Lepton Pair, W, and Z Production

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Lecture II

1. Comparison to lepton pair data
   - quantitative description
   - role in global fits
2. W and Z production
   - different couplings
   - comparison with lepton pair calculations
3. p_T distributions
   - Sudakov suppression
   - resummation
4. W production
   - W rapidity distribution
   - charged lepton distributions
   - roles in global fits and W mass measurements
5. Conclusions
6. References

Lepton pair production data

Database available at durpdg.dur.ac.uk/HEPDATA. Data available for a wide variety of beams, targets, and kinematic coverage.

- Data for $p, \overline{p}$, and $\pi^{\pm}$ beams on hydrogen and nuclear targets
- CM energies from 8.6 GeV to 630 GeV (data also available at 1.8 TeV from DØ and CDF, but not listed yet in this database. See the individual CDF and DØ web sites.)
- Data sets include both longitudinal and transverse momentum distributions.

Comparison to data

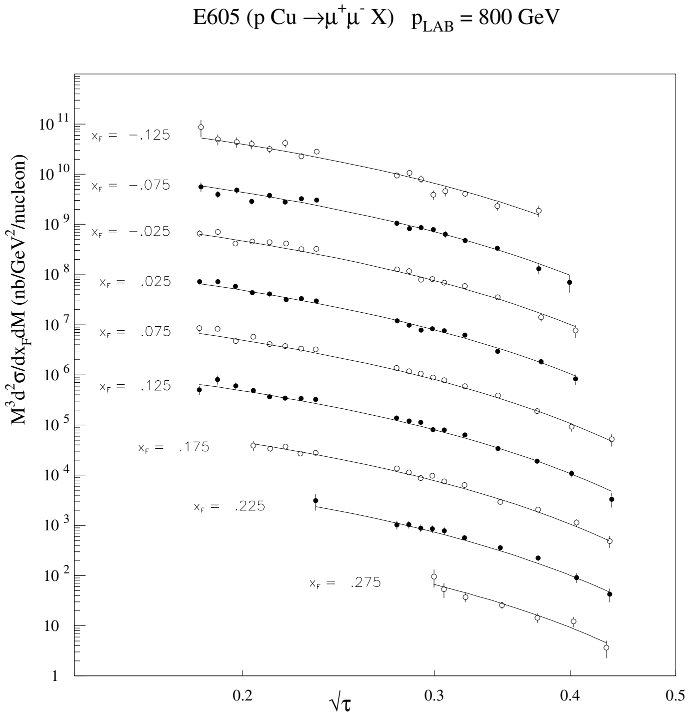
- Lepton pair production data used extensively in global fits for parton distribution functions

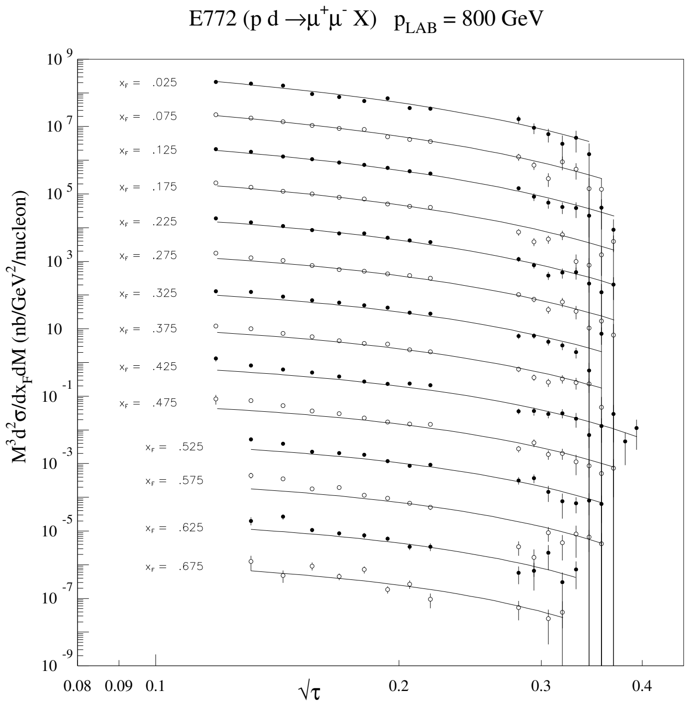
- Sensitive to antiquark distributions in pp, pN collisions

$$\sigma \sim \sum_q e_q^2 [q_a(x_a)\overline{q}_b(x_b) + a \leftrightarrow b]$$

- Corrections through $\mathcal{O}(\alpha_s^2)$ available (although most fits use only through $\mathcal{O}(\alpha_s)$)

- Excellent fits to $d\sigma/dQ^2dy$ and related distributions

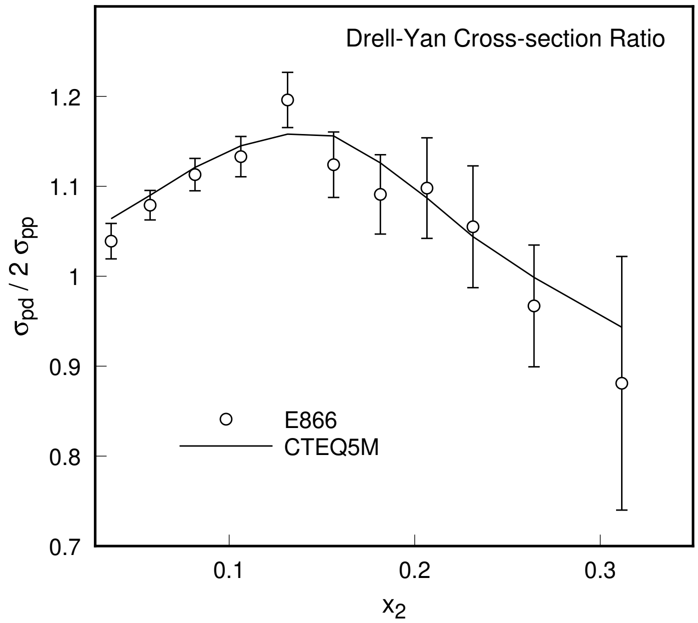




Typical fits to lepton pair production data (from MRST, Martin, Roberts, Stirling, and Thorne, hep-ph/9803445, Eur. Phys. J. C4 (1998) 463)

- πN collisions enable one to determine pion pdf's.

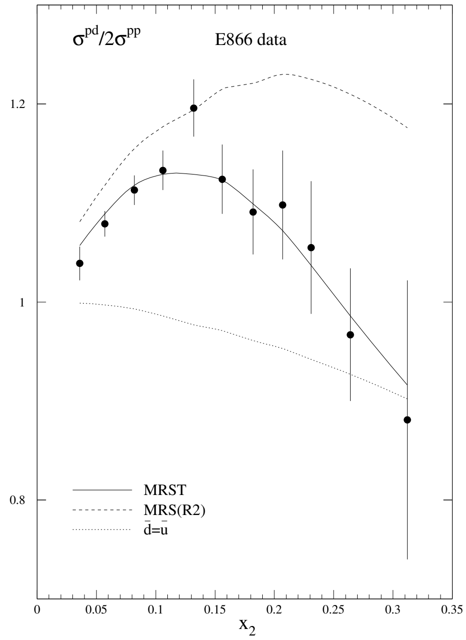
- Data with sufficiently broad kinematic coverage using pp, pd processes can help constrain $\overline{d}/\overline{u}$ (E-866).



Easy to understand impact of the E-866 data. Consider just u and d quarks and antiquarks and let R denote the cross section ratio $\sigma(pd)/2\sigma(pp)$. Then

$$\begin{aligned} R &= \frac{4u_1(\overline{u}_2+\overline{d}_2)+4\overline{u}_1(u_2+d_2)+d_1(\overline{u}_2+\overline{d}_2)+\overline{d}_1(u_2+d_2)}{2[4u_1\overline{u}_2+d_1\overline{d}_2+4\overline{u}_1u_2+\overline{d}_1d_2]} \\ &= \frac{1}{2}+\frac{4u_1\overline{d}_2+d_1\overline{u}_2+4\overline{u}_1d_2+\overline{d}_1u_2}{2[4u_1\overline{u}_2+d_1\overline{d}_2+4\overline{u}_1u_2+\overline{d}_1d_2]} \end{aligned}$$

Here the subscripts denote that the argument is x_1 or x_2. Also, the above equations have utilized isospin invariance for the pdf's (u in a proton is d for a neutron, etc.)



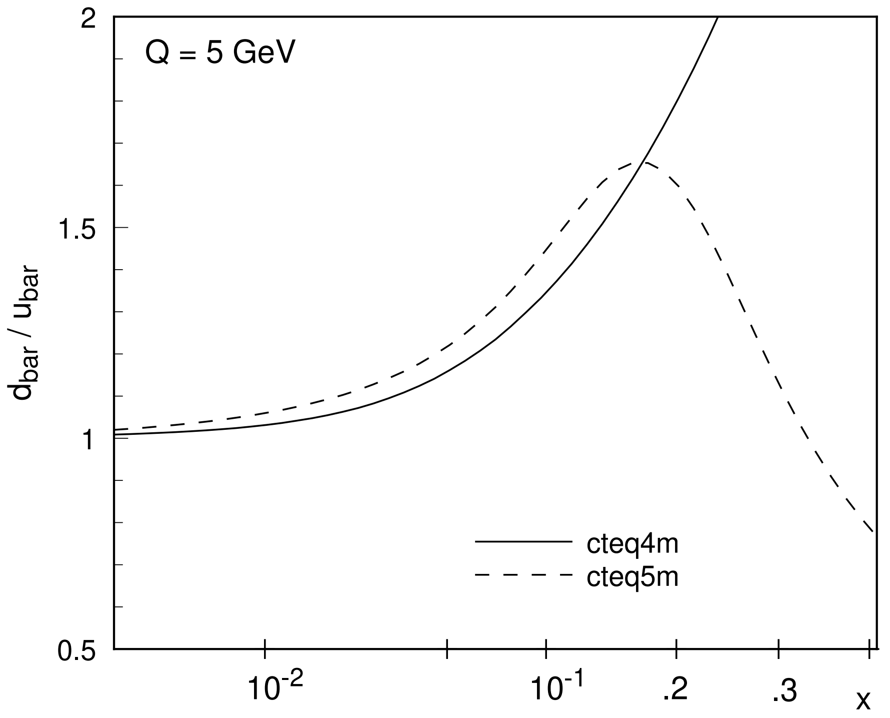
Let $r = \overline{d}/\overline{u}$. Then, we can write

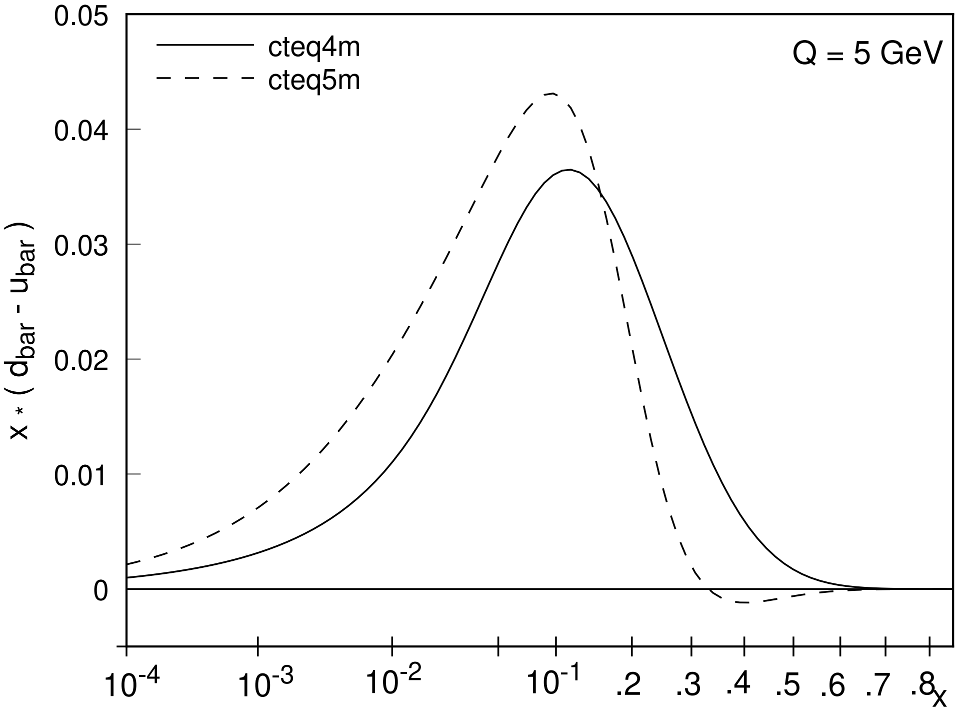
$$R = \frac{1}{2} + \frac{4u_1 r_2 + d_1 + 4d_2 + r_1 u_2}{2[4u_1 + d_1 r_2 + 4u_2 + r_1 d_2]}.$$

Now, suppose that $r = 1$, *i.e.*, $\overline{d} = \overline{u}$. Then

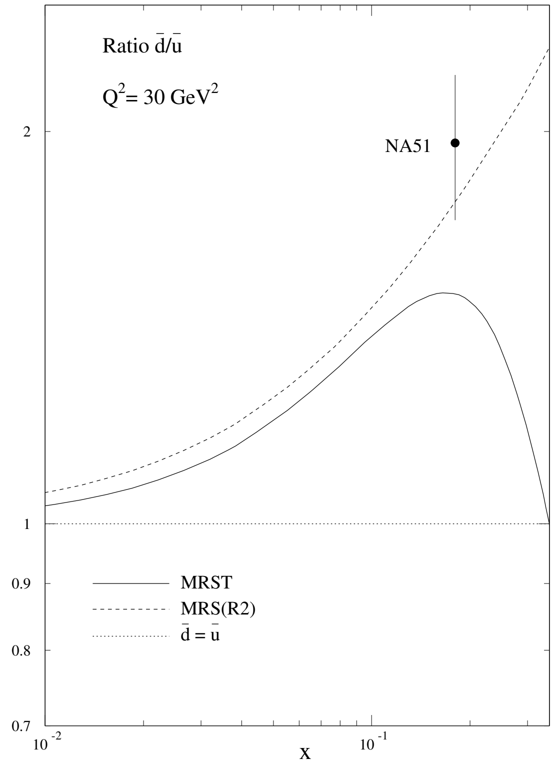
$$R = \frac{1}{2} + \frac{4u_1 + d_1 + 4d_2 + u_2}{2[4u_1 + d_1 + 4u_2 + d_2]}.$$

- At small x the sea quarks dominate the valence contributions so, for $r = 1$ we get $u \approx d$ and we see that $R \approx 1$ (see lower curve in figure above).
- For larger x we have $u > d$ so $R < 1$ (see lower curve above).
- If $r > 1$ then $R > 1$.
- Data indicate $R > 1$ and increasing at first, then decreasing as x gets larger. This implies a similar behavior for r.





CTEQ5M results from H. L. Lai *et al.*, hep-ph/ 9903282, Eur. Phys. J. C12 (2000) 375.



- CTEQ4M vs CTEQ5M and MRST vs MRS(R2) show impact of the E-866 data.

- $\overline{d}/\overline{u}$ curves shows behavior anticipated previously.

- Lepton pair production data providing strong constraints on the flavor dependence of the non-strange sea.

W and Z Production

- Thus far we have seen that fixed target pp and pN lepton pair production experiments provide important information concerning $\overline{u}$ and $\overline{d}$ distributions in the nucleon.

- W and Z production involve subprocesses which are very similar to lepton pair production, e.g., $q\overline{q}' \to W$ and $q\overline{q} \to Z$.

- To date, W/Z production by hadron beams has been studied at high energy $p\overline{p}$ colliders. Since the valence partons in an antiproton are antiquarks, the dominant subprocesses probe the u and d distributions. As we shall see, this places strong constraints on the d/u ratio.

- We shall be working in the narrow width approximation, i.e., the vector bosons will be treated as stable particles of fixed mass. All of the previous lepton pair production results can be easily utilized, providing that we make some changes in the couplings.

Consider $q(p_1)\overline{q}'(p_2) \to W(p)$, for which the matrix element is

$$M = -iV_{qq'}\frac{g}{\sqrt{2}}\epsilon_\alpha \overline{v}(p_2)\gamma^\alpha \frac{1}{2}(1-\gamma_5)u(p_1)$$

where $V_{qq'}$ is the appropriate element of the CKM matrix. The spin/color averaged squared matrix element is given by

$$\begin{aligned}
\overline{\sum}|M|^2 &= |V_{qq'}|^2\frac{g^2}{96}2\mathrm{Tr}\left[\not{p}_1(1-\gamma_5)\not{p}_2(1-\gamma_5)\right] \\
&= |V_{qq'}|^2\frac{g^2}{24}\mathrm{Tr}[\not{p}_1\not{p}_2] = |V_{qq'}|^2\frac{g^2}{6}p_1\cdot p_2 \\
&= |V_{qq'}|^2\frac{G_F M_W^4}{\sqrt{2}}\frac{2}{3}
\end{aligned}$$

where $g^2 = \frac{8G_F M_W^2}{\sqrt{2}}$. The hadronic cross section σ is given by convoluting the parton level cross section $\hat{\sigma}$ with the appropriate parton distributions:

$$\begin{aligned}
\sigma &= \int dx_a dx_b \sum_{qq'} q(x_a)\overline{q}'(x_b)\hat{\sigma} \\
\hat{\sigma} &= \frac{1}{2\hat{s}}\frac{2}{3}\frac{G_F M_W^4}{\sqrt{2}}|V_{qq'}|^2\int\frac{d^3p}{(2\pi)^3 2E}(2\pi)^4\delta^4(p-p_1-p_2).
\end{aligned}$$

The integrand of the phase space integral can be rewritten as

$$2\pi\ d^4p\ \delta^4(p-p_1-p_2)\delta(\hat{s}-M_W^2)$$

which yields

$$\hat{\sigma} = \frac{2\pi}{3}|V_{qq'}|^2\frac{G_F M_W^2}{\sqrt{2}}\delta(\hat{s}-M_W^2).$$

Compare this to our lepton pair production result

$$\hat{\sigma}_{\gamma^*} = \frac{4\pi^2\alpha}{3} e_q^2 \delta(\hat{s} - Q^2)$$

which shows that

$$4\pi\alpha e_q^2 \leftrightarrow 2|V_{qq'}|^2 \frac{G_F M_W^2}{\sqrt{2}}.$$

Z Production

Here the subprocess is $q(p_1)\overline{q}(p_2) \to Z(p)$, with the matrix element given by

$$M = -ig\epsilon_\alpha \overline{v}(p_2)\gamma^\alpha(g_V + g_A\gamma_5)u(p_1).$$

The partonic cross section is given by

$$\hat{\sigma}_Z = \frac{8\pi}{3}\frac{G_F M_W^2}{\sqrt{2}}(g_V^2 + g_A^2)\delta(\hat{s} - M_Z^2)$$

where

$$g_V^2 + g_A^2 = \frac{1}{8}(1 - 4|e_q|\sin^2\theta_W + 8e_q^2\sin^4\theta_W).$$

Apart from changing the coupling, we can treat W and Z production just like lepton pair production at a fixed value of Q^2.

Rapidity dependence in W production

According to our earlier calculations, the rapidity dependence of W production is given by the x dependence of the pdf's, since the W longitudinal momentum is, in lowest order, given by $\frac{\sqrt{s}}{2}(x_a - x_b)$. Of course, the leptonic decay $W \to l\nu$, by which the W is detected, poses a problem because of the unobserved ν. Nevertheless, the rapidity dependence of the charged lepton gives some useful information. To begin with, consider the rapidity dependence of the W and define an asymmetry $A(y)$ by:

$$A(y) = \frac{\frac{d\sigma}{dy}(W^+) - \frac{d\sigma}{dy}(W^-)}{\frac{d\sigma}{dy}(W^+) + \frac{d\sigma}{dy}(W^-)}.$$

In lowest order

$$\begin{aligned}
\frac{d\sigma}{dy}(W^+) &= \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}\sum_{q\overline{q}'}|V_{qq'}|^2[q(x_a)\overline{q}'(x_b) + a \leftrightarrow b] \\
&\approx \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}u(x_a)d(x_b) \text{ for } p\overline{p} \text{ collisions}
\end{aligned}$$

and similarly

$$\frac{d\sigma}{dy}(W^-) \approx \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}d(x_a)u(x_b) \text{ for } p\overline{p} \text{ collisions.}$$

To this level of approximation we can write the asymmetry A in terms only of the parton distributions.

$$\begin{aligned}
A &\approx \frac{u(x_a)d(x_b) - d(x_a)u(x_b)}{u(x_a)d(x_b) + d(x_a)u(x_b)} \\
&= \frac{R_{du}(x_b) - R_{du}(x_a)}{R_{du}(x_b) + R_{du}(x_a)}
\end{aligned}$$

where

$$R_{du}(x) = \frac{d(x)}{u(x)}.$$

Now,

$$x_{\substack{a\\b}} = \frac{M_W}{\sqrt{s}} e^{\pm y} \approx x_0(1 \pm y)$$

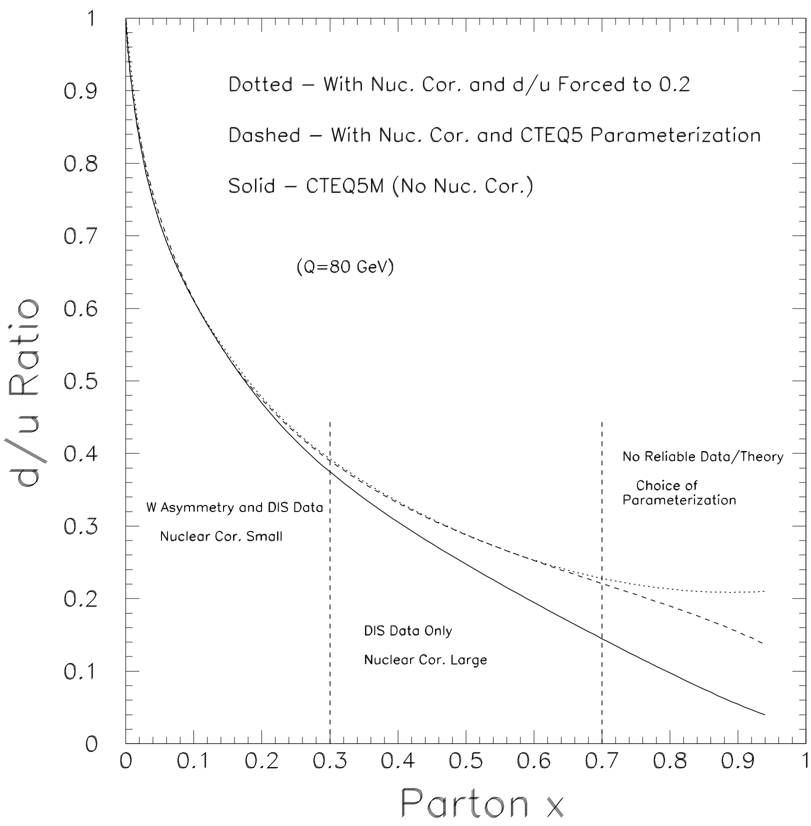
for small y. Here $x_0 = M_W/\sqrt{s}$. Then,

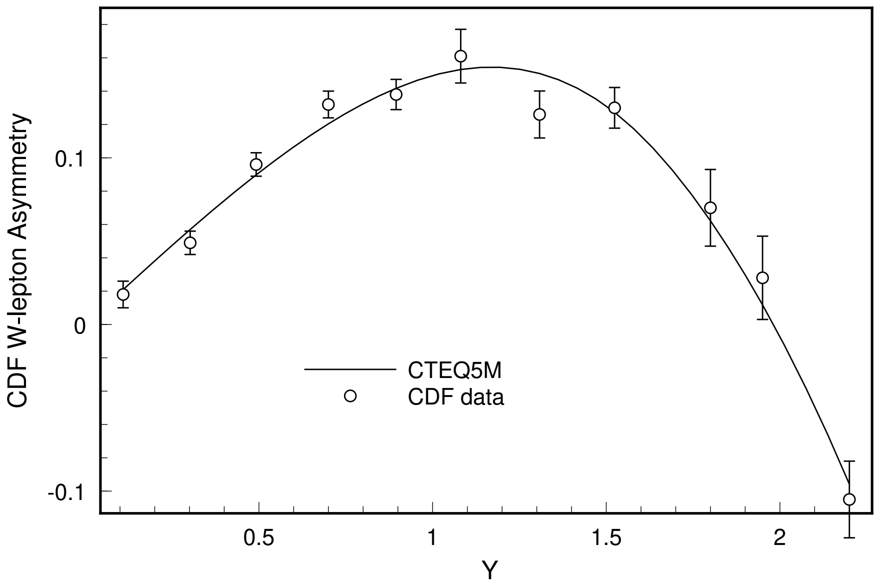
$$R_{du}(x_{\substack{a\\b}}) \approx R_{du}(x_0) \pm y x_0 R'_{du}(x_0).$$

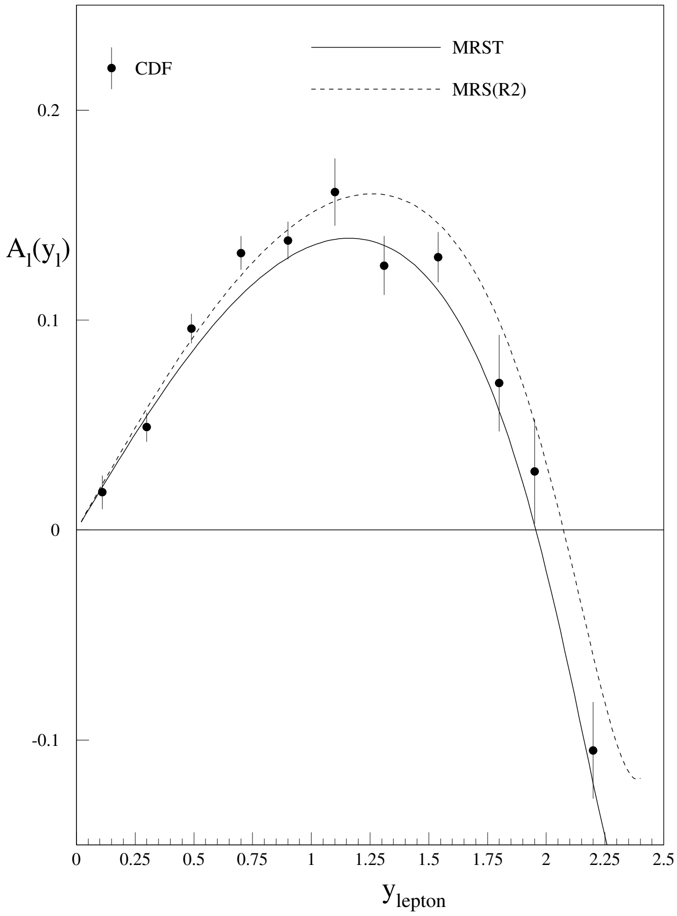
Therefore, in this approximation we obtain

$$A(y) \approx -x_0 y \frac{R'_{du}(x_0)}{R_{du}(x_0)}.$$

So, A gives us information about the *slope* of the d/u ratio. We'll see shortly that we can do the same for the charged lepton asymmetry. This yields valuable constraints in the low- to moderate-x range as shown below (from S. Kuhlmann *et al.*, hep-ph/9912283, Phys. Let. B76 (2000) 291:

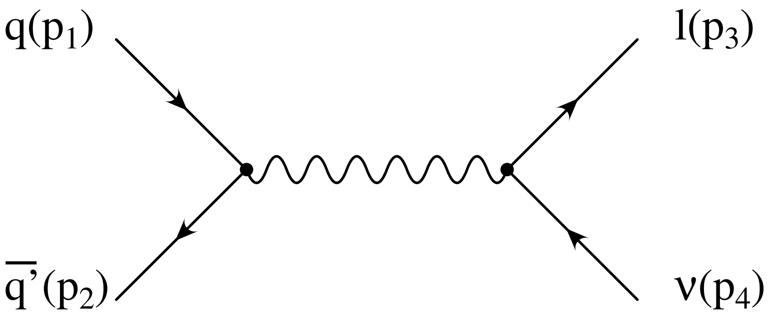



CDF W-lepton Asymmetry
0.1
0
-0.1
0.5
1
1.5
2
Y
CTEQ5M
CDF data


CDF
MRST
MRS(R2)
$A_l(y_l)$
0.2
0.1
0
-0.1
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
y_{lepton}

As mentioned above, the presence of the ν in the leptonic decay of the W complicates matters somewhat. Let's look at the distribution of the *charged lepton* in W production. We'll work in lowest order, but the same ideas apply in higher order. The subprocess is

$$q(p_1)\overline{q}'(p_2) \to l(p_3)\nu(p_4).$$



The matrix element is

$$\begin{aligned} M &= V_{qq'}\frac{g^2}{2}\overline{v}(p_2)\gamma_\mu\frac{1}{2}(1-\gamma_5)u(p_1) \\ &\times \frac{1}{\hat{s}-M_W^2+iM_W\Gamma_W}\overline{u}(p_3)\gamma^\mu\frac{1}{2}(1-\gamma_5)v(p_4). \end{aligned}$$

The squared matrix element is given by

$$\begin{aligned} \overline{\sum}|M|^2 &= \frac{1}{4}\frac{1}{3}|V_{qq'}|^2\frac{g^4}{4}\frac{1}{(\hat{s}-M_W^2)^2+M_W^2\Gamma_W^2} \\ &\times \frac{1}{4}\mathrm{Tr}[\not{p}_1\gamma_\nu(1-\gamma_5)\not{p}_2\gamma_\mu(1-\gamma_5)] \\ &\times \frac{1}{4}\mathrm{Tr}[\not{p}_4\gamma^\nu(1-\gamma_5)\not{p}_3\gamma^\mu(1-\gamma_5)] \end{aligned}$$

To simplify the traces, use the following relations:

$$\begin{aligned}
\mathrm{Tr}[\not{p}_1\gamma_\nu\not{p}_2\gamma_\mu] &= 4[p_{1\nu}p_{2\mu} + p_{1\mu}p_{2\nu} - g_{\mu\nu}p_1 \cdot p_2] \\
\mathrm{Tr}[\not{p}_1\gamma_\nu\not{p}_2\gamma_\mu\gamma_5] &= -4i\epsilon_{\alpha\nu\beta\mu}p_1^\alpha p_2^\beta \\
\epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\sigma\tau} &= -2(g_\sigma^\mu g_\tau^\nu - g_\tau^\mu g_\sigma^\nu)
\end{aligned}$$

and note that ϵ contracted with a symmetric function gives zero. The result is

$$\overline{\sum}|M|^2 = \frac{1}{3}|V_{qq'}|^2\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2 \frac{4\hat{s}^2(1+\cos\theta)^2}{(\hat{s}-M_W^2)^2 + M_W^2\Gamma_W^2}.$$

Next, note that phase space gives $d\cos\theta/16\pi$ and there is a flux factor of $1/2\hat{s}$. Thus,

$$\frac{d\hat{\sigma}}{d\cos\theta} = \frac{1}{24\pi}|V_{qq'}|^2\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2 \frac{4\hat{s}^2(1+\cos\theta)^2}{(\hat{s}-M_W^2)^2 + M_W^2\Gamma_W^2}.$$

Comment: If the width $\Gamma \ll M$ one can use the narrow width approximation wherein

$$\int_{-\infty}^{\infty} \frac{ds}{(s-M^2)^2 + M^2\Gamma^2} = \frac{\pi}{M\Gamma}$$

(let $\frac{s-M^2}{M\Gamma} = \tan\theta$. Then the integral is elementary.) Thus,

$$\frac{1}{(s-M^2)^2 + M^2\Gamma^2} \approx \frac{\pi}{M\Gamma}\delta(s-M^2) \text{ for } \Gamma \ll M.$$

We are interested in the hadron=hadron cm rapidity y. This is related to the parton-parton cm rapidity $\widehat{y}$ by

$$y = \widehat{y} + \frac{1}{2}\ln\frac{x_a}{x_b}.$$

Furthermore, $\widehat{y} = \ln\cot\theta/2 = \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta}$. Then,

$$\frac{d\widehat{y}}{d\cos\theta} = \frac{1}{\sin^2\theta} \text{ and } \frac{d\widehat{\sigma}}{d\widehat{y}} = \frac{d\widehat{\sigma}}{d\cos\theta}\sin^2\theta.$$

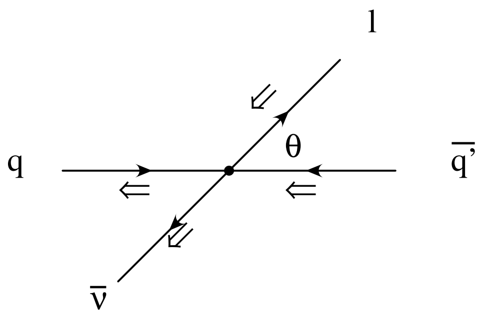
We therefore obtain

$$\frac{d\sigma}{dy} = \sum_q \int dx_a dx_b \frac{d\widehat{\sigma}}{d\cos\theta}\sin^2\theta[q(x_a)\overline{q}'(x_b) + a \leftrightarrow b].$$

Note: given x_a, x_b, y, and s one gets $\widehat{s}$ and $\widehat{y}$ and then uses $\sin\theta = \frac{1}{\cosh\widehat{y}}$. From here one can get the lepton p_T if a cut on p_T is desired.

This type of calculation can be extended to higher orders. Thus, one can calculate the rapidity dependence of the *charged lepton* and, therefore, the lepton rapidity asymmetry. This, in turn can be used to constrain pdf's in global fits.

Comment: In the calculation if the $\cos\theta$ dependence we encountered a factor of $(1+\cos\theta)^2$. The source of this is easy to understand. The W couples to left-handed particles and right-handed antiparticles.



When $\theta \to \pi$ the cross section must vanish since angular momentum would not be conserved. However, $\theta = 0$ is allowed. The $(1 + \cos\theta)$ factor ensures this.

p_T distributions revisited

The p_T distribution for W production is of vital importance for the precision measurement of the W mass. In lecture I we saw how to calculate the high-p_T *tail* of the distribution. Now, we need to reexamine this issue with an eye towards calculating the *full* distribution. First, consider the $q\bar{q} \to l^+ l^- g$ subprocess discussed in Lecture I.

$$\begin{aligned}\frac{d\sigma}{dQ^2 dy dp_T^2} &= \frac{\alpha^2 \alpha_s}{sQ^2} \frac{8}{27} \frac{1}{p_T^2} \int_{x_{amin}}^1 \frac{dx_a}{x_a - x_1} \sum_q H_q(x_a, x_b, \mu^2) e_q^2 \\ &\times \left[1 + \frac{\tau^2}{(x_a x_b)^2} - \frac{x_T^2}{2 x_a x_b}\right]\end{aligned}$$

where

$$\begin{aligned} x_b &= \frac{x_a x_2 - \tau}{x_a - x_1} \\ x_{amin} &= \frac{x_1 - \tau}{1 - x_2} \\ x_1 &= -(u - Q^2)/s = \sqrt{Q^2 + p_T^2}\ e^y/\sqrt{s} \\ x_2 &= -(t - Q^2)/s = \sqrt{Q^2 + p_T^2}\ e^{-y}/\sqrt{s} \end{aligned}$$

Now, consider the limit as $p_T \to 0$:

$$x_b \to \sqrt{\tau} e^{-y} = x_b^0 \text{ and } x_{amin} \to \sqrt{\tau} e^y = x_a^0$$

so

$$(x_{amin} x_b)^2 \sim \tau^2.$$

As $p_T \to 0$ the [ ] term above goes to 2. Near $p_T = 0$ we can integrate the $\frac{1}{x_a - x_1}$ term, approximating the rest of the integrand as $\sim$ constant.

Keeping the most singular terms, we get

$$
\begin{aligned}
\frac{d\sigma}{d\tau dy dp_T^2} &\approx \frac{\alpha^2\alpha_s}{Q^2}\frac{8}{27}\frac{\ln s/p_T^2}{p_T^2} 2\sum_q H_q(x_a^0, x_b^0) \\
&\approx \frac{4\alpha_s}{3\pi}\left(\frac{d\sigma}{d\tau dy}\right)_{Born}\frac{\ln s/p_T^2}{p_T^2}
\end{aligned}
$$

Next, following the arguments of Parisi and Petronzio, Nucl. Phys. B154, 427 (1979), we know that the integral over all p_T^2 is finite, so

$$
\int_0^s \frac{d\sigma}{d\tau dy dp_T^2}\, dp_T^2 = \left(\frac{d\sigma}{d\tau dy}\right)_{Born} + \mathcal{O}(\alpha_s)
$$

and, using the above results,

$$
\begin{aligned}
\int_0^{p_T^2} \frac{d\sigma}{d\tau dy dp_T^2}\, dp_T^2 &= \left(\frac{d\sigma}{d\tau dy}\right)_{Born}\left(1 - \int_{p_T^2}^s \frac{4\alpha_s}{3\pi}\frac{\ln s/p_T^2}{p_T^2}\, dp_T^2\right) \\
&= \left(\frac{d\sigma}{d\tau dy}\right)_{Born}\left[1 - \frac{2\alpha_s}{3\pi}\ln^2 s/p_T^2\right]
\end{aligned}
$$

Extended to higher orders, it can be shown that the square bracketed term exponentiates. Hence,

$$
\int_0^{p_T^2} \frac{d\sigma}{d\tau dy dp_T^2}\, dp_T^2 = \left(\frac{d\sigma}{d\tau dy}\right)_{Born} \exp(-\frac{2\alpha_s}{3\pi}\ln^2 s/p_T^2).
$$

For more details, see Dokshitzer, D'yakanov, and Troyan, Phys. Rep. 58, 271 (1980) and Curci, Greco, and Srivastava, Phys. rev. Lett 43, 834 (1979) and Nucl. Phys. B159, 451 (1979).

Differentiating the above results yields

$$\frac{d\sigma}{d\tau dy dp_T^2} = \left(\frac{d\sigma}{d\tau dy}\right)_{Born} \frac{4\alpha_s}{3\pi} \frac{\ln s/p_T^2}{p_T^2} \exp\left(-\frac{2\alpha_s}{3\pi} \ln^2 s/p_T^2\right).$$

The exponential is referred to as a Sudakov form factor. It represents the summation of the leading double-log terms. Notice that the exponential kills the divergence at $p_T = 0$. Physically, this represents the fact that the probability to produce a massive lepton pair with no additional radiation is zero.

- In this approximation the gluon emissions are treated as *uncorrelated*. If the lepton pair is to have zero p_T, then *all* the gluons must have zero p_T.

- This suppression ia actually too strong. One can have two or more gluons whose $\vec{p}_T$ adds to zero. Thus, configurations with balancing gluons should be included. However, these are subleading terms, even though they may be dominant at sufficiently small values of p_T (see Parisi and Petronzio, Nucl. Phys. B154, 427 (1979)).

- Resummation techniques exist which include these subleading terms and which give non-zero cross sections at $p_T = 0$ (see Collins, Soper, and Sterman, Nucl. Phys. B250, 199 (1985)).

Exponentiating in impact parameter space is a way of ensuring conservation of transverse momentum which includes the previously mentioned subleading terms.

$$
\begin{aligned}
\frac{d\sigma}{d\tau dy dp_T^2} &\sim \sum_q \frac{\sigma_0^{q\bar{q}}}{2} \int_0^\infty b db J_0(bp_T) exp(-S(b,Q)) \\
&\times \left[q(x_a, b_0/b)\bar{q}(x_b, b_0/b) + a \leftrightarrow b\right] \\
S(b,Q) &= \int_{(b_0/b)^2}^{Q^2} \frac{dq^2}{q^2} \left[A \ln Q^2/q^2 + B\right] \text{ with } x_{\substack{a\\b}} = \sqrt{\tau} e^{\pm y}
\end{aligned}
$$

A and B have perturbative expansions. Additional non-leading contributions can be systematically included.

Note: the above expressions must be supplemented with a prescription for treating the large b region where non-perturbative effects will come into play.

Technical aside

To avoid the large b region one technique is to replace

$$b \to b^* = \frac{b}{\sqrt{1 + (b/b_{max})^2}}$$

in $S(b, Q)$ and in the pdf's.

In addition, one adds a term S_{np} to S:

$$S(b, Q) \to S(b^*, Q) + S_{np}(b, Q)$$

where S_{np} is often parametrized as

$$S_{np} = b^2 \left[g_1 + g_2 \ln \frac{Qb_{max}}{2} \right].$$

Note: S_{np} parametrizes the large b non-perturbative region. Its form is suggested by requiring it to smoothly tie on to $S(b, Q)$.

Note: The gaussian term in b (g_1) corresponds to a gaussian in p_T since

$$\int_0^\infty b db J_0(bp_T) e^{-\alpha b^2} = \frac{1}{2\alpha} e^{-p_T^2/4\alpha}.$$

Thus, this form for the resummation contains the gaussian smearing ansatz originally used to describe the lepton pair p_T distribution at low values of p_T.

An alternative formalism exists which avoids the use of impact parameter space - see Ellis and Veseli, Nucl. Phys. B511, 649 (1998). The approach follows the original work of Dokshitzer, D'yakanov and Troyan (DDT formula).

$$\frac{d\sigma}{d\tau dy dp_T^2} = \sum_q \sigma_0^{q\bar{q}} \frac{d}{dp_T^2} \{[q(x_a, p_T)\bar{q}(x_b, p_T) + a \leftrightarrow b] \times \exp\left(-\int_{p_T^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln Q^2/\mu^2 + B\right]\right)\}.$$

Here, too, A and B have perturbative expansions.

- The physics behind this expression is the same as in the $b-$space form. Adjustments are made so that the leading, next-to-leading, and next-to-next-to-leading logs are the same.

- In this case some adjustment must be made in the small-p_T region, since the pdf's can not be evolved to arbitrarily small p_T.

Comparison to data

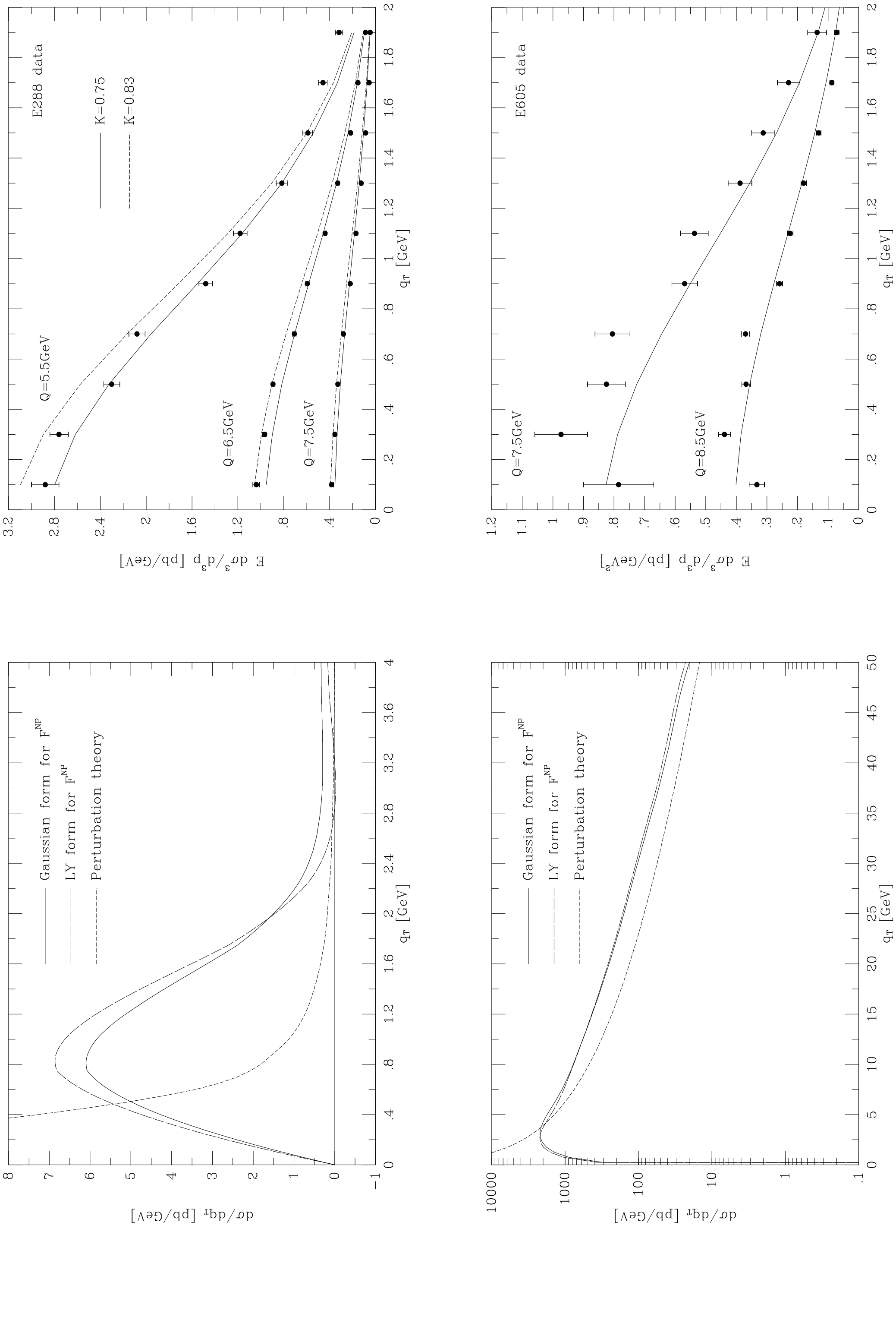
The resummation formalisms discussed for lepton pair production can equally well be applied to W and Z production.

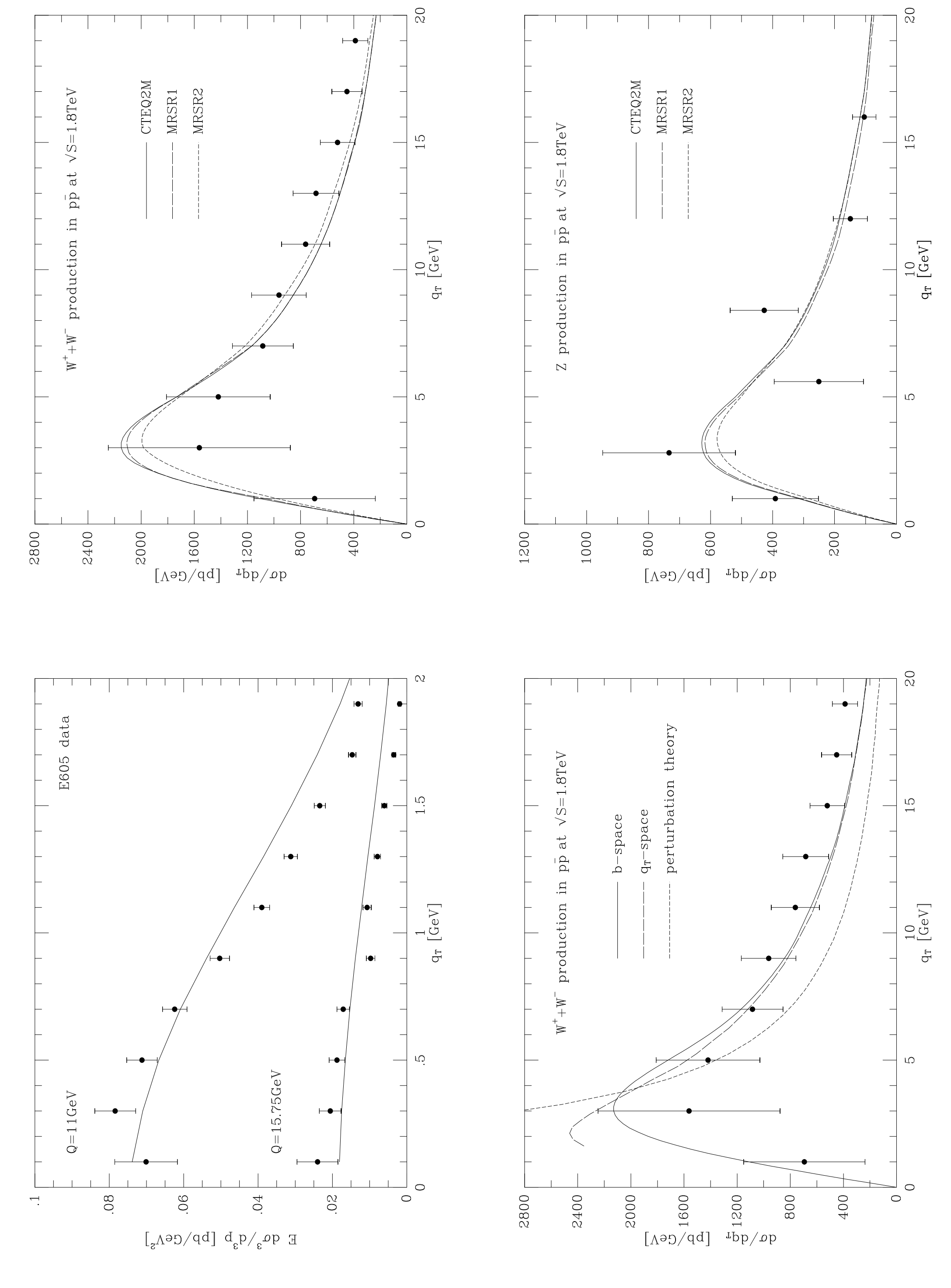
Some references

1. Altarelli, Ellis, Greco, and Martinelli, Nucl. Phys. B246, 12 (1984) (first detailed comparison of resummed perturbation theory with vector boson production data.)

2. Ellis and Veseli, hep-ph/9706526, Nucl. Phys. B511, 649 (1998)

3. Ellis, Ross, and Veseli, hep-ph/9704239, Nucl. Phys. B503, 309 (1997)


E288 data
K=0.75
K=0.83
Q=5.5GeV
Q=6.5GeV
Q=7.5GeV
E $d\sigma^3/d^3p$ [pb/GeV]
q_T [GeV]
E605 data
Q=7.5GeV
Q=8.5GeV
E $d\sigma^3/d^3p$ [pb/GeV^2]
Gaussian form for F^{NP}
LY form for F^{NP}
Perturbation theory
$d\sigma/dq_T$ [pb/GeV]



E605 data
Q=11GeV
Q=15.75GeV
E dσ³/d³p [pb/GeV²]
q_T [GeV]
W⁺+W⁻ production in pp̄ at √S=1.8TeV
CTEQ2M
MRSR1
MRSR2
dσ/dq_T [pb/GeV]
q_T [GeV]
W⁺+W⁻ production in pp̄ at √S=1.8TeV
b-space
q_T-space
perturbation theory
dσ/dq_T [pb/GeV]
q_T [GeV]
Z production in pp̄ at √S=1.8TeV
CTEQ2M
MRSR1
MRSR2
dσ/dq_T [pb/GeV]
q_T [GeV]

Okay, we now have a method of calculating the vector boson p_T distribution. It requires a number of parameters for the small-p_T region (this is true whether one uses b-space of k_T-space resummation.) One can try fitting these to the fixed target lepton pair production data (see the Ellis, Ross, and Veseli reference, as well as Landry, Brock, Ladinsky, and Yuan, hep-ph/9905391)

Why is such a description important?

- Intrinsic interest in the nonperturbative parameters
  - differences between quark and gluon parameters
  - impact on other production processes, *e.g.*, Higgs production
- impact on precision W mass measurements

Why is the W p_T distribution important for the measurement of the W mass?

The W is detected via its leptonic decay, as discussed earlier. The ν is not detected; rather, one detects the charged lepton. The transverse momentum distribution for the charged lepton shows a Jacobian peak which depends on the W mass.

Lepton p_T distribution

At the parton level, the total cross section at lowest order is

$$\hat{\sigma} = \int d\cos\theta \frac{d\sigma}{d\cos\theta} = \frac{|V_{qq'}|^2}{9\pi}\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2 \frac{\hat{s}}{(\hat{s}-M_W^2)^2 + M_W^2\Gamma^2}.$$

$\cos\theta$ and p_T are related by

$$p_T^2 = \frac{\hat{s}}{4}\sin^2\theta \text{ so that } \cos\theta = \sqrt{1-\frac{4p_T^2}{\hat{s}}}.$$

This means that

$$\frac{dp_T^2}{d\cos\theta} = \frac{\hat{s}}{2}\cos\theta$$

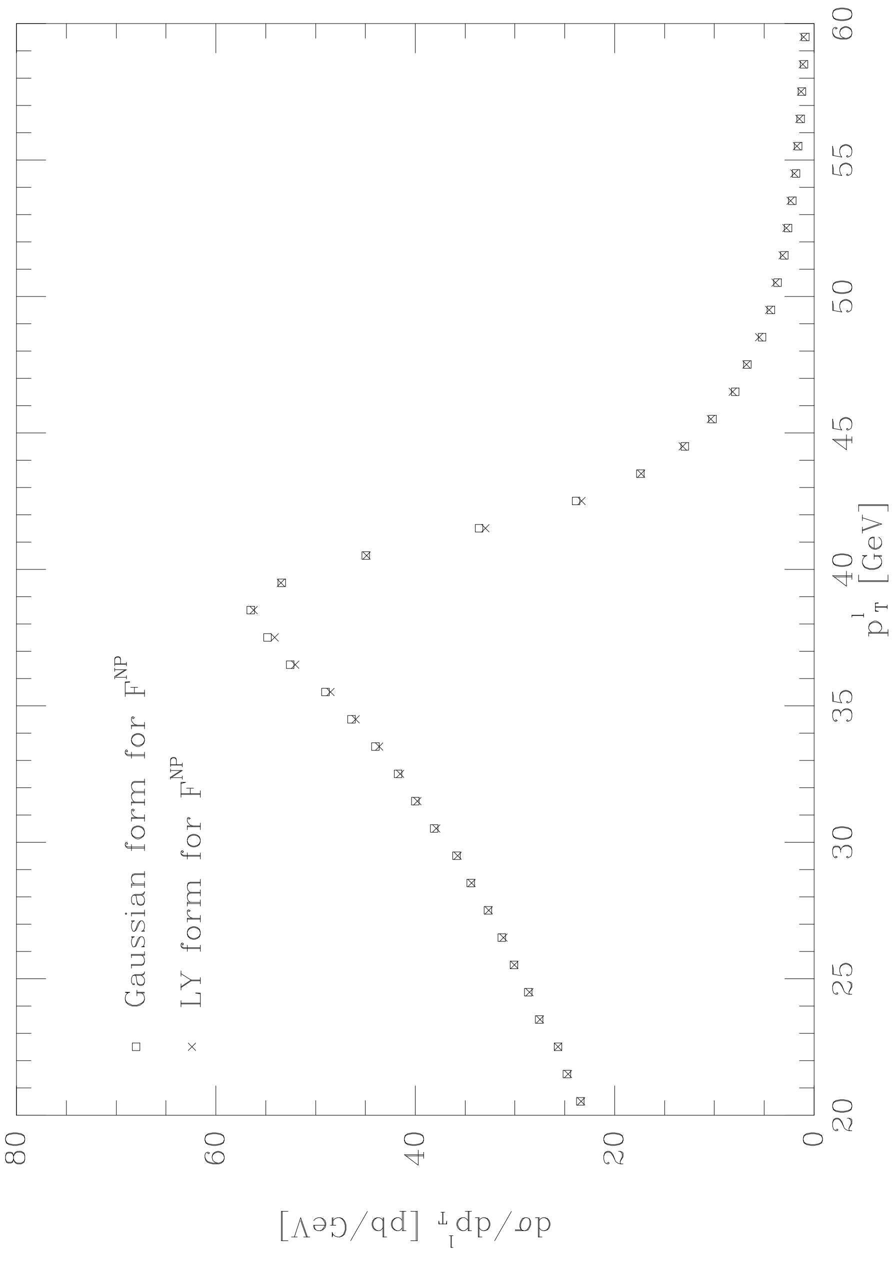
and, therefore, that

$$\frac{d\hat{\sigma}}{dp_T^2} = \frac{2}{\hat{s}\cos\theta}\frac{d\hat{\sigma}}{d\cos\theta} = \frac{3}{4}\hat{\sigma}\frac{(1+\cos\theta)^2}{\hat{s}\ \cos\theta}.$$

This can be rewritten as

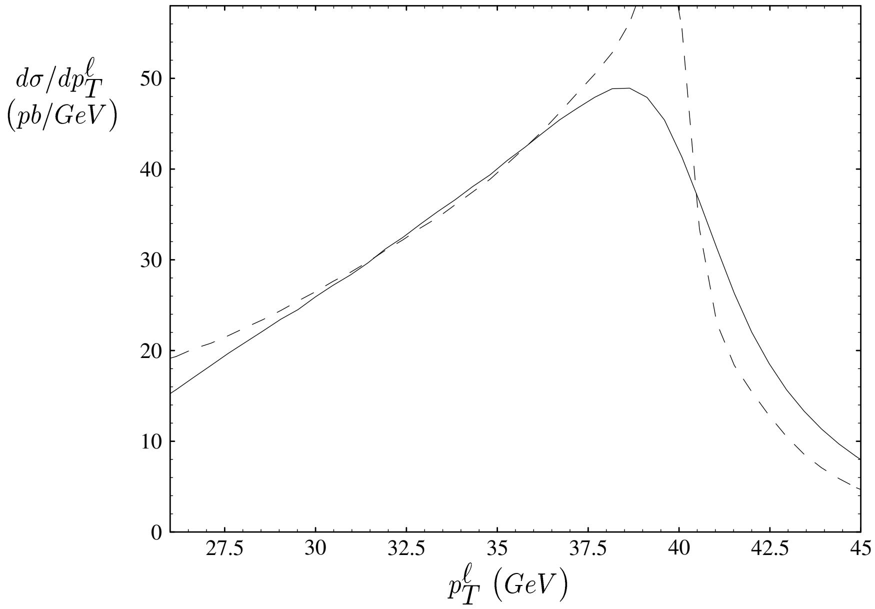
$$\frac{d\hat{\sigma}}{dp_T^2} = \frac{3\hat{\sigma}}{4}\frac{\left[1+\sqrt{1-4p_T^2/\hat{s}}\right]^2}{\sqrt{1-4p_T^2/\hat{s}}}.$$

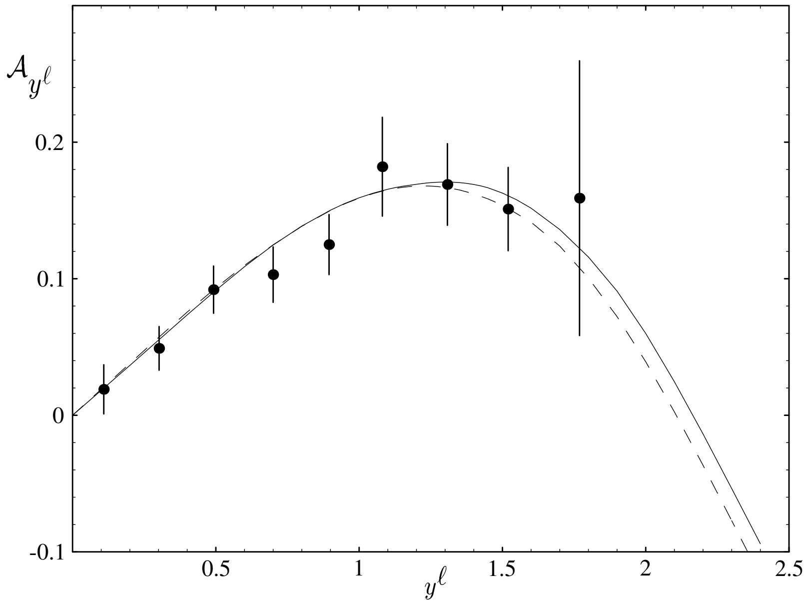
There is a Jacobian peak at $p_T = \sqrt{\hat{s}}/2 \approx M_W$. The peak is smeared out by the Breit-Wigner and by the W p_T distribution.



Gaussian form for F^NP
LY form for F^NP
dσ/dp^l_T [pb/GeV]
p^l_T [GeV]

Resummation effects on the decay lepton distributions have also been studied by Balazs, Qiu, and Yuan, hep-ph/9505203, Phys. Lett. B355, 548 (1995). Comparison with NLO calcualtions shows significant effects on the transverse momentum and rapidity asymmetry distributions, as shown below.





Transverse Mass

The invariant mass of the (l, ν) pair is given by

$$
\begin{aligned}
M^2(l,\nu) &= (E_l + E_\nu)^2 - (\vec{p}_l + \vec{p}_\nu)^2 \\
&= (|\vec{p}_l| + |\vec{p}_\nu|)^2 - (\vec{p}_l + \vec{p}_\nu)^2
\end{aligned}
$$

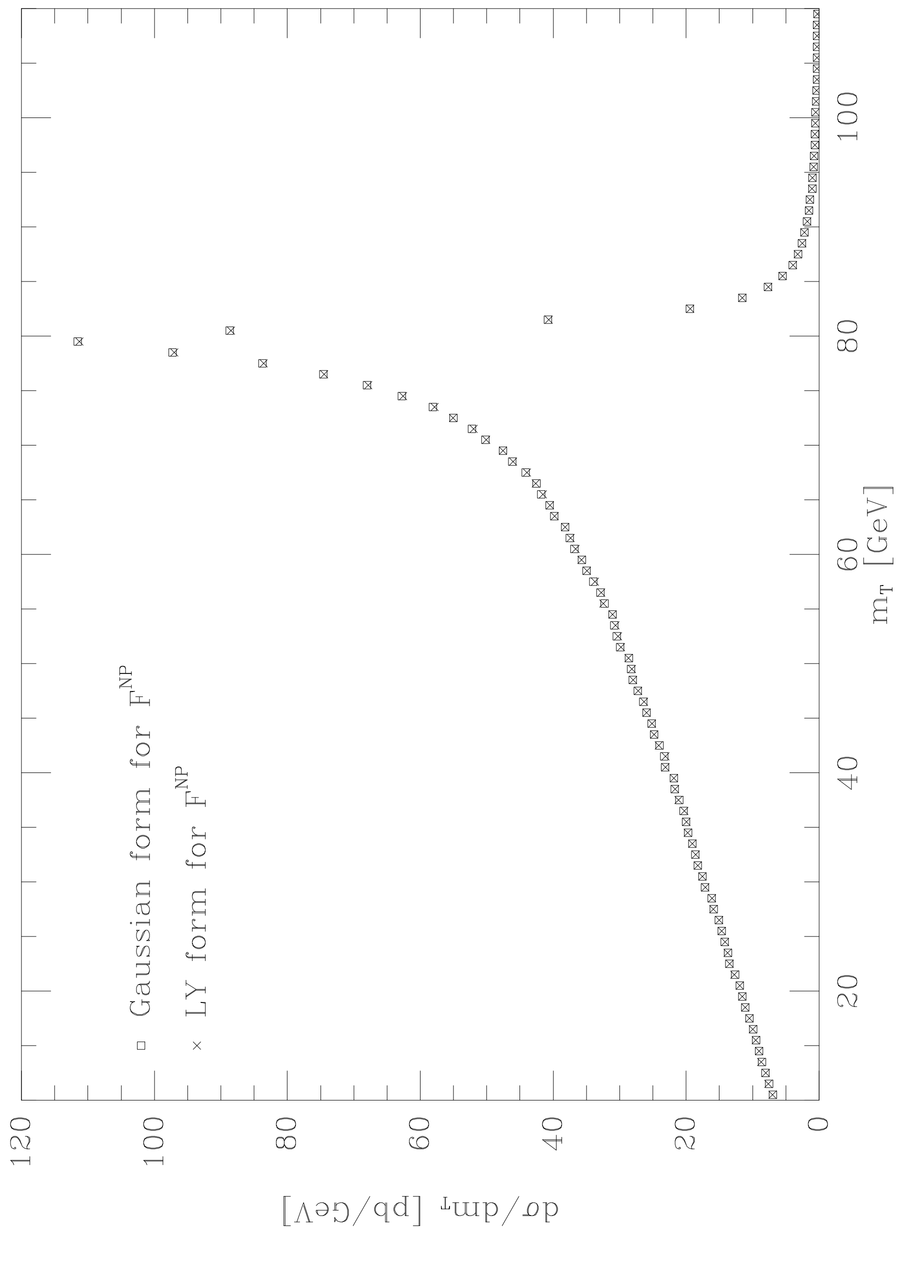
However, the longitudinal momentum of the ν isn't measured. It is, therefore, useful to define a transverse mass m_T given by

$$
\begin{aligned}
M_T^2(l,\nu) &= (|\vec{p}_{Tl}| + |\vec{p}_{T\nu}|)^2 - (\vec{p}_{Tl} + \vec{p}_{T\nu})^2 \\
&= 2|\vec{p}_{Tl}||\vec{p}_{T\nu}|(1 - \cos\phi)
\end{aligned}
$$

This has the virtue that $m^2 - m_T^2 \geq 0$. Furthermore, suppose that the W is produced with some p_{TW}. Then

$$
\begin{aligned}
\vec{p}_{Tl} &\to \vec{p}_{Tl} + \vec{p}_{TW}/2 \\
\vec{p}_{T\nu} &\to -\vec{p}_{Tl} + \vec{p}_{TW}/2
\end{aligned}
$$

Inserting these into the above definition, one finds that the $\mathcal{O}(p_{TW})$ terms cancel. Hence, the transverse mass distribution has a reduced sensitivity to $\vec{p}_{TW}$. Nevertheless, resummed predictions are necessary if one wants to perform a precision mass determination of the W. An example from Ellis, Ross, and Veseli is shown on the following page.


Gaussian form for F^{NP}
LY form for F^{NP}
m_T [GeV]
$d\sigma/dm_T$ [pb/GeV]
20
40
60
80
100
0
20
40
60
80
100
120

M_W and pdf uncertainties

At hadron colliders one attempts to determine M_W by examining various distributions such as the charged lepton p_T distribution or that of the transverse mass.

One detects the lepton in a limited range of rapidity and transverse momentum. Recall that $y = \hat{y} + \ln x_a/x_b$ so limiting y restricts $\hat{y}$ in an x_a, x_b dependent way. Furthermore, $\sin\theta = 1/\cosh\hat{y}$, so limiting $\hat{y}$ affects the p_T of the lepton and vice versa.

Therefore, the kinematic cuts constrain the range of x_a, x_b one integrates over and the shapes of the pdf's can affect the $p_T, y, m_T, \ldots$ distributions and, therefore, the mass one measures for the W.

- Use W p_T and lepton y asymmetry data to constrain the production models (*e.g.*, the non-perturbative input to the resummation calculations) *and* to help constrain the pdf's.

- The two stages are correlated!

Conclusions

- Lepton pair production has a long history of serving as a physics probe of hadronic interactions and new physics.
- QCD corrections to the basic parton model picture have been calculated though $\mathcal{O}(\alpha_s^2)$.
- Resummation techniques to handle the two-scale problem (p_T, Q) have been developed.
- W and Z production serve as sources of information on Standard Model physics and beyond
- Production properties serve to constrain pdf's needed to refine measurements of the W mass
- Technology developed for lepton pair production is directly applicable to W and Z production

References

The following references may prove helpful for further studies of vector boson production. Each reference contains numerous references to the original literature.

- Applications of Perturbative QCD, R.D. Field, published by Addison-Wesley (originally published in 1989; a 1995 edition is available)

- Collider Physics, V. Barger and R. Phillips, published by Addison-Wesley (originally published in 1987; a 1996 edition is available)

- QCD and Collider Physics, R.K. Ellis, W.J. Stirling, and B.R. Webber, published by Cambridge University Press (1996)

In addition to these references, an article based on notes from Jack Smith's 1995 DESY-CTEQ lectures on the Drell-Yan process is available from Björn Pötter at

- www.desy.de/~ poetter or
- www.phys.psu.edu/~cteq/schools/summer95/dy.ps.

This writeup covers the calculation of the $\mathcal{O}(\alpha_s)$ corrections using dimensional regularization. Several useful appendices covering calculational techniques in n dimensions are included.