pQCD Calculation IV: Fragmentation, hadronization and jets

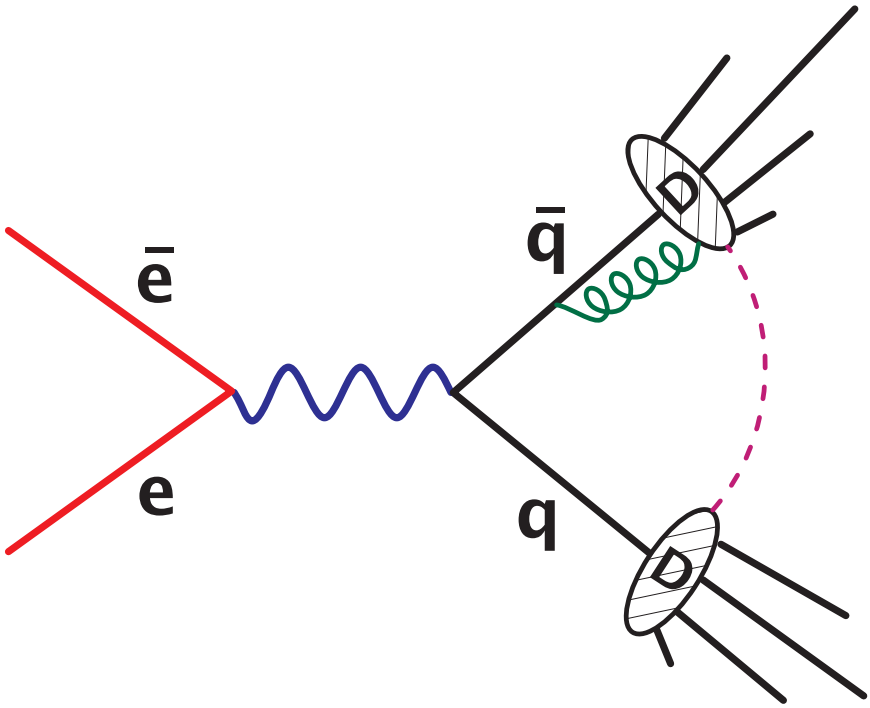
(see N. Varelas)

Revisit $e^+e^- \to$ hadrons – (note the color connection - - - -)

Expect:

⇒ collinear singularities just as for the distribution functions

⇒ Fragmentation functions acquire μ^2 dependence similarly to the distribution function



Start with distribution of hadrons *h* as a function of the fraction of the total energy

$$F^h\left(x,Q^2\right)=\frac{1}{\sigma_{TOT}}\frac{d\sigma}{dx}\left(e^+e^- \to h+X\right);\ x=\frac{2E_h}{Q}$$

which satisfies

$$\left\langle n_h\left(Q^2\right)\right\rangle=\int_0^1 dx\,F^h\left(x,Q^2\right)$$

$$\sum_h \int_0^1 dx\,x\,F^h\left(x,Q^2\right)=2$$

Naively this distribution arises from a sum over the contributions from the various primary partons, produced at the short distance scale Q^2

$$F^h\left(x,Q^2\right)=\sum_i \int_x^1 \frac{dz}{z} C_i\left(Q^2;z,\alpha_s\right) D_i^h\left(\frac{x}{z},Q^2\right)$$

The indices of the fragmentation function *D* denote *i*→*h*.

The coefficient function *C* describes the short distance production of the primary partons.

$$C_q \sim e_q^2\left[\delta(1-z)+O\left(\alpha_s\right)+\ldots\right]$$

assuming only photon exchange (no *Z*'s). Gluons will only start to contribute at order α_s.

- The function D^h is *not* calculable perturbatively (hadron formation is intrinsically long distance and non-perturbative).

- The fragmentation (or time-like evolution) of parton *i* → parton j is treatable perturbatively. The collinear divergences are factorizable (just as for the parton distribution functions).

$$D_i^j\left(x,\mu^2\right)=\sum_k \int_x^1 dz\, K_i^k\left(z,\mu^2,\mu_0^2\right) D_k^j\left(\frac{x}{z},\mu_0^2\right)$$

where *K* is calculable (for μ^2 and μ_0^2 large).

Note - Having once factored the collinear singularities into the renormalized *D*, there is no problem doing the same with confinement and setting *j* → *h* again.

Warning: This picture suggests that in $e^+e^- \to$ hadrons

- Each hadron is associated with one specific initial parton
- Each hadron is associated with a jet

Analysis of the data has proceeded with this in mind.

BUT – it ain't so! The soft hadrons (at least) must be associated with the coherent interactions of color singlet *combinations* of partons (the color connection – strings?).

Back to Fragmentation Function

In summary - pQCD tells us that

- the renormalized fragmentation functions evolve
- the form of the evolution is calculable

$$\mu^2 \frac{\partial}{\partial \mu^2} D_i\left(x,\mu^2\right) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} D_j\left(\frac{x}{z},\mu^2\right) P_{ji}\left(z,\alpha_s\right)$$

just like DGLAP except for the different order of the indices (*i* is the initial parton and *j* the final).

$$P_{ji}\left(z,\alpha_s\right) = P_{ji}^{(0)} + \frac{\alpha_s}{2\pi} P_{ji}^{(1)}\left(z\right) + \ldots$$

The lowest order splitting functions $P^{(0)}$ are identical to those introduced earlier, the higher order ones are not.

$$P_{qq}^{(0)}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

$$P_{qg}^{(0)}(z) = T_R\left[z^2 + (1-z)^2\right]$$

$$P_{gq}^{(0)}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right]$$

$$P_{gg}^{(0)}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{(1-z)}{z} + z(1-z)\right] + \delta(1-z)\frac{11C_A - 4n_f T_R}{6}$$

Define moments as before

$$D\left(j,\mu^2\right) = \int_0^1 dz\, z^{j-1}\, D\left(z,\mu^2\right)$$

where, for example,

$$\mu^2\frac{\partial}{\partial\mu^2} D_{NS}\left(j,\mu^2\right) = \frac{\alpha_s\left(\mu^2\right)}{2\pi}\gamma_{qq}\left(j,\alpha_s\left(\mu^2\right)\right) D_{NS}\left(j,\mu^2\right)$$

$$\gamma_{qq}\left(j,\alpha_s\left(\mu^2\right)\right) = \int_0^1 dz\, z^{j-1} P_{qq}\left(z,\alpha_s\left(\mu^2\right)\right)$$

Define a singlet function as

$$D_S = \sum_i \left(D_{q_i} + D_{\bar{q}_i}\right)$$

which mixes with the gluon function and satisfies

$$\mu^2 \frac{\partial}{\partial \mu^2}\begin{pmatrix} D_S\left(j,\mu^2\right) \\ D_g\left(j,\mu^2\right)\end{pmatrix} = \frac{\alpha_s\left(\mu^2\right)}{2\pi}\times$$
$$\begin{pmatrix} \gamma_{qq}\left(j,\alpha_s\left(\mu^2\right)\right) & 2n_f\gamma_{qg}\left(j,\alpha_s\left(\mu^2\right)\right) \\ \gamma_{gq}\left(j,\alpha_s\left(\mu^2\right)\right) & \gamma_{gg}\left(j,\alpha_s\left(\mu^2\right)\right)\end{pmatrix}\begin{pmatrix} D_S\left(j,\mu^2\right) \\ D_g\left(j,\mu^2\right)\end{pmatrix}$$

$$\gamma_{qq}^{(0)}(j) = C_F\left[-\frac{1}{2}+\frac{1}{j(j+1)}-2\sum_{k=2}^{j}\frac{1}{k}\right]$$

$$\gamma_{qg}^{(0)}(j) = T_R\left[\frac{2+j+j^2}{j(j+1)(j+2)}\right]$$

$$\gamma_{gq}^{(0)}(j) = C_F\left[\frac{2+j+j^2}{j(j^2-1)}\right]$$

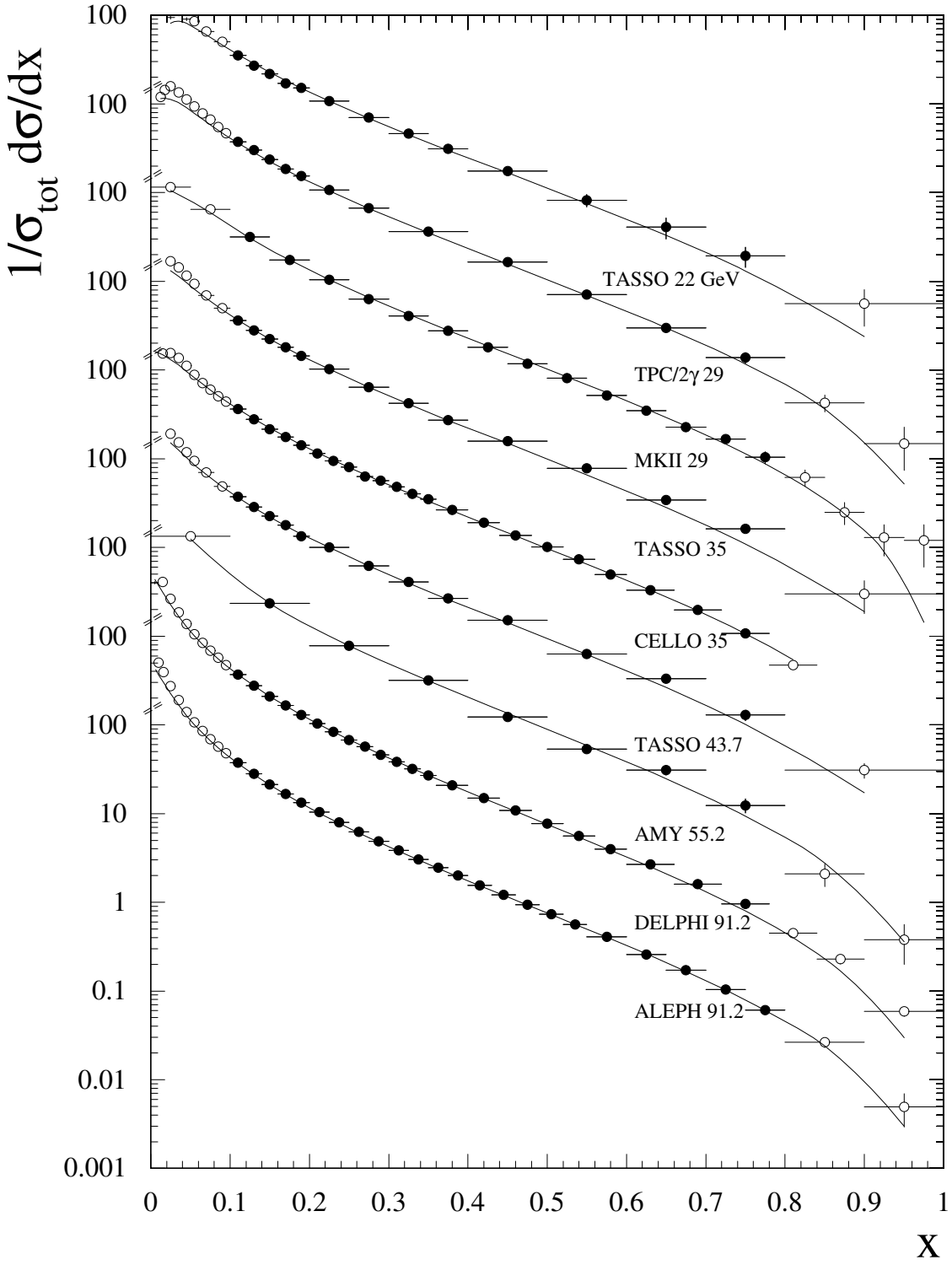
$$\gamma_{gg}^{(0)}(j) = 2C_A\left[-\frac{1}{12}+\frac{1}{j(j-1)}+\frac{1}{(j+1)(j+2)}-\sum_{k=2}^{j}\frac{1}{k}\right]-\left(\frac{2}{3}\right)n_f T_R$$

Fragmentation function have scaling violations just like parton distribution function

- as μ^2 increases

- **the fragmentation function decreases at large *z***
- **the fragmentation function increases at small *z***

ALEPH comparison of NLO scaling violations (and a simple 1/Q correction) yields



Pretty good (if somewhat uncertain) !

What can we learn about (so far ill-defined*) jets, especially quarks versus gluons?

Since the (coupling)^2 of a gluon to a gluon is

$$\frac{C_A}{C_F} = \frac{9}{4}$$

times stronger than a gluon to a quark, we expect 9/4 times more radiation in a gluon initiated jet than a quark initiated one.

We see exactly this result in the ratio

$$\frac{\gamma_{gg}^{(0)}(j)}{\gamma_{gq}^{(0)}(j)} \xrightarrow[j\to 1]{} \frac{C_A}{C_F}$$

which we expect to control the particle multiplicity ratio**

$$\frac{\langle n_X \rangle_g}{\langle n_X \rangle_q} \xrightarrow[\mu^2\to\infty]{} \frac{C_A}{C_F} \qquad X = h,\, q,\, g$$

Gluon jets have higher multiplicities than quark jets

OPAL finds $\dfrac{\langle n_{h^\pm} \rangle_g}{\langle n_{h^\pm} \rangle_q} = 2.29 \pm 0.09 \pm 0.15$ **for the ratio of "soft" charged hadrons, where gluon jets are those in the hemisphere opposite 2 tagged quark jets. (The ratio for the full multiplicity in the jets is smaller – see N. Varelas.)**

*We will define jets more carefully shortly.

**Since both anomalous dimensions are singular at j=1, the analysis of the full multiplicity distributions is more complicated than discussed here.

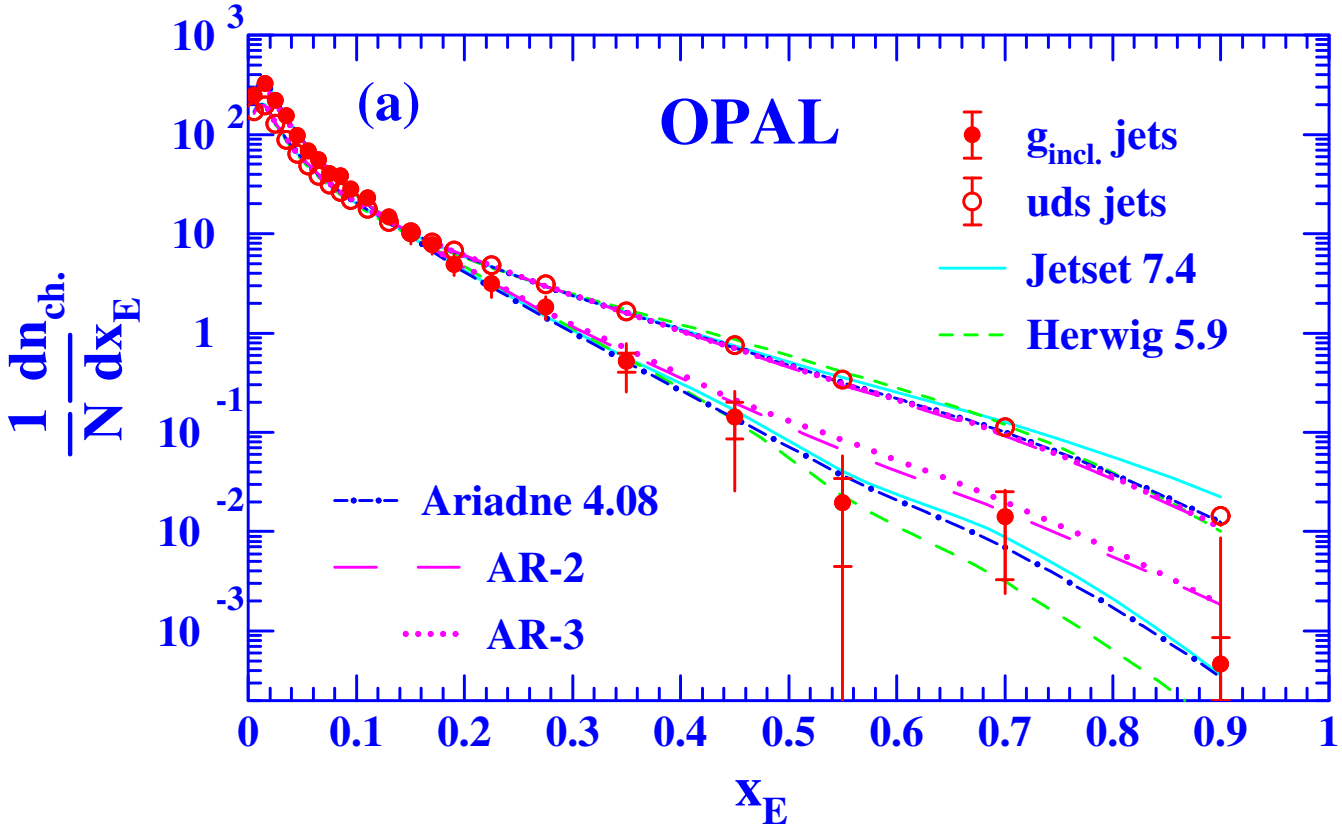
The same ratio appears in the ln j behavior of $\gamma_{gg}^{(0)}$ compared to $\gamma_{qq}^{(0)}$ for j>>1.

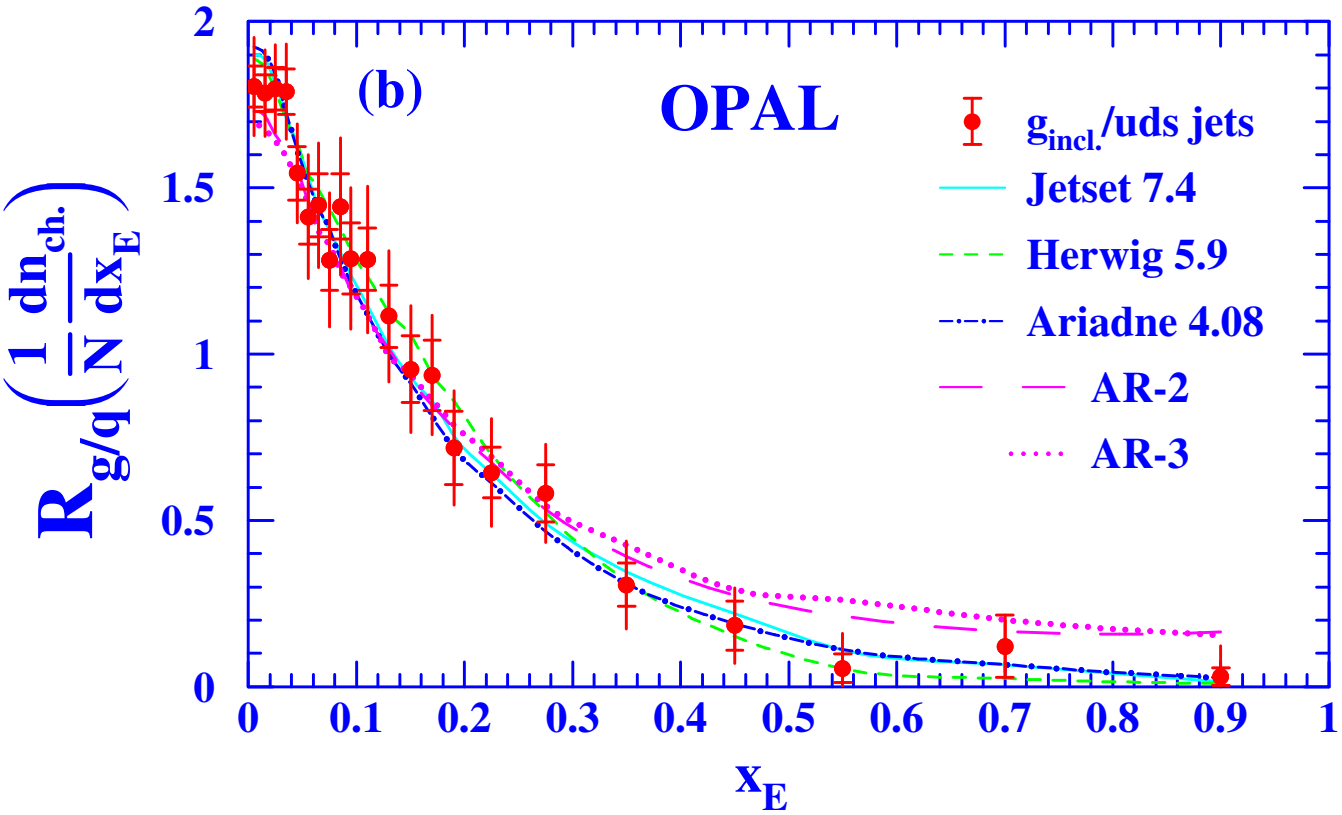
$$\frac{\gamma_{gg}^{(0)}}{\gamma_{qq}^{(0)}} \xrightarrow[j\to\infty]{} \frac{-2\,C_A \ln j}{-2\,C_F \ln j}$$

Relating this to the exponent of (1-z) (as we did for (1-x) in the parton distribution function discussion) suggests that evolution leads to gluon fragmentation functions that are softer than quark fragmentation functions

$$\frac{D_g^g\left(z,\mu^2\right)}{D_q^q\left(z,\mu^2\right)} \xrightarrow[z\to 1\;\mu^2\to\infty]{} (1-z)^{4(C_A-C_F)\ln\left(\ln\left(\mu^2/\Lambda_{QCD}^2\right)\right)/\beta_0+a\left(\mu_0^2\right)}$$

The softer gluon fragmentation is in fact seen, as in the following OPAL analysis -


(a)
OPAL
$g_{incl.}$ jets
uds jets
Jetset 7.4
Herwig 5.9
Ariadne 4.08
AR-2
AR-3
$\frac{1}{N}\frac{dn_{ch.}}{dx_E}$
x_E


(b)
OPAL
$g_{incl.}$/uds jets
Jetset 7.4
Herwig 5.9
Ariadne 4.08
AR-2
AR-3
$R_{g/q}\left(\frac{1}{N}\frac{dn_{ch.}}{dx_E}\right)$
x_E

pQCD Calculation V: Hadron – Hadron scattering

- With our tools in hand we can attack any process that provides a calculable short distance interaction, with the long distance complexity factored into the parton distribution and fragmentation functions.

- Of course, since they are not predicted by QCD, it is best to avoid them – and we can avoid the fragmentation function.

Examples:

$p\bar{p} \to \gamma + X$ at large p_T (see J. Huston)

$p\bar{p} \to \gamma^*, W, Z + X$ (see J. Owens)

$p\bar{p} \to jet + X$ at large p_T with an appropriate (IRS) jet definition

This last process receives contributions from a vast number of channels, even at LO

$$gg \to gg;\ qq \to qq;\ gg \to q\bar{q};\ q\bar{q} \to q\bar{q};\ gq \to gq;\ etc.$$

At NLO the bookkeeping issue is even more demanding. Here I will only note that, with an appropriate infrared safe jet definition to be discussed below, the NLO results exhibit

- the required cancellation of all singularities*

- remarkably good agreement with data

- the expected residual dependence on the factorization scale and the renormalization scale (typically set equal)

*The initial state collinear singularities are factored into the parton distributions.

First some kinematics – jets at hadron colliders are characterized by their

- **transverse momentum $\vec{P}_T$ or their scalar transverse energy**

  $E_T = E\sin\theta$ ($\left|\vec{P}_T\right| \approx E_T$ **for a particle or a "narrow" jet)**

- **pseudorapidity** $\eta_J = -\ln\left(\tan\left(\theta_J/2\right)\right)$ **or true rapidity**

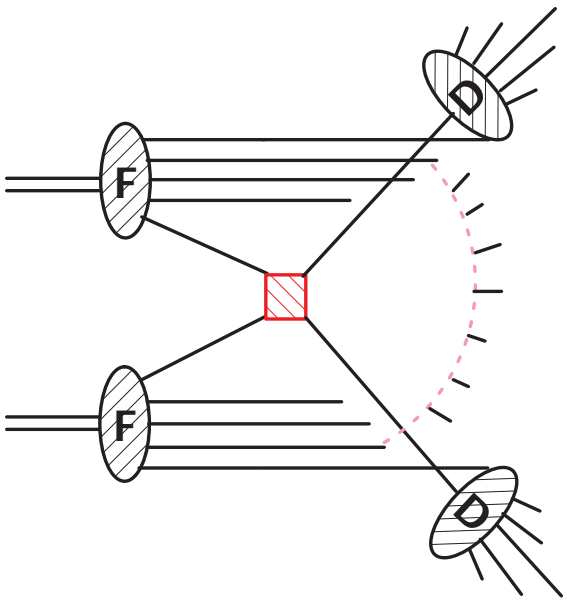
  $y_J = \ln\left(\left(E_J + P_{z,J}\right)/\left(E_J - P_{z,J}\right)\right)$, ***i.e.,*** $E_J = \sqrt{M_J^2 + P_{T,J}^2}\cosh y_J$,

  $P_{z,J} = \sqrt{M_J^2 + P_{T,J}^2}\sinh y_J$ ($\eta \cong y$, $E >> M$)*

Without detailed information on masses, *etc.*, η has been a variable of choice as it requires only an angle measurement.

In a typical hadron-hadron collision (minimum bias event) final state particles are fairly uniformly distributed in η (an original motivation for the "wee partons" with a *dx/x* ~ dη distribution).

Even in an event with a "hard" interaction the soft interactions of the spectator partons ⇒ Underlying Event ~ Min-Bias event, which contributes to the jet



*You should verify this limiting result.

So how to define a jet? Use a jet algorithm - Based on some measure of the localization of the expected collinear spray of particles

- **Start with a list of particles (4-vectors) and/or calorimeter towers (energies and angles)**
- **End with lists of particles/towers, one list for each jet**
- **And a list of particles/towers not in any jet – the spectators – remnants of the initial hadrons not involved in the short distance physics**

Goals of IDEAL ALGORITHM (motherhood)

Fully Specified: **including defining in detail any preclustering, merging, and splitting issues**

Theoretically Well Behaved: **the algorithm should be infrared and collinear safe (and insensitive) with no *ad hoc* clustering parameters (*e.g.*, R_{SEP})**

Detector Independent**: there should be no dependence on cell type, numbers, or size**

Order Independent: **The algorithms should behave equally at the parton, particle, and detector levels.**

Uniformity**: everyone uses the same algorithms (to the best possible approximation)**

Theory –

- **Boost invariant results – use variables with appropriate boost properties**
- **Kinematic boundary stability – use variables with appropriate energy conservation to allow resummation calculations**

Experiment –

- Minimize resolution smearing and angle biases
- Stability with luminosity – not sensitive to multiple collisions
- Efficient use of computer resources – but do not let this drive problems with physics issues (e.g., seeds and preclustering)

Easy to calibrate – not so worried about size of corrections as with accuracy of corrections

Easy to use!

Basic types of Jet Algorithms:

- Geometric or cone – select particles nearby in configuration space (*i.e.*, nearby in the detector) – hadron-hadron

- k_T – select particles nearby in momentum space (small relative transverse moment) – e^+e^-

The Cone Algorithm – formalized in the 1990 "Snowmass Accord" – cone of size *R*

CONE center - (η^C,φ^C)

CONE i $\subset$ C *iff* $\sqrt{\left(\eta^i-\eta^C\right)^2+\left(\varphi^i-\varphi^C\right)^2}\le R$

Energy $E_T^C=\sum_{i\subset C}E_T^i$

Centroid $\overline{\eta}^C=\sum_{i\subset C}E_T^i*\eta^i/E_T^C;\overline{\varphi}^C=\sum_{i\subset C}E_T^i*\varphi^i/E_T^C$

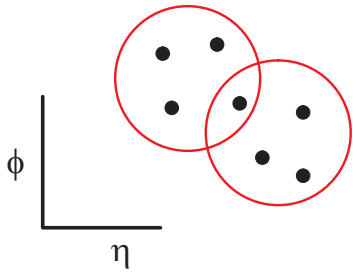
"Flow vector" $\vec{F}^C=\left(\overline{\eta}^C-\eta^C,\overline{\varphi}^C-\varphi^C\right)$

Jet is defined by "stable" cone:

$$\eta^J=\eta^C=\overline{\eta}^C;\varphi^J=\varphi^C=\overline{\varphi}^C;\vec{F}^C=0$$

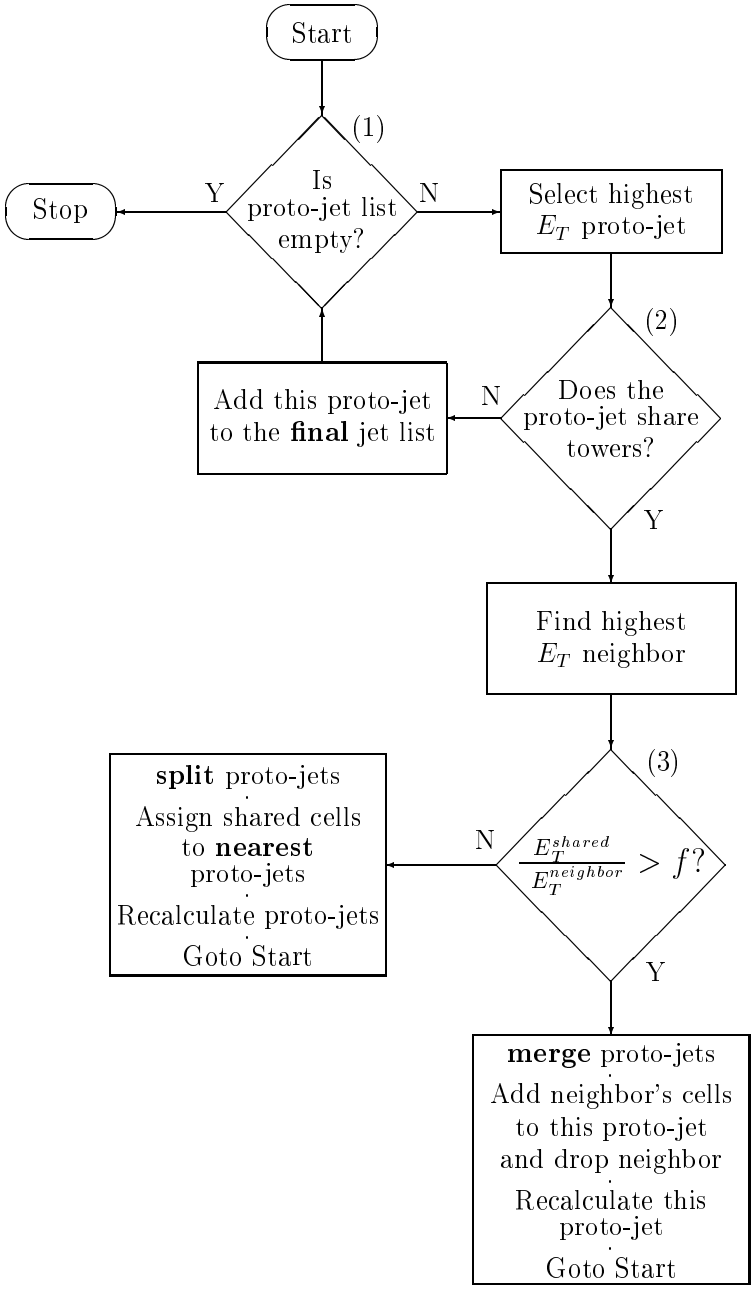
Stable cones found by iteration: start with cone anywhere (and, in principle, everywhere), calculate the centroid of this cone, put new cone at centroid, iterate until cone stops "flowing", i.e., stable

ISSUES for cone algorithm: -



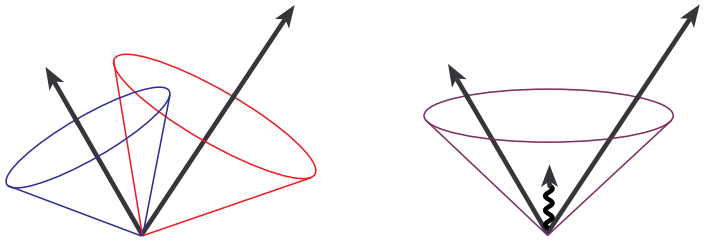
- **Stable cones can and do overlap! Need rules for merging and splitting (protojet = stable cone)**

Typical rule



- Theoretically can look "everywhere" and find all stable cones

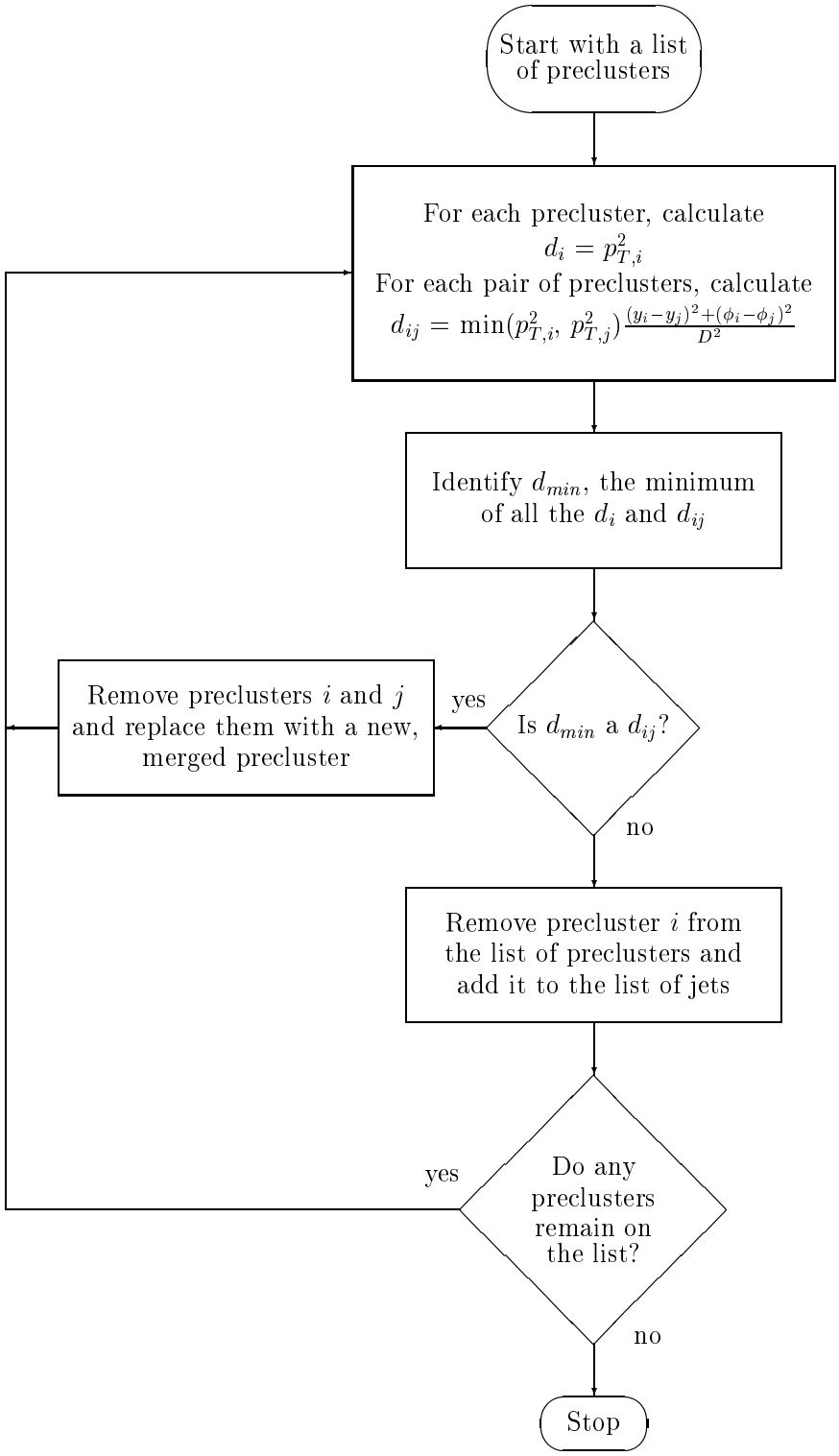
- Experimentally reduce size of analysis by putting initial cones only at seeds – energetic towers or clusters of towers – thus introducing undesirable IR sensitivity and missing certain possible 2-jets-in-1 configurations - fix theory with an *ad hoc* parameter R_{SEP} - do not allow 2 partons in the same jet if $\sqrt{(\eta_1-\eta_2)^2+(\varphi_1-\varphi_2)^2} \geq R_{SEP}$ even if satisfies the algorithm



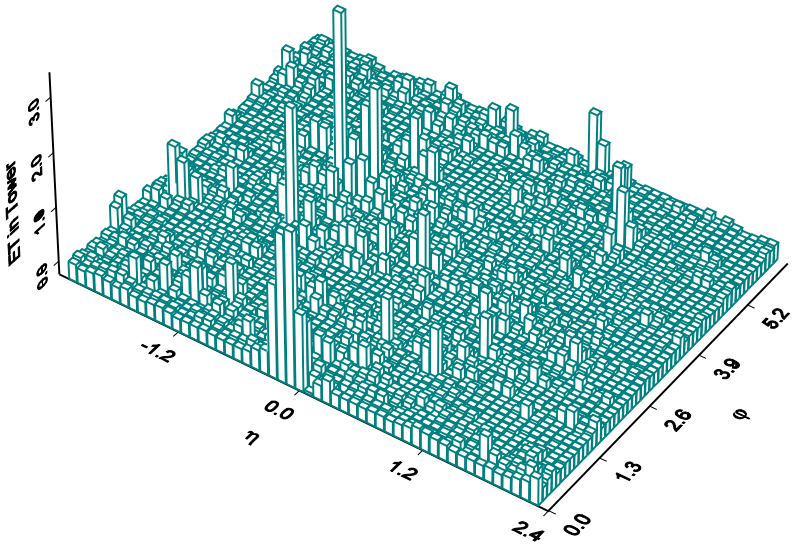
- "Easy" to correct for underlying event contribution

k_T algorithm – originally used in e+e- - all particles in a jet, no overlap issues, computer intensive (N^3 analysis, N=number of particles) – more difficult to correct for underlying event

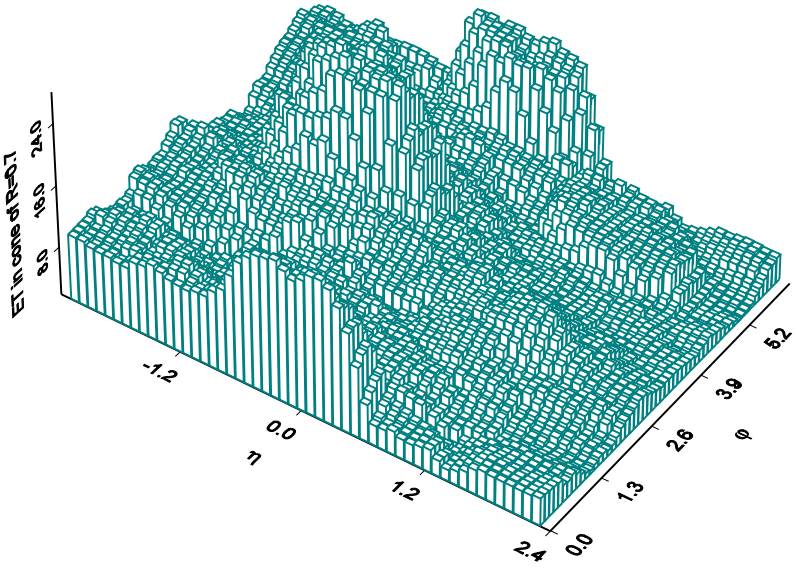
(d ~ relative transverse momentum^2)



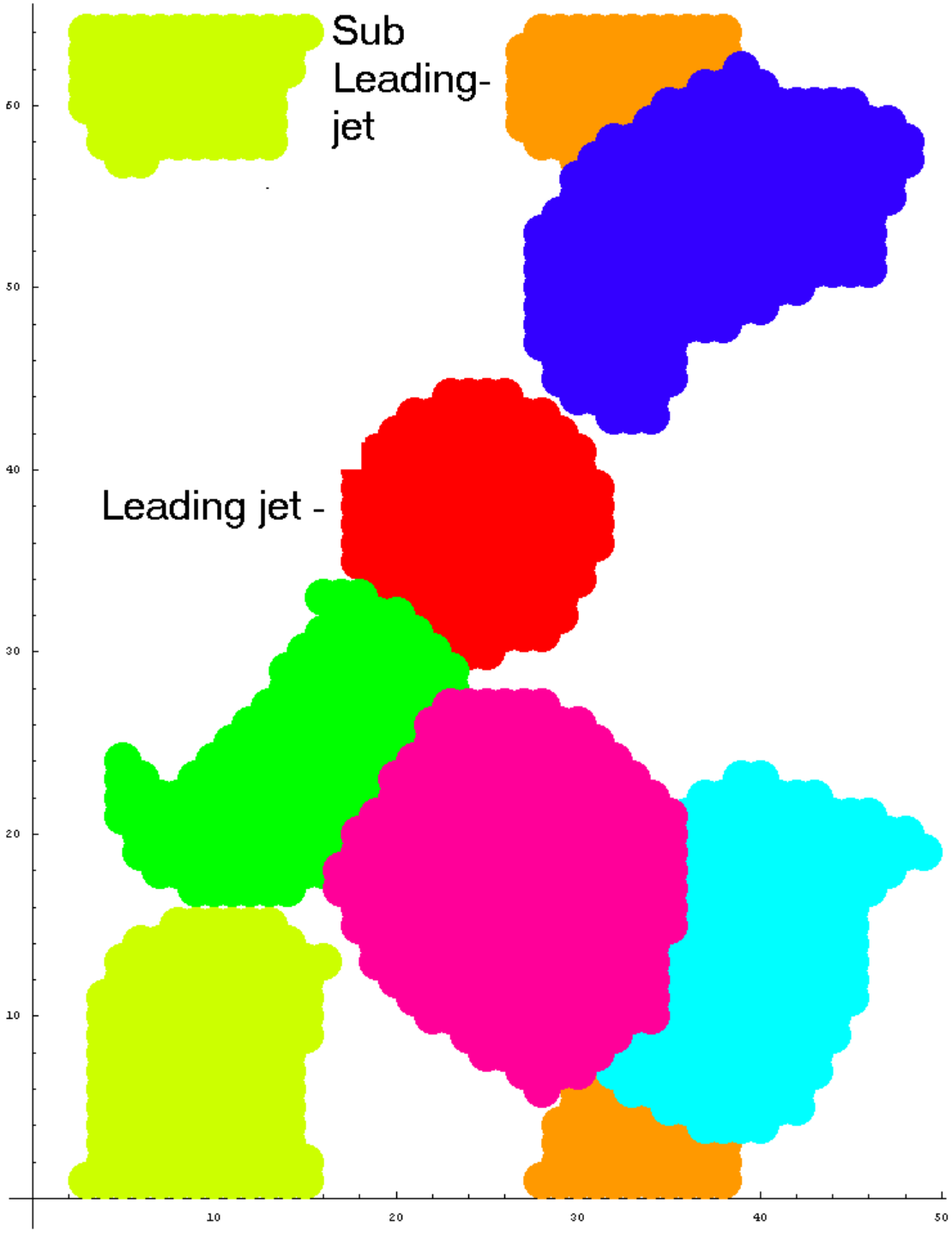
Look at an example first – D0 event with jets – E_T in central calorimeter towers



E_T in cone of R=0.7 centered on each tower



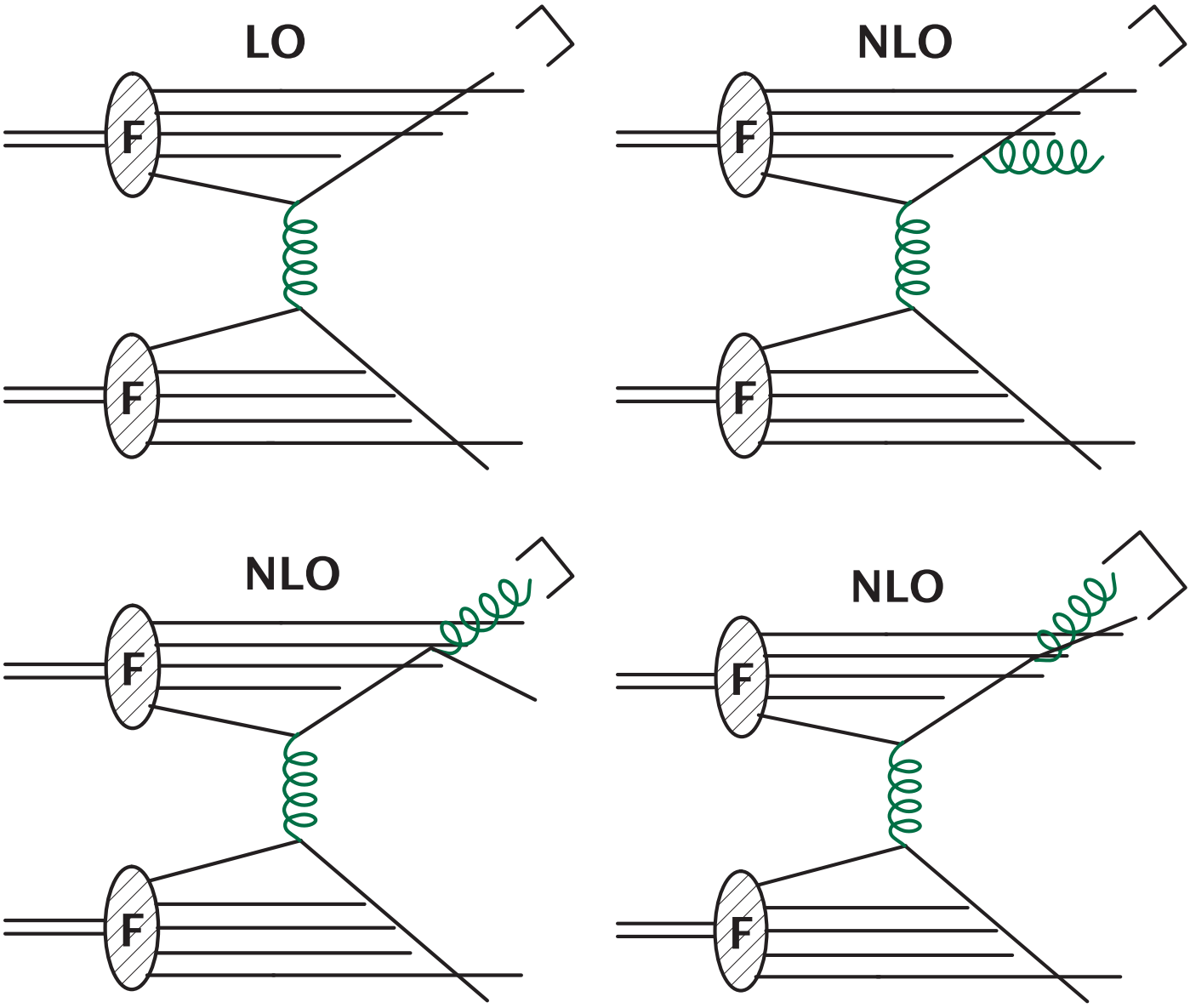
Jets found –



ϕ tower #

η tower #

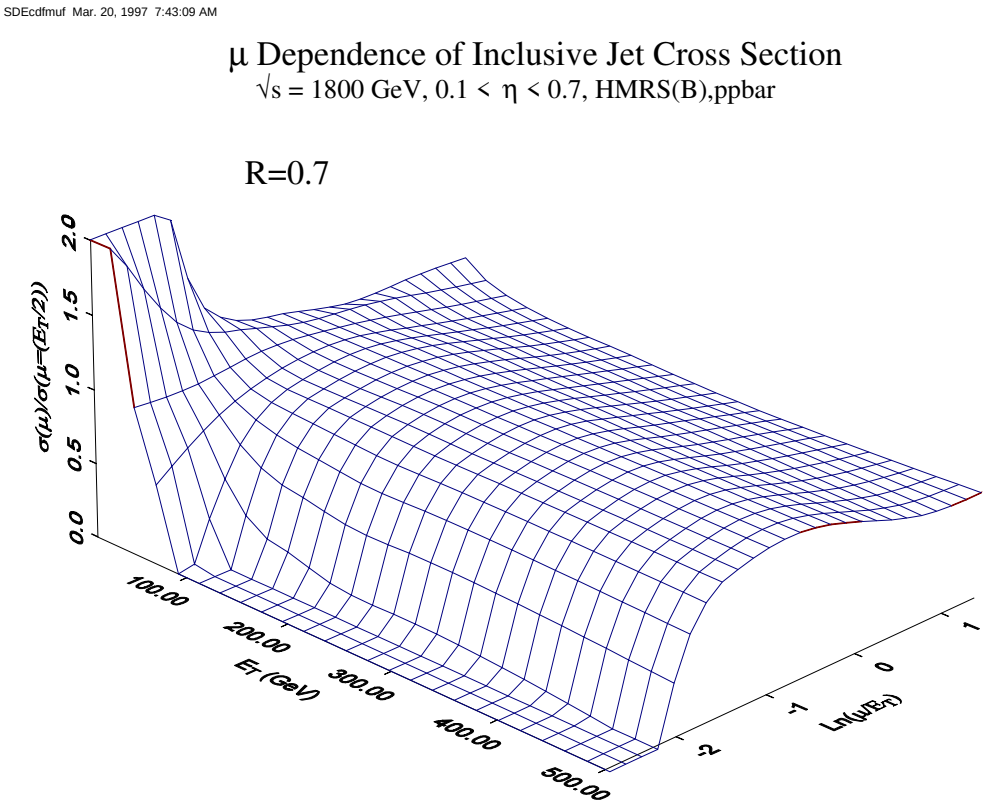
Back to NLO Jet Cross section – sample graphs –



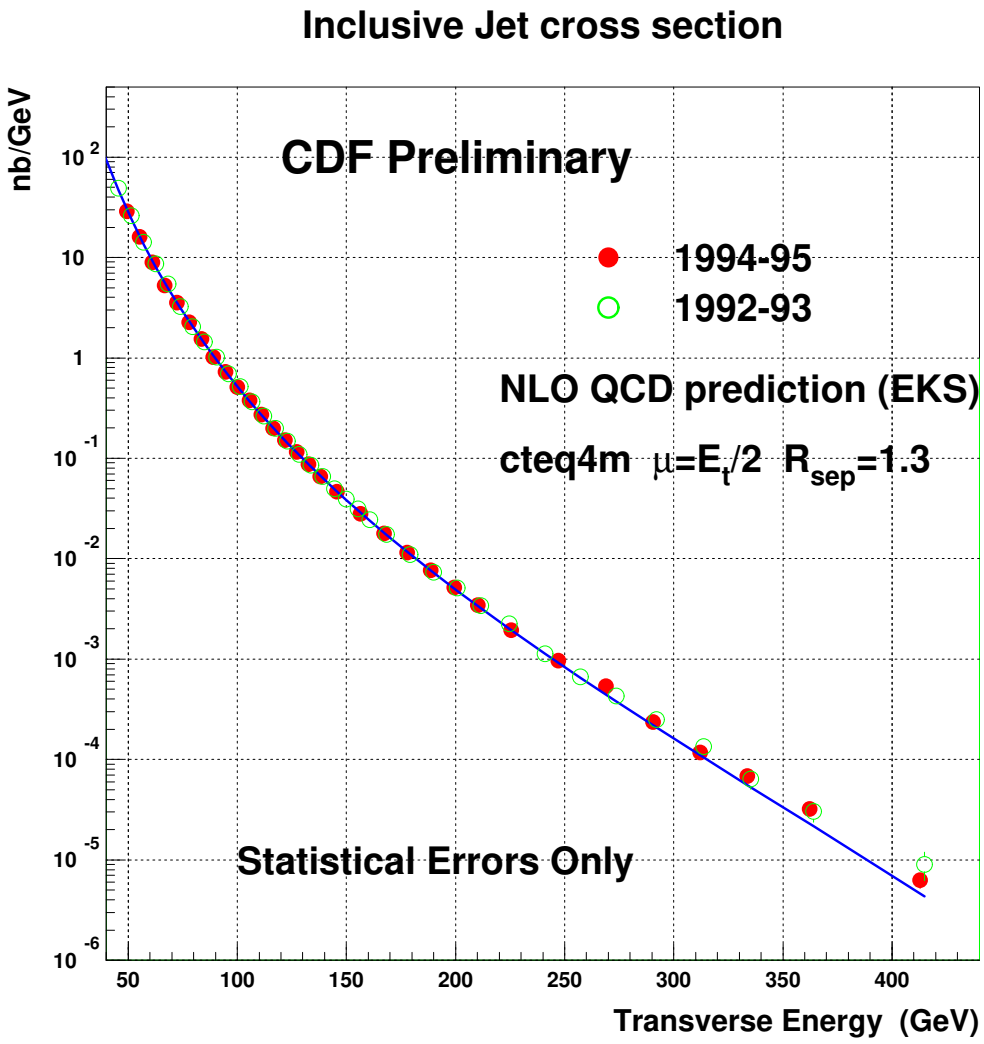
+ … + Virtual

⇒ Improved accuracy (smaller μ dependence) and gives structure to the jet.

NLO jet cross section μ dependence ($\mu_{renorm} = \mu_{fact}$)



Jet Cross section – CDF – compared to NLO, note the range



Final Outlook: QCD and pQCD work well

Much more to do – goal is strong interactions to < ± 1%!

- NNLO calculations for everything
- Resuming logarithms everywhere possible
- Better understanding of uncertainties – Global fits (see J. Stirling)
- Theory for the power corrections
- Nonperturbative physics from lattice work
- Determine what really happens at truly short distances
  (see S. Dawson)