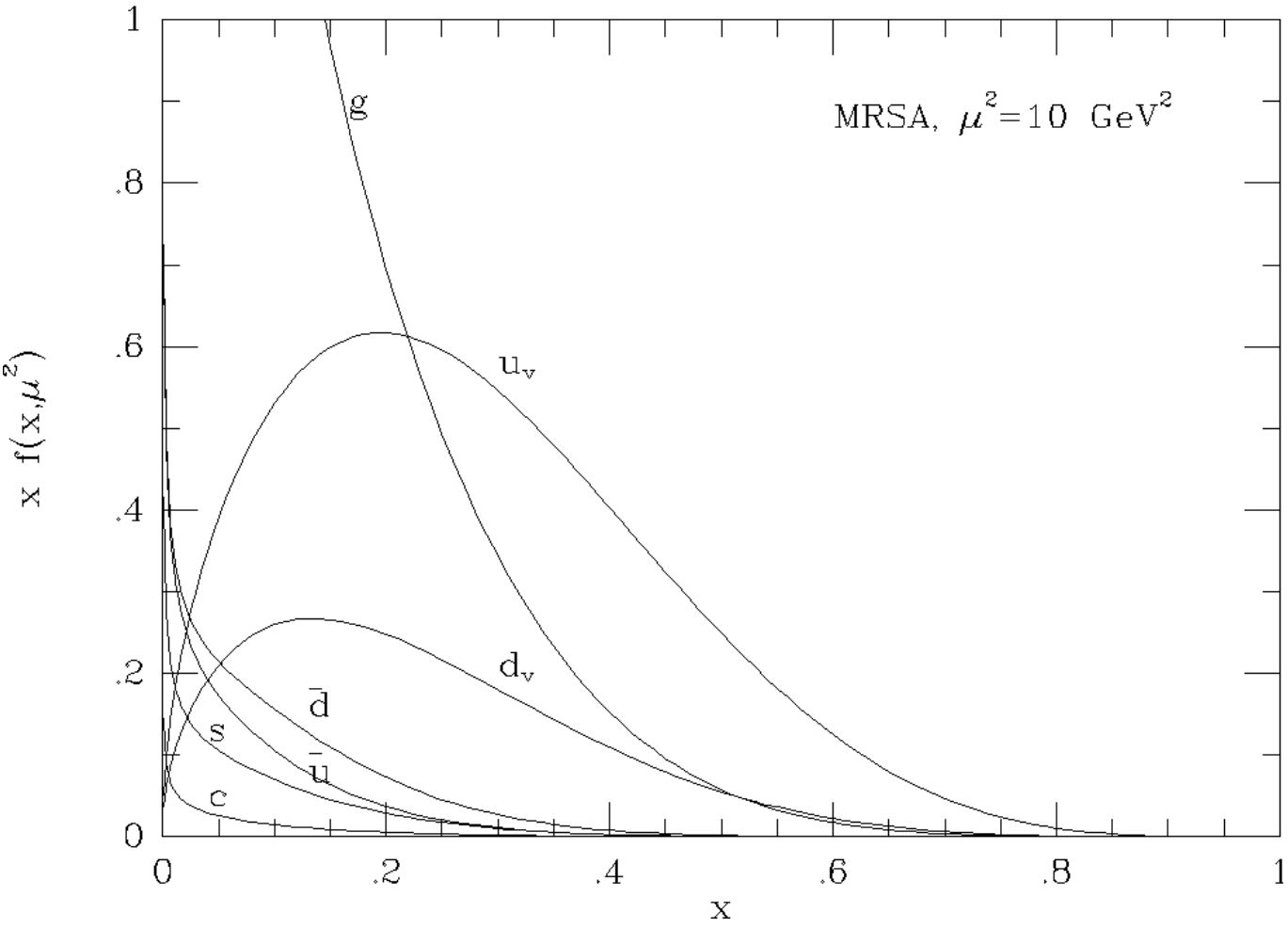
$$\int_0^1 dx\, u_V(x) = 2; \int_0^1 dx\, d_V(x) = 1$$

but total momentum -

$$\sum_q \int_0^1 dx x\left[q(x)+\bar{q}(x)\right] = \int_0^1 dx x\left[u_V(x)+d_V(x)+6S(x)\right]\cdots\left[\approx 0.5\right]_{data}$$

Only 50% of the momentum is carried by quarks!?

Typical distribution parton distribution functions look like (ignore the one labeled "g" for now)

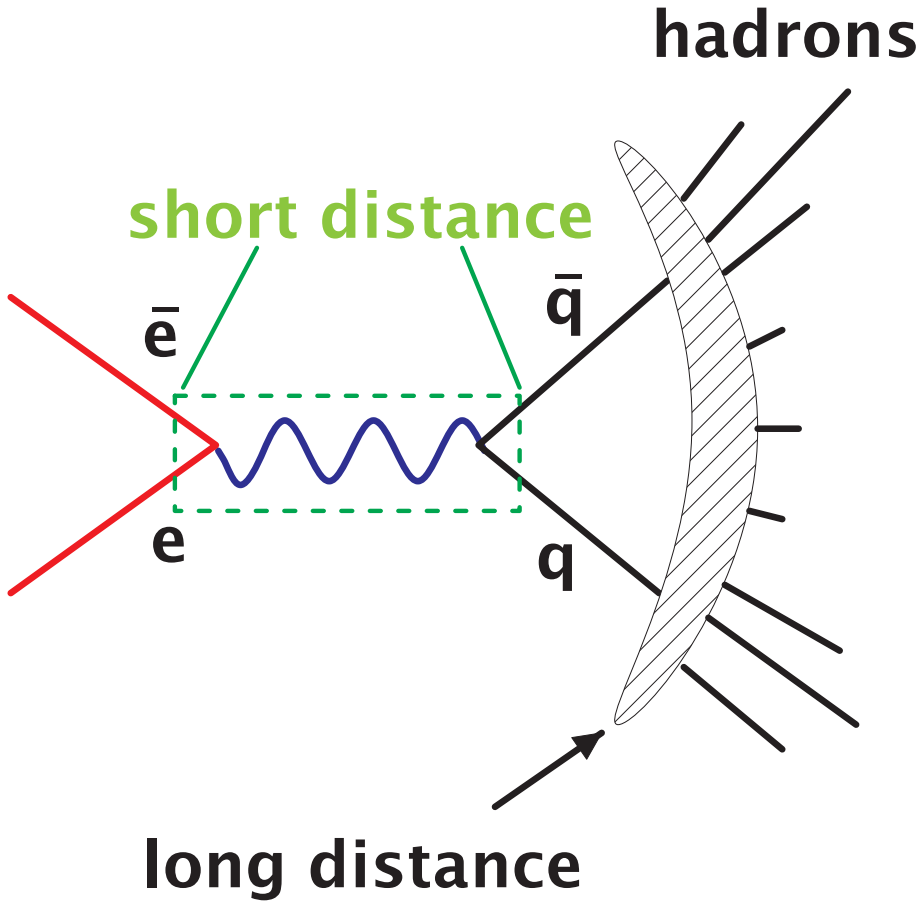


Note that the sea is NOT SU(3) or even SU(2) symmetric.

Similarly the process $e^+e^- \to$ hadrons (inclusively) can be thought of as $e^+e^- \to q\bar{q}$. The total cross section is thus

$$\sigma_0 = \frac{4\pi\alpha^2}{3Q^2}\sum_q e_q^2; \qquad R = \frac{\sigma\left(e^+e^- \to q\bar{q}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \sum_q e_q^2 = \frac{2}{3}$$

The picture looks like – again factor short and long distances



At "long distances" the scattered quarks pull further quarks and anti-quarks out of the vacuum that somehow reassemble into hadrons. This gives us two new concepts –

- fragmentation
- jets

The fragmentation function $D_{h/q}(z)$ describes the probability to find a hadron h in the collinear debris of the fragmenting quark q with momentum fraction z of the original quark.

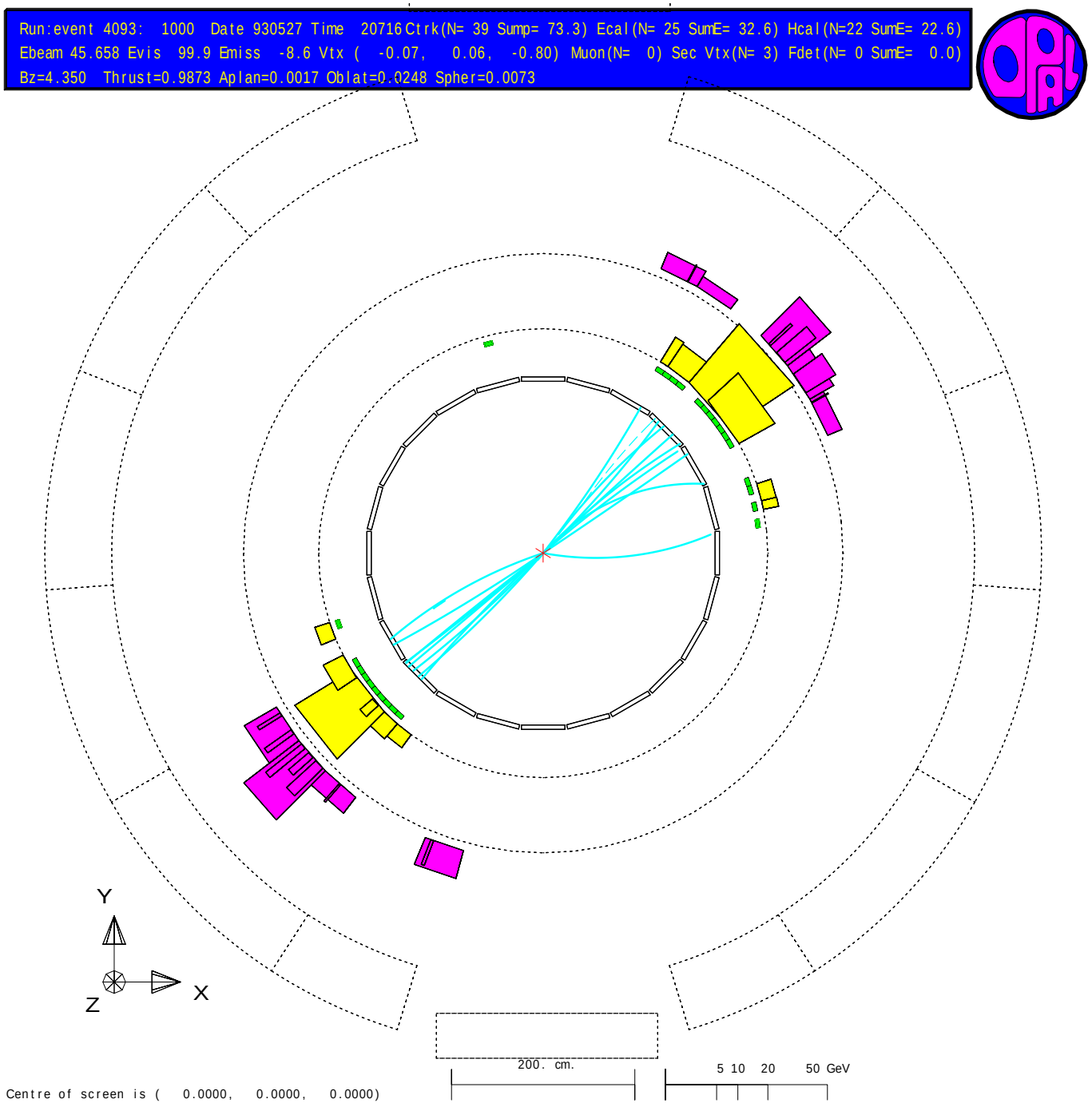
A jet is a "spray" of essentially collinear hadrons whose total momentum and even flavor quantum numbers track those of the fragmenting quark – the "footprint" of the quark.

Study the kinematics of the produced quarks by studying the kinematics of the leading hadrons – forming 2 jets.

The angular cross section for electrons to quarks, i.e., spin ½ fermions, should track the angular distribution of the jets (or at least the leading hadrons) –

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2Q^2}\left(1+\cos^2\theta\right)\sum_q e_q^2$$

and it agrees with the data! This is another indication that the partons are really quarks!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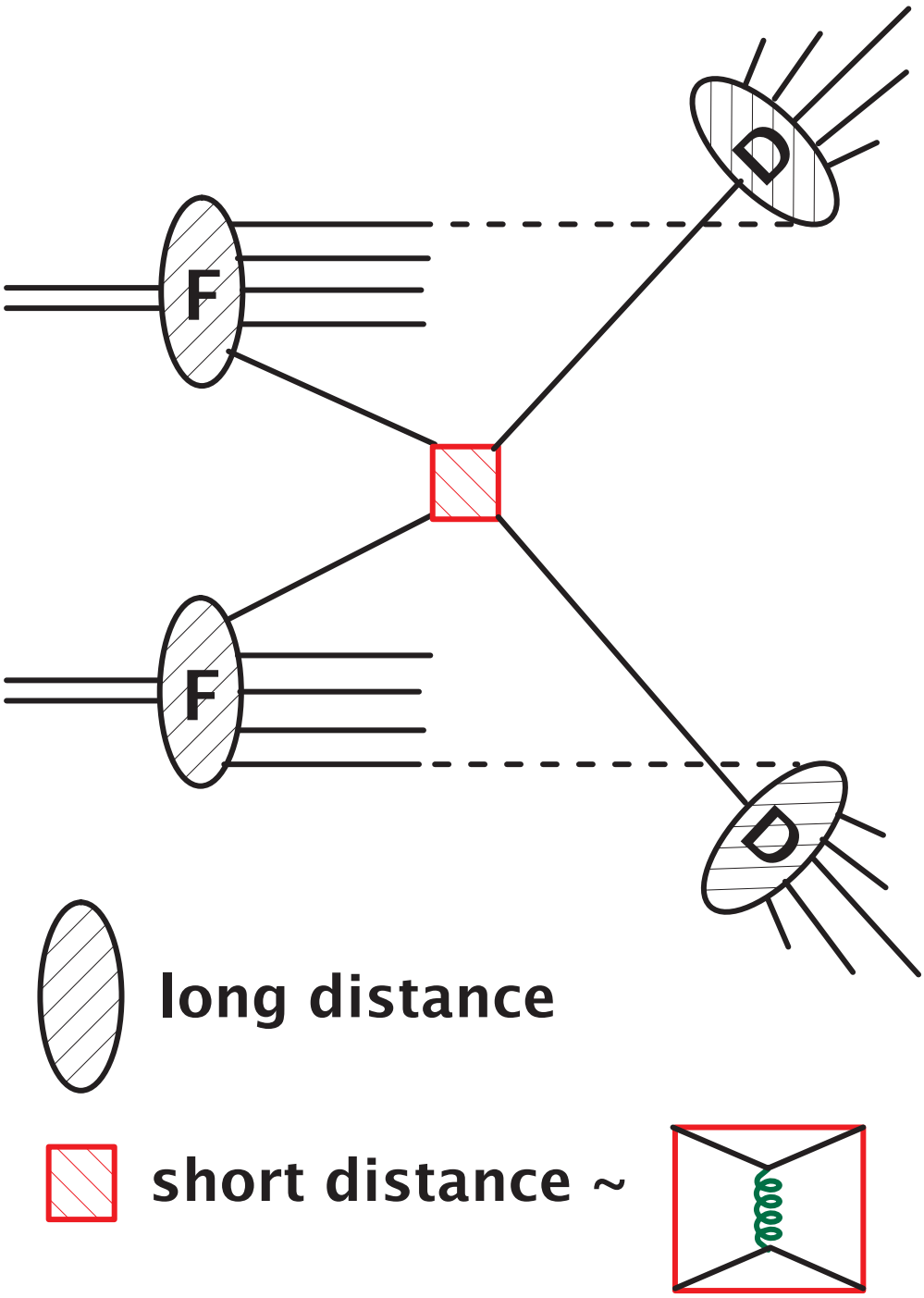
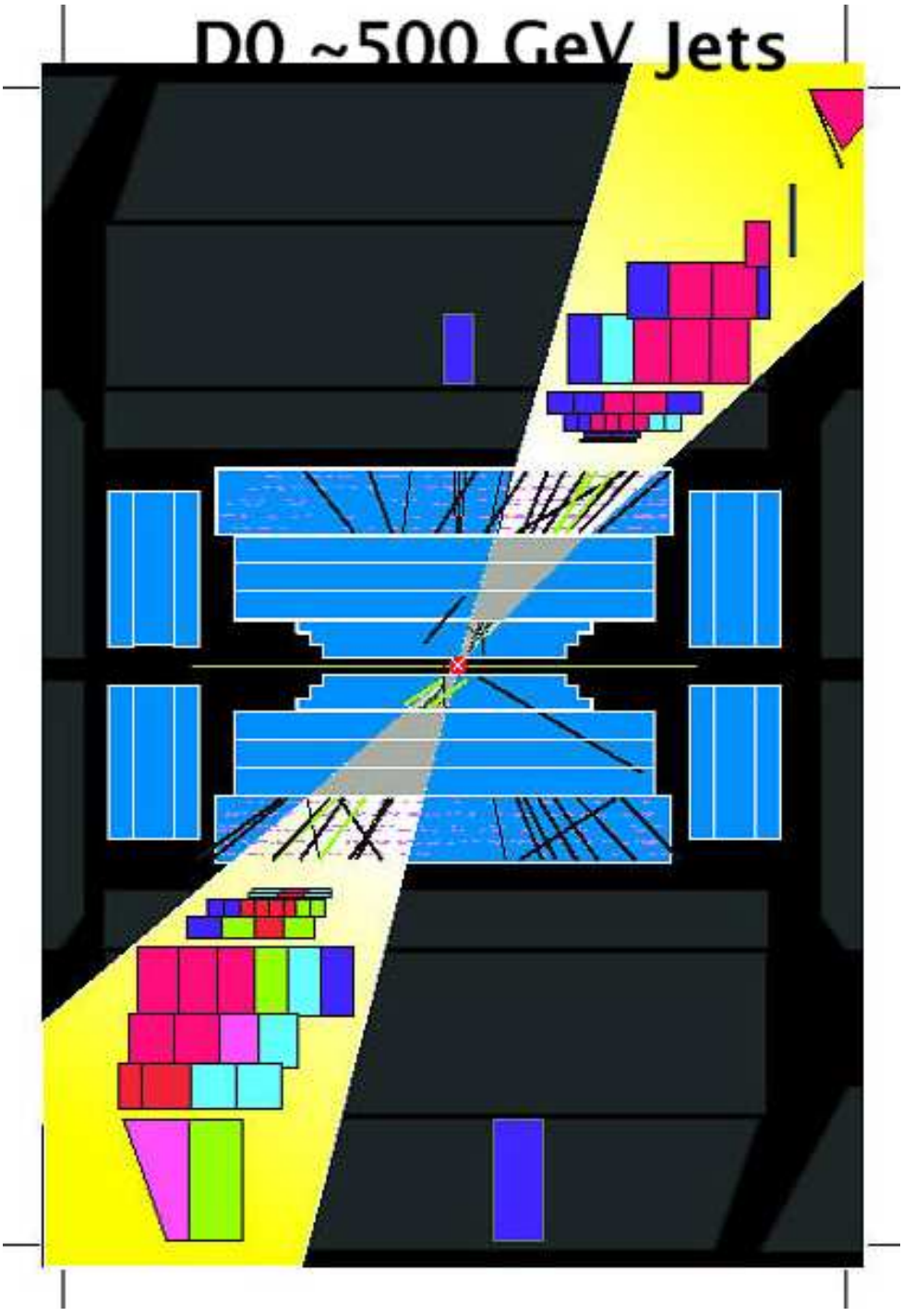
The picture for large P_T hadron-hadron scattering –



$$\sigma \sim \iiint F(x_1)\, F(x_2)\, \hat{\sigma}\, D(z_1) D(z_2)$$



This is all very nice …

BUT…

- Baryons have 3 quarks in a purely symmetric state
- The calculated e^+e^- cross section is small by a factor 3
- ~50% of the proton's momentum is not in quarks

⇒ Simple fix – quarks have a previously hidden quantum number, COLOR, that comes in 3 values (quarks are a $\underline{3}$ under the corresponding SU(3)) so that

- Color singlet ground state is anti-symmetric
- Must sum over colors in e^+e^- final state → factor of 3, R→2
- Extra partons holding proton together → gluons, carrying the rest of the momentum

But still no one has ever seen an isolated quark and we had NO theory of what this was all about!

In the mid-1970's the next step occurred!

QCD era (>~1974…) - The Yang-Mills Revolution

Lagrangians are back!!

Quantum field theory can describe a unified version of the Electro-Weak interactions based on a gauged (but broken) SU(2) ⊗U(1) symmetry.

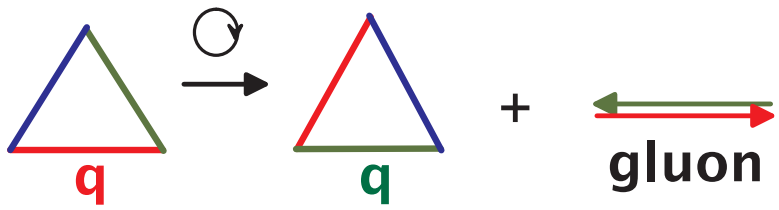
QCD Field Theory – unbroken SU(3)

- **quarks in the fundamental representation of– the triplet, Δ**
- **anti-quarks in the complex conjugate representation, ∇**

Recall that we now have 6 quarks (with masses spread over an enormous range – NOT explained by QCD – why not?)

quark	u	d	s	c	b	t
charge	2/3	-1/3	-1/3	2/3	-1/3	2/3
mass	~ 4 MeV	~ 7 MeV	~ 135 MeV	~ 1.5 GeV	~ 5 GeV	~ 175 GeV

Local **SU(3) gauge invariance requires the theory to have a vector field (the gluon) that is in the adjoint representation 8.**



Why a vector field?

Quarks have both a flavor and a color index - $q_{f,a}$

Color singlet states (1) - just what we wanted!!

- $q_a \bar{q}_a$ - **mesons**
- $\varepsilon_{abc} q_a q_b q_c$ - **baryons**

The classical Lagrangian has the form

$$L_{QCD} = -\frac{1}{4} F^B_{\alpha\beta} F_B^{\alpha,\beta} + \sum_f \bar{q}_{f,a} \left(iD_\mu \gamma^\mu - m_f \right)_{ab} q_{f,b}$$

$$F^B_{\alpha\beta} = \left[\partial_\alpha A^B_\beta - \partial_\beta A^B_\alpha - g f^{BCD} A^C_\alpha A^D_\beta \right]$$

Acting on the triplet and octet, respectively, the covariant derivative is

$$\left(D_\mu\right)_{ab} = \partial_\mu \delta_{ab} + ig\left(t^C A^C_\mu\right)_{ab} ; \left(D_\mu\right)_{CD} = \partial_\mu \delta_{CD} + ig\left(T^B A^B_\mu\right)_{CD}$$

The matrices for the fundamental (t_{ab}^B) and adjoint (T_{CD}^B) representations carry the information about the Lie algebra

$$\left[t^B, t^C\right] = if^{BCD} t^D ; \left[T^B, T^C\right] = if^{BCD} T^D ; \quad \left(T^B\right)_{CD} = -if^{BCD};$$

$$Tr\left[t^B t^C\right] = \frac{\delta^{BC}}{2} \equiv T_R \delta^{BC} ; \; t^B_{ab} t^B_{bc} = \frac{4}{3} \delta_{ac} \equiv C_F \delta_{ac} ;$$

$$Tr\left[T^B T^C\right] = 3\delta^{BC} \equiv C_A \delta^{BC}$$

(f^{BCD} is the structure constant of the group)