B Meson Oscillations

- $B^0\bar{B}^0$ oscillations discovered in 1987 by ARGUS
- $B^0\bar{B}^0$ mixing well established phenomenon: $\sigma < 4\%$
- B^0 system: 2 CP-conjugate states:

  $|B^0\rangle = |\bar{b}d\rangle$ and $|\bar{B}^0\rangle = |b\bar{d}\rangle$

- Mass eigenstates $\neq$ Flavour eigenstates

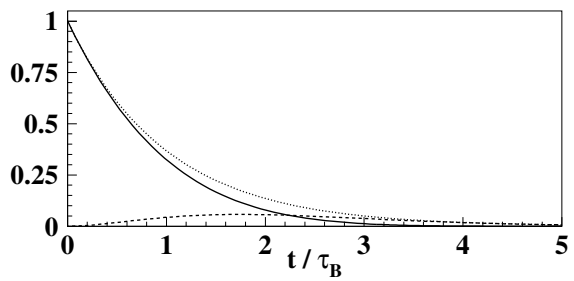
  $|B^0\rangle = 1/\sqrt{2}\,(|B^0_L\rangle + |B^0_H\rangle)$ H: heavy

  $|\bar{B}^0\rangle = 1/\sqrt{2}\,(|B^0_L\rangle - |B^0_H\rangle)$ L: light

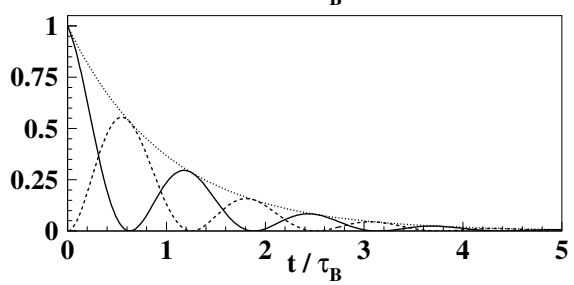
- Δm = mass difference between both B^0 mass eigenstates B^0_H-B^0_L
- If initial state is B^0:

$$\mathcal{P}_{\rm mix}(t) = 1/2 \cdot \Gamma\, e^{-\Gamma t} \cdot (1 - \cos \Delta m\, t)$$
$$\mathcal{P}_{\rm unmix}(t) = 1/2 \cdot \Gamma\, e^{-\Gamma t} \cdot (1 + \cos \Delta m\, t)$$

- Measure Δm = oscillation frequency
- Mixing parameter $x = \Delta m/\Gamma$: describes oscillation period relative to B meson lifetime $\tau_B = \hbar/\Gamma$



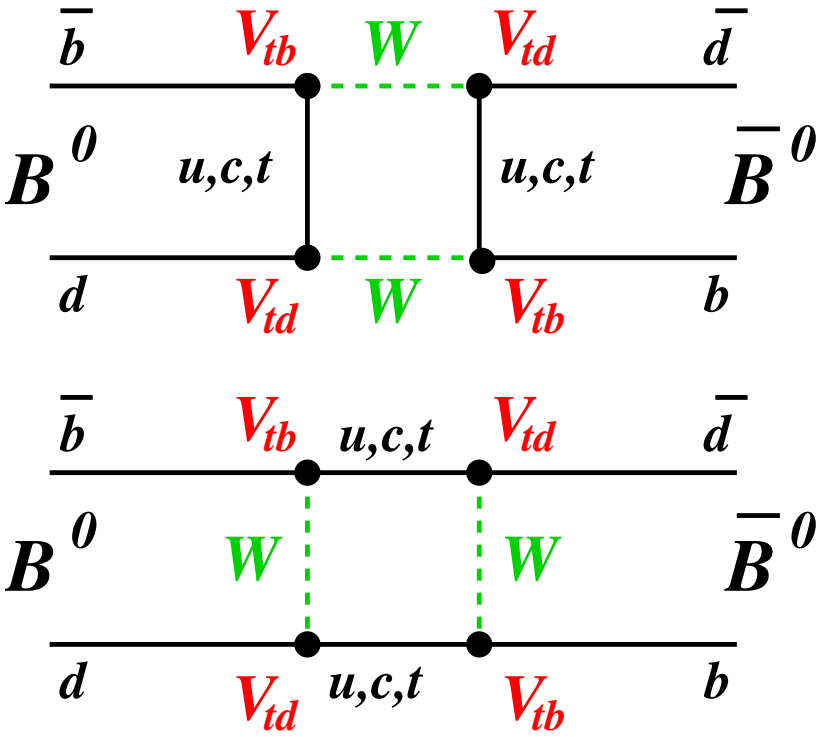
small mixing parameter



larger mixing parameter

B Meson Oscillations

Theory:



- In Standard Model:

$$\Delta m_d = \frac{G_F^2}{6\pi^2}\, m_B\, m_t^2\, (f_B^2 B_B)\, \eta_{QCD}\, F\left(\frac{m_t^2}{m_W^2}\right)\, |V_{tb}|^2\, |V_{td}|^2$$

  - $f_b = 180 \pm 40$ MeV (B^0 decay constant)
  - $B_B = 1.0 \pm 0.05$ (B bag parameter)
  - $\eta_{QCD} = 0.55 \pm 0.01$ (QCD correction factor)
  - $m_t = 175.6 \pm 5.5$ GeV/c^2 (top quark mass)
  - F slowly varying function of m_t^2 and m_W^2 (~ 1)

- Many uncertainties cancel in ratio:

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_S^0}\eta_{QCD}^{B_S^0}(f_{B_S^0}^2 B_{B_S^0})}{m_{B^0}\eta_{QCD}^{B^0}(f_{B^0}^2 B_{B^0})}\left|\frac{V_{ts}}{V_{td}}\right|^2 \approx (1.15 \pm 0.05)^2 \times \left|\frac{V_{ts}}{V_{td}}\right|^2$$

B Mixing Measurement

Do not discuss time integrated mixing (χ_d - CLEO)

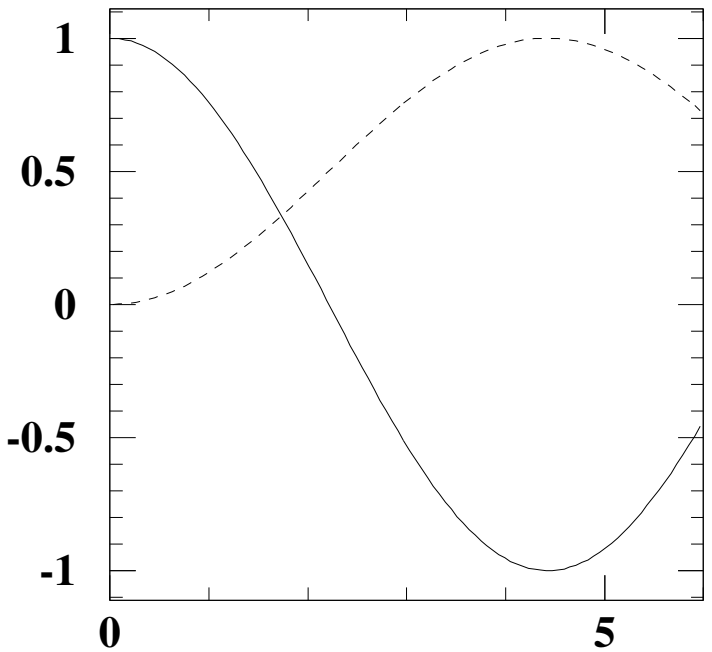
Ingredients for time evolution measurement:

- Need proper decay time info (decay vertex)
- Need B flavour at decay
  - If decay final states are distinct: $B^0 \to f$, $\bar{B}^0 \to \bar{f}$, $f \neq \bar{f}$
  - Use e.g. charge of ℓ from semileptonic B decay
- Need B flavour at production
  - Study B flavour tagging

- Quantities used in B mixing analyses:

$$\text{Mixed fraction}: \quad \frac{\mathcal{P}_{\text{mix}}(t)}{\mathcal{P}_{\text{unmix}}(t) + \mathcal{P}_{\text{mix}}(t)} = \frac{1}{2}(1 - \cos \Delta m t)$$

$$\text{Asymmetry}: \quad \mathcal{A} = \frac{\mathcal{P}_{\text{unmix}}(t) - \mathcal{P}_{\text{mix}}(t)}{\mathcal{P}_{\text{unmix}}(t) + \mathcal{P}_{\text{mix}}(t)} = \cos \Delta m t$$



Time Dependent B Mixing

Time dependent $B^0\bar{B}^0$ mixing measurement:

- Need proper decay time t of B meson

$$t = \frac{L_{xy}}{\beta\gamma} = L_{xy} \cdot \frac{m_B}{p_B}$$

- Uncertainty on decay time:

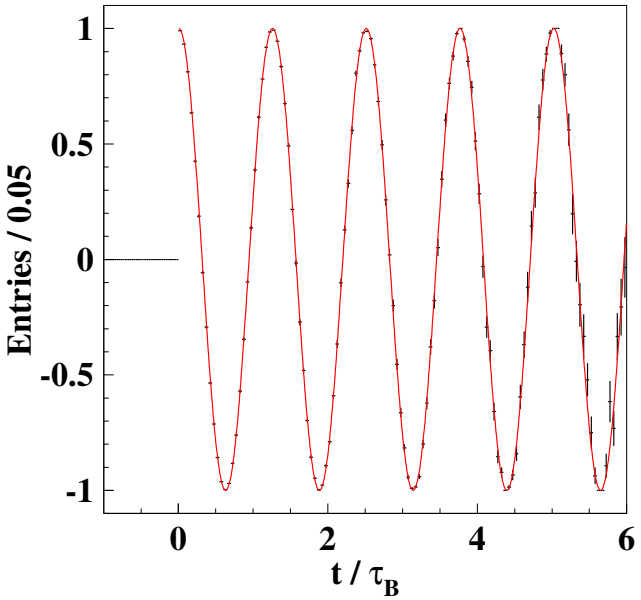
$$\frac{\sigma_t}{\tau} = \sqrt{\left(\frac{\Delta L_{xy}}{L^0_{xy}}\right)^2 + \left(\frac{t}{\tau}\frac{\Delta p_B}{p_B}\right)^2} \qquad (L^0_{xy} = \frac{p_B}{m_B} \cdot c\tau)$$

  - ΔL_{xy} : Vertexing resolution (includes uncertainty on primary & secondary vertex)
    $\sigma_{VX} \sim$ 0.1-0.15 ps (LEP/CDF), $\sim$ 0.06 ps (SLD)

  - Δp_B : Momentum uncertainty;
    Resolution is time dependent!
    - Fully reconstructed B: small effect
    - Partially reconstructed B: depending on boost resolution ($\sigma_P/p \sim$ 10-20%)
    - Good σ_t crucial for B^0_S mixing:
      $\Delta m = 15\ \text{ps}^{-1} \Leftrightarrow 0.3$ ps / cycle

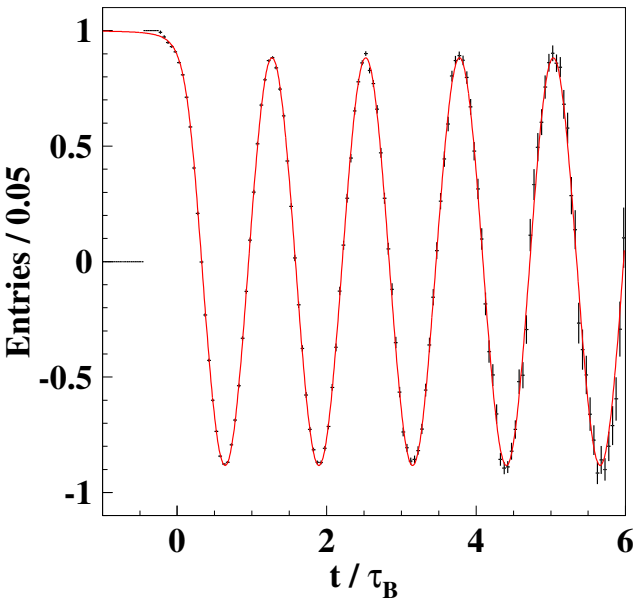
Time Dependent B Mixing

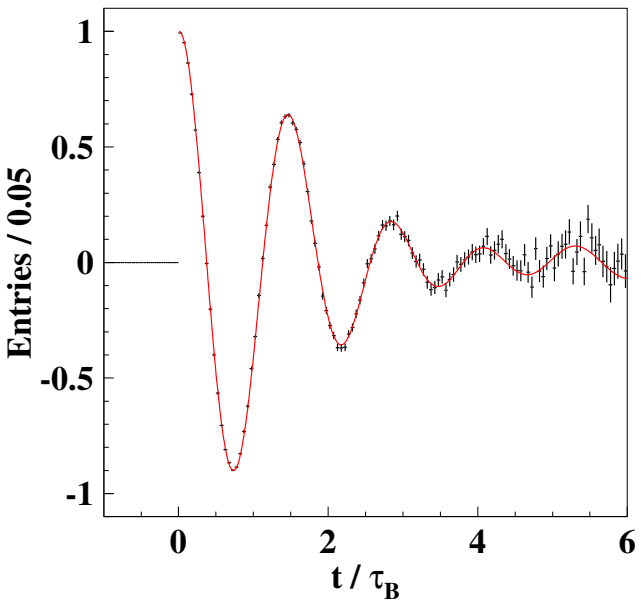
Effect of vertexing resolution on Δm measurement:



perfect resolution

vertex resolution

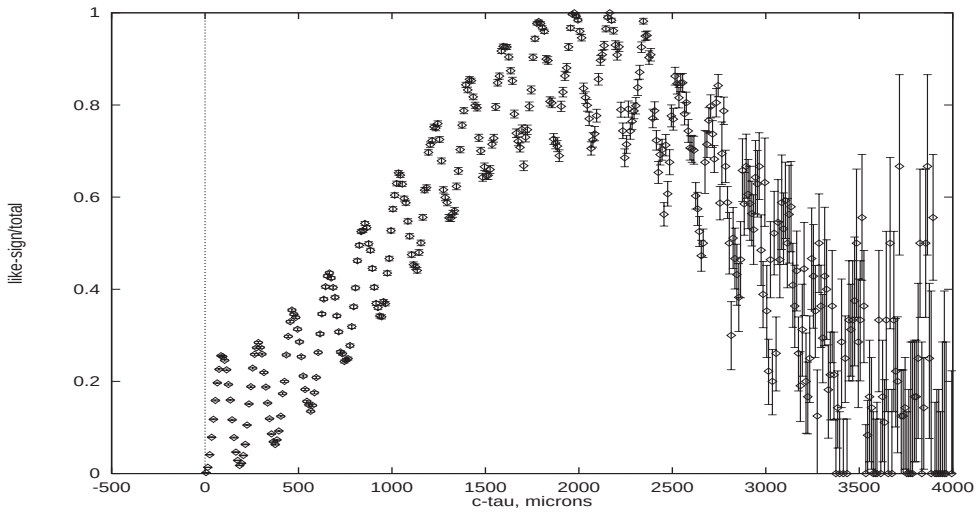




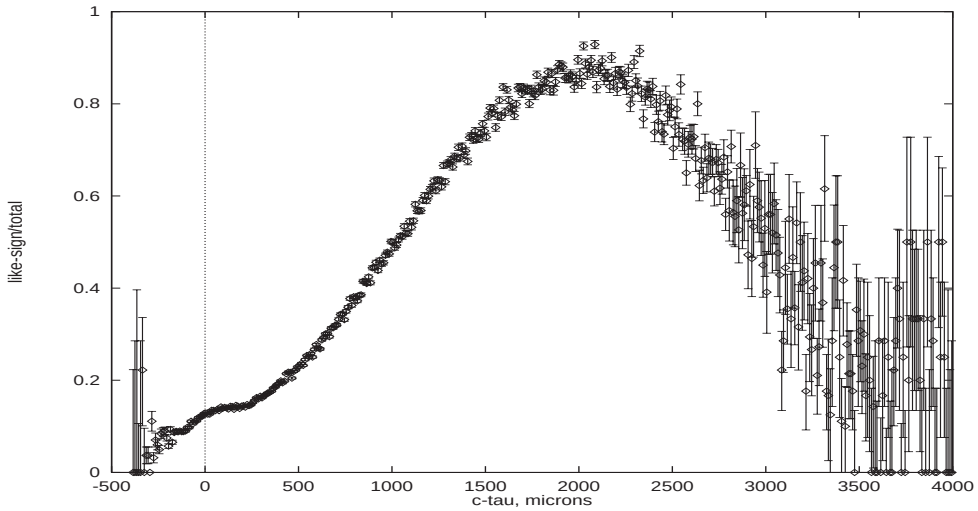
momentum resolution

Time Dependent B Mixing

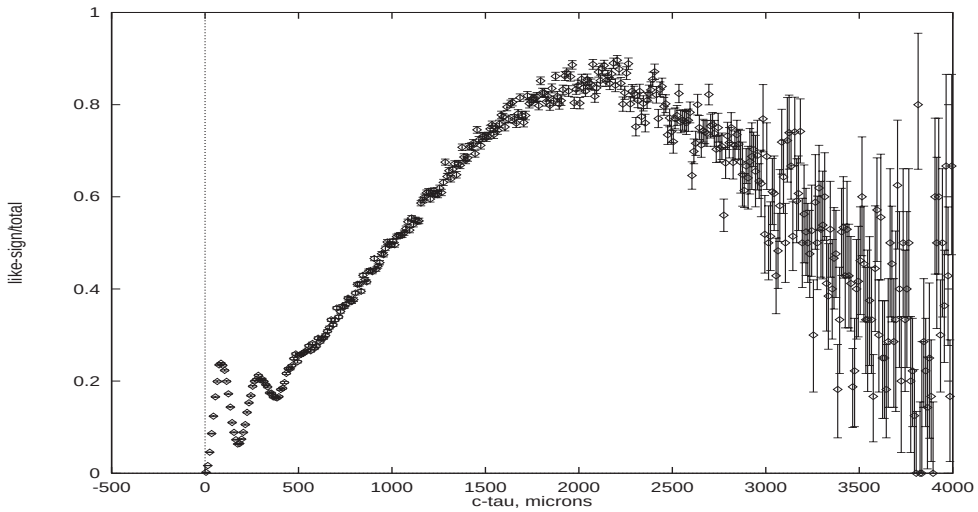
Effect of vertexing resolution on rapid oscillations:



perfect resolution



vertexing resolution



momentum resolution

B Flavour Tagging

Flavour tagging = determination of B production flavour

Figure of merit to compare tags:

* Effective tagging efficiency: $\varepsilon \mathcal{D}^2$
  - $\varepsilon :=$ efficiency
  - $\mathcal{D} = (N_R - N_W)/(N_R + N_W) :=$ dilution
    $\mathcal{D} = 1 - 2\,w$ ($w =$ 'mistag fraction')

- For CP violation measurement:

$$\mathcal{A}_{CP}^{\text{obs}} = \mathcal{D} \cdot \mathcal{A}_{CP}$$

- Flavour tag purity: $N_{\text{eff}} = N_{\text{tagged}} \cdot \mathcal{D}^2$
  - 100 tagged events with $\mathcal{D} = 0.2$ is worth 4 events
  - with time resolution added: $\mathcal{D}_{\text{eff}} = \mathcal{D}_{\text{tag}} \cdot e^{-(\Delta m \sigma_t)^2/2}$

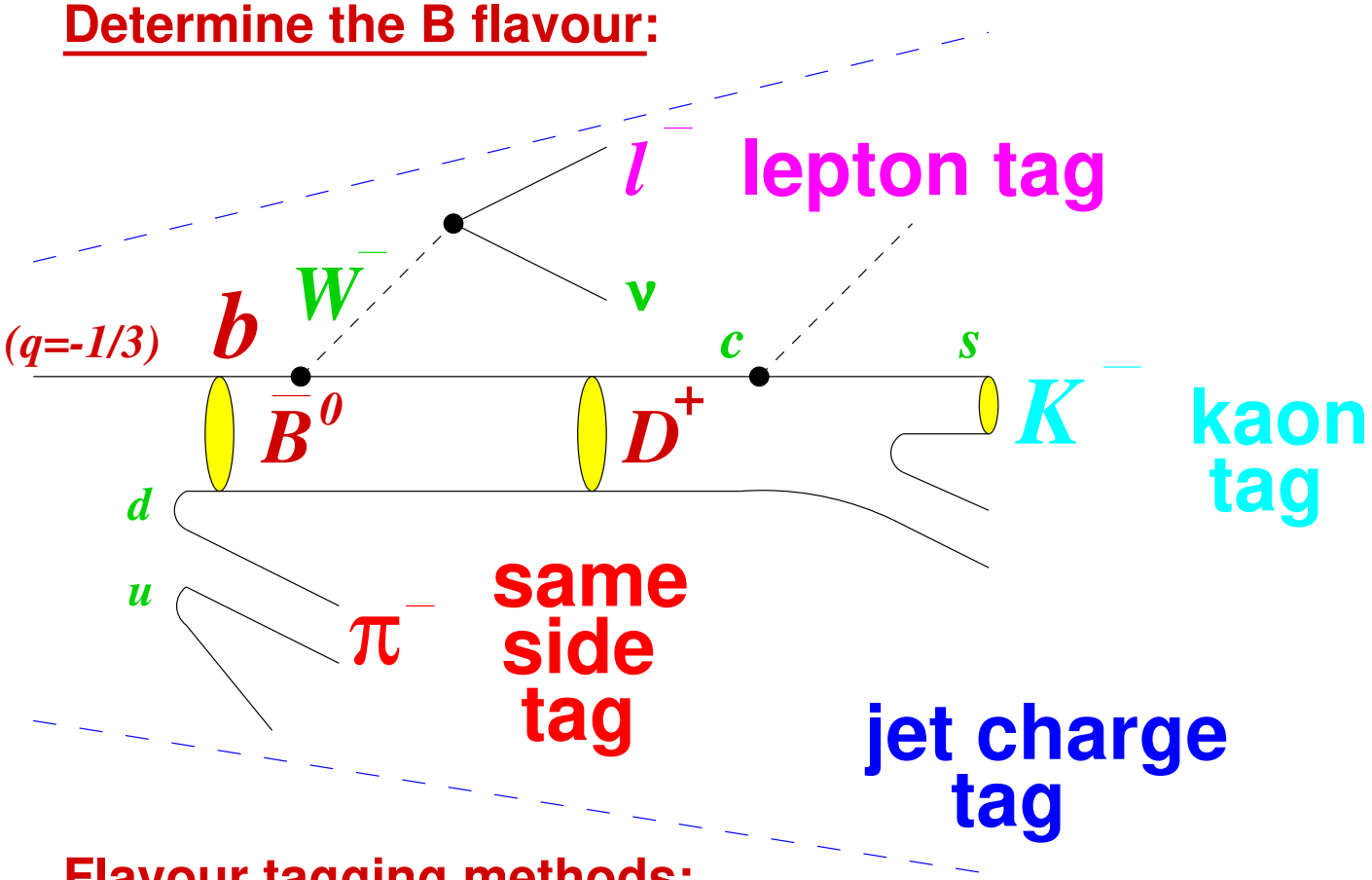
- Plot time dependent asymmetry

$$\mathcal{A}(t) = D \cdot \cos \Delta m_d t = (1 - 2w) \cdot \cos \Delta m_d t$$

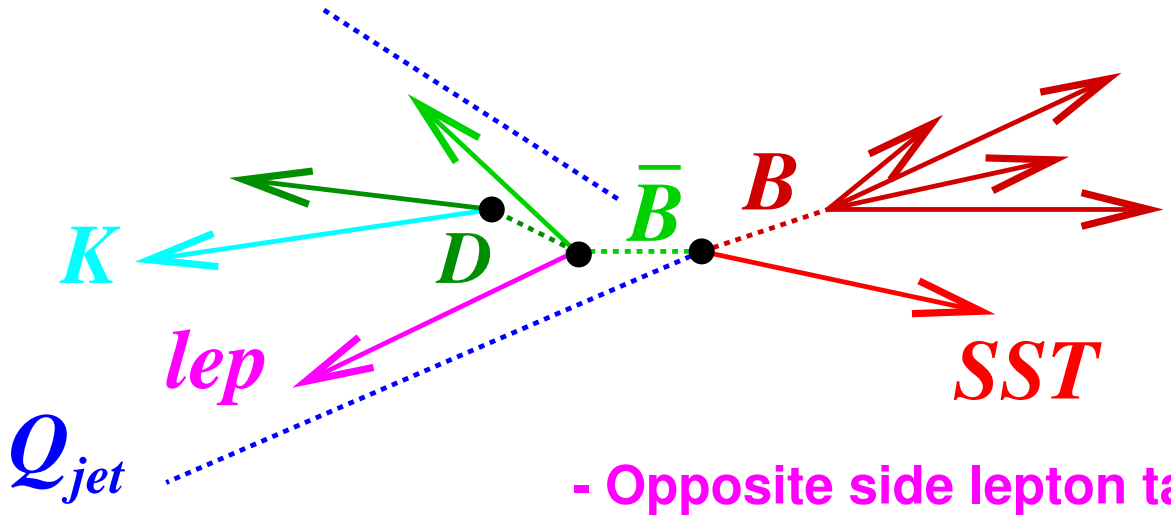
- Time dependent $B^0 \bar{B}^0$ mixing measurement is like B lifetime measurement, but divide up sample in mixed and unmixed portion
  - mixed: decay flavour $\neq$ production flavour
  - unmixed: decay flavour $=$ production flavour

B Flavour Tagging

Determine the B flavour:



Flavour tagging methods:

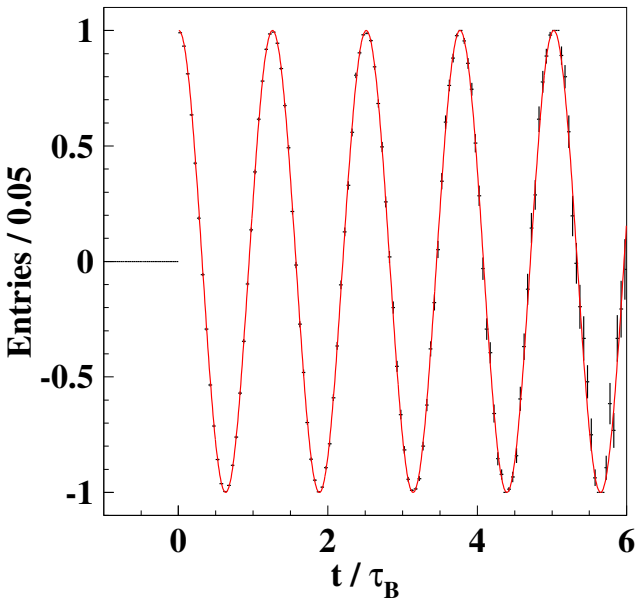


- Opposite side lepton tag
- Jet charge tag
- Same side tag
- Opposite side kaon tag

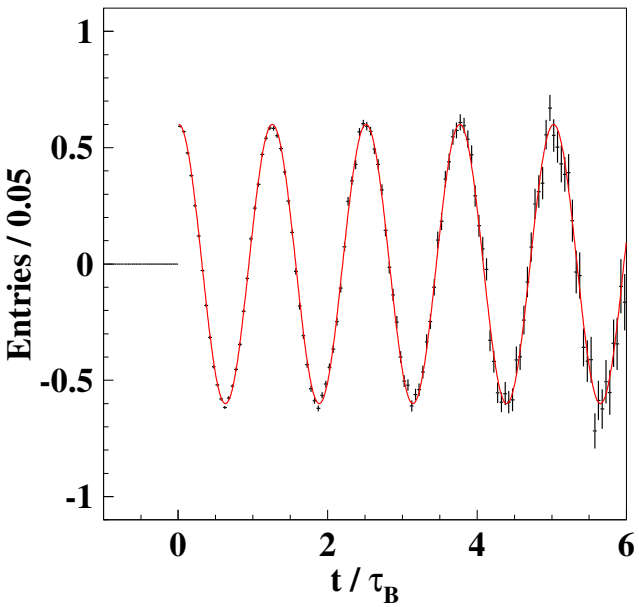
- Forward-backward asymmetry (SLD)

Time Dependent B Mixing

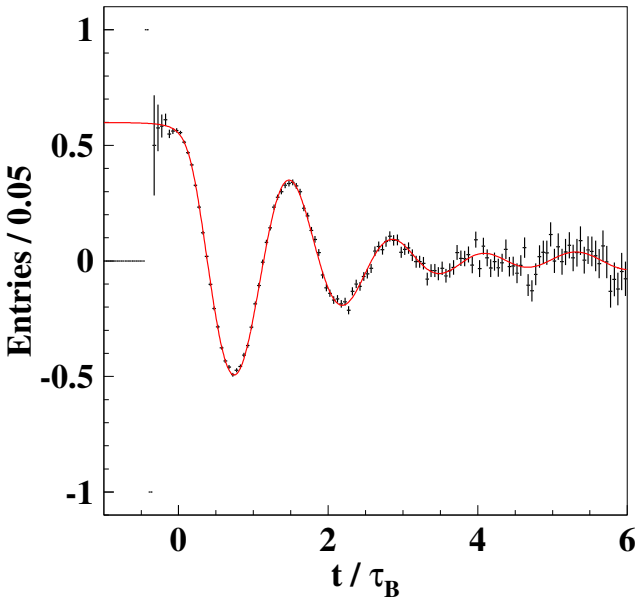
Effect of flavour tagging on Δm measurement:



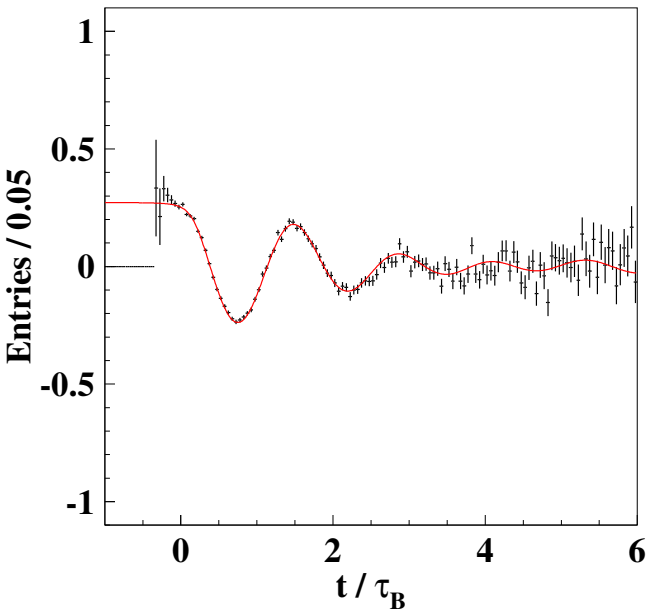
perfect resolution



mistag probability



vertex and momentum resol.
plus mistag probability



include background

Lepton and Jet Charge Tagging

Lepton tag:

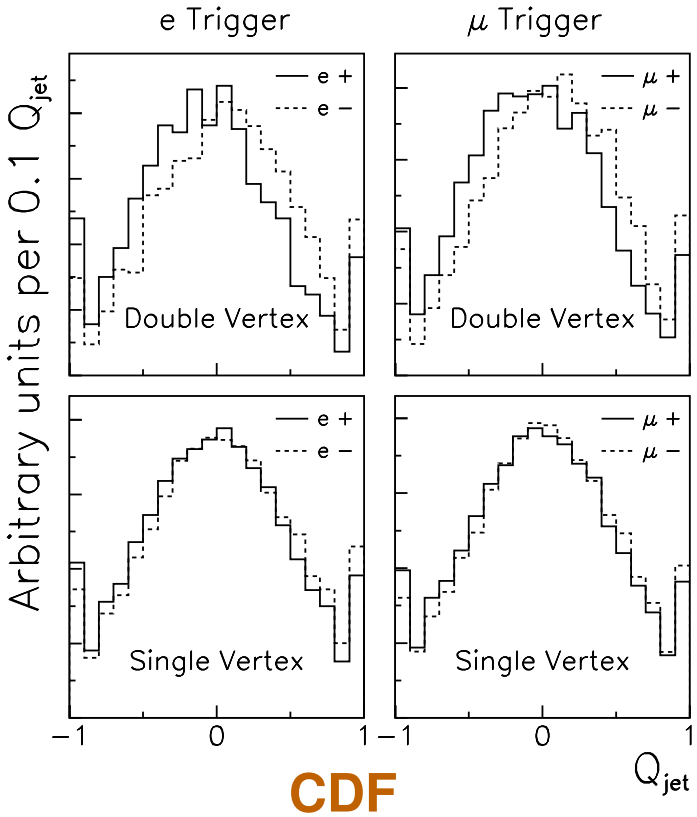
- Find lepton from opposite side b hadron
  - Low efficiency ($BR \sim 10\%$)
  - Good purity, limited by mixing ($\chi_b \sim 12\%$), sequential decays $b \to c \to \ell$ and fakes

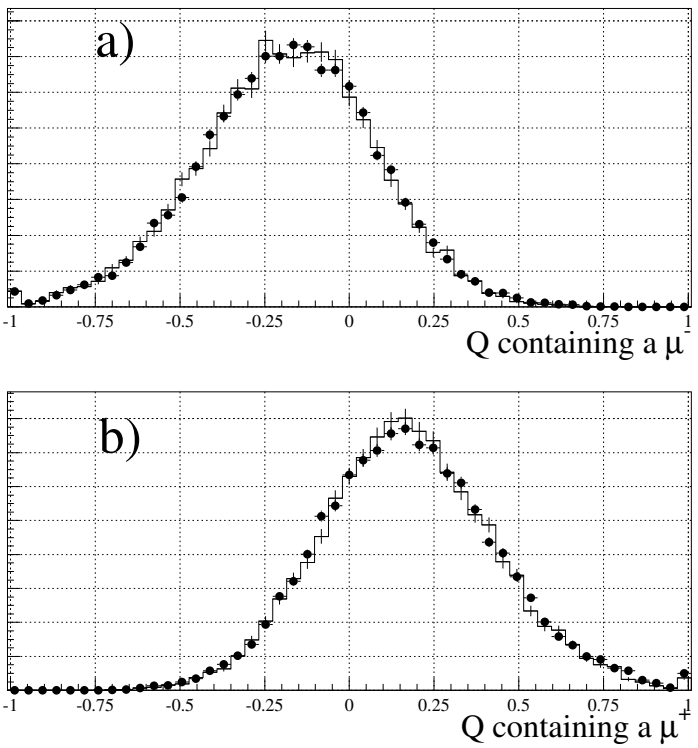
Kaon tag:

- Similar to lepton tag
  - Efficiency higher, purity lower

Jet charge tag:

- Find b jet on opposite side
  - Use track based jet clustering and p_t weighting
  - High efficiency but low purity
  - Not affected by mixing, $B^0 \to \bar{B}^0$ preserves net charge





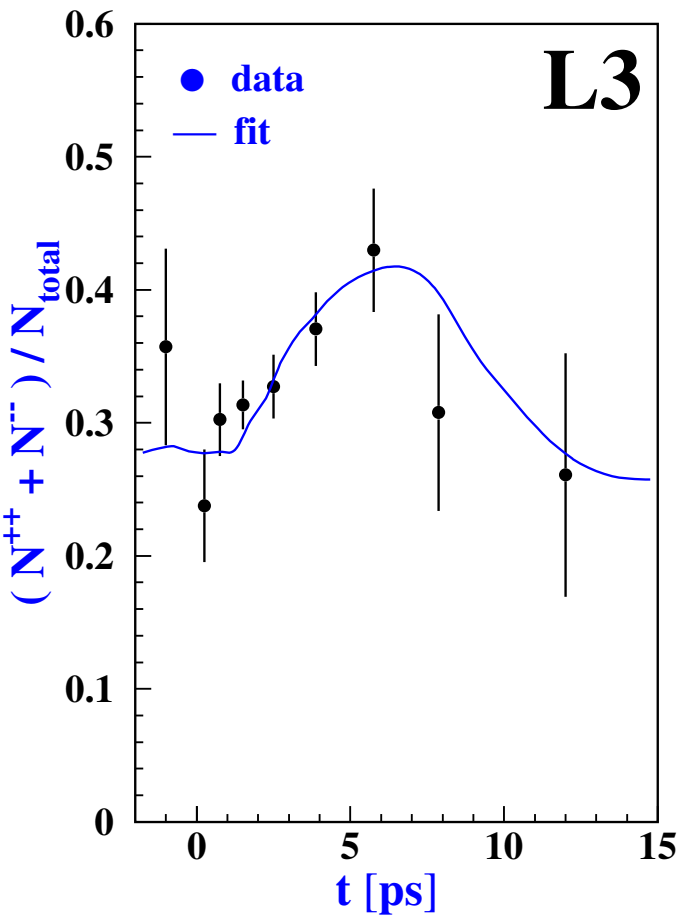
Lepton and Jet Charge Tagging

- Measurement of $B^0\bar{B}^0$ oscillations = Demonstration that tagging algorithm works

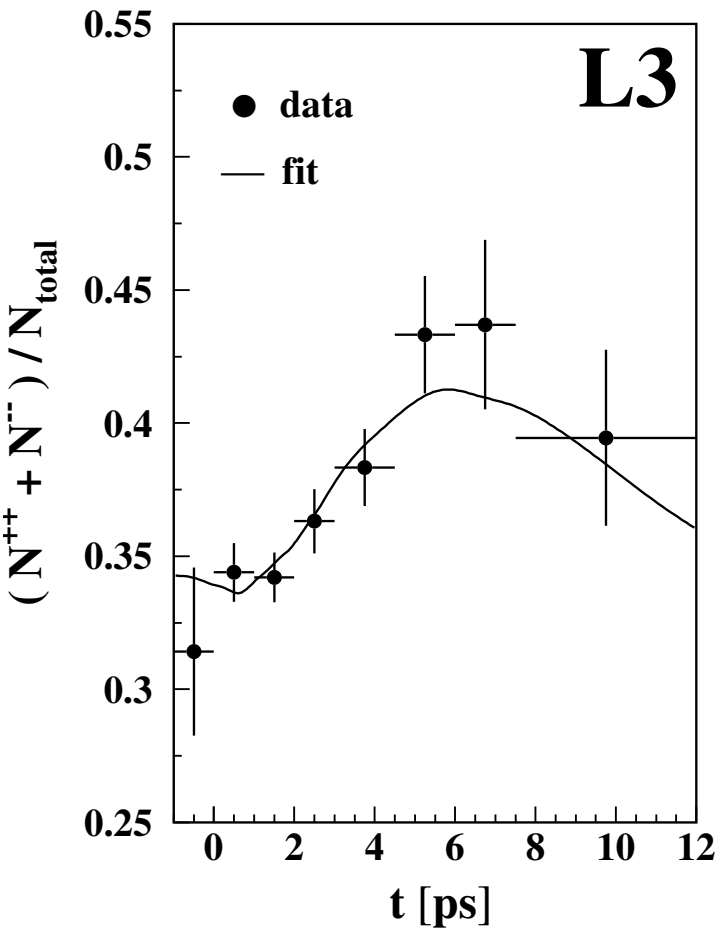
  e.g. L3: $\Delta m_d = (0.444 \pm 0.028 \pm 0.028)\ \mathrm{ps}^{-1}$

- Lepton tag: $\varepsilon\mathcal{D}^2$: $\sim 1\%$ (CDF), $\sim$ 4-8% (BaBar)

- Jet charge tag: $\varepsilon\mathcal{D}^2$: $\sim 1\%$ (CDF), $\sim$ 5-10% (LEP)



Lepton tag



Add jet charge tag

Same Side Tagging

- Use charge correlation between B flavour and fragmentation tracks

  Classic example: $D^{*+} \to D^0\pi^+$

  $B^* \to B\gamma$ not much of use

- Use P-wave B^{**} states [Gronau, Nippe, Rosner PRD47 (1993)]

  $B^{**0} \to B^+\pi^-$, $B^{**+} \to B^0\pi^+$

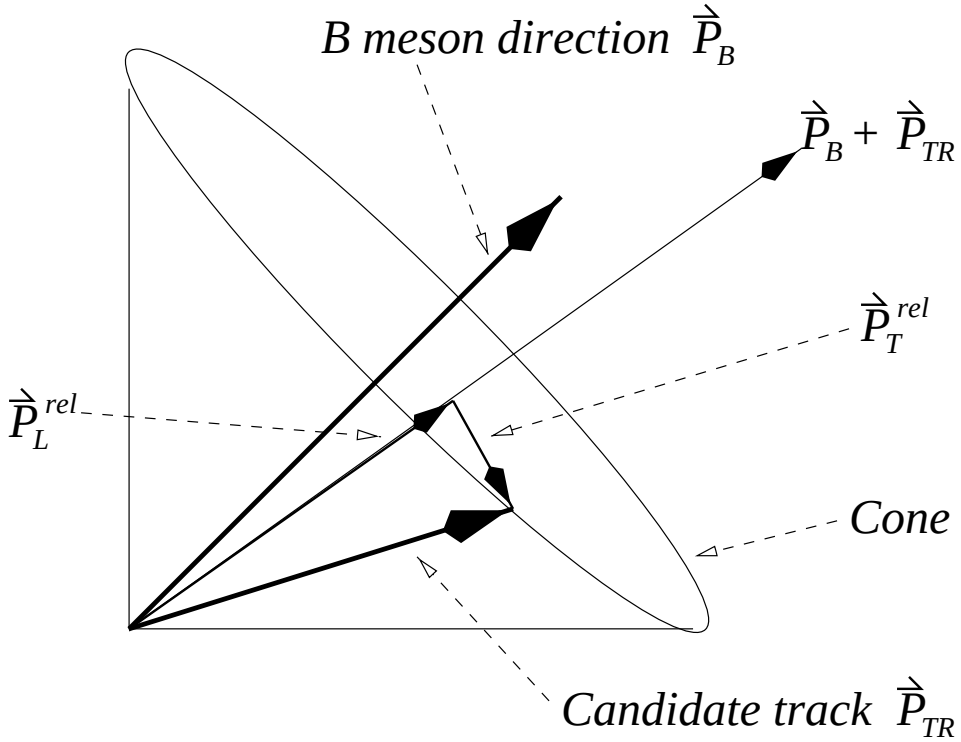
- Fragmentation gives same correlation as B^{**}

  $\bar{b} \to B^+\pi^-$, $\bar{b} \to B^0\pi^+$

  B^{**} account for 28-38% of b quark fragmentation

- High efficiency, fair purity ($\mathcal{D} \sim 0.2$-0.3)

- E.g. CDF: Choose track with minimum $p_t^{\rm rel}$:

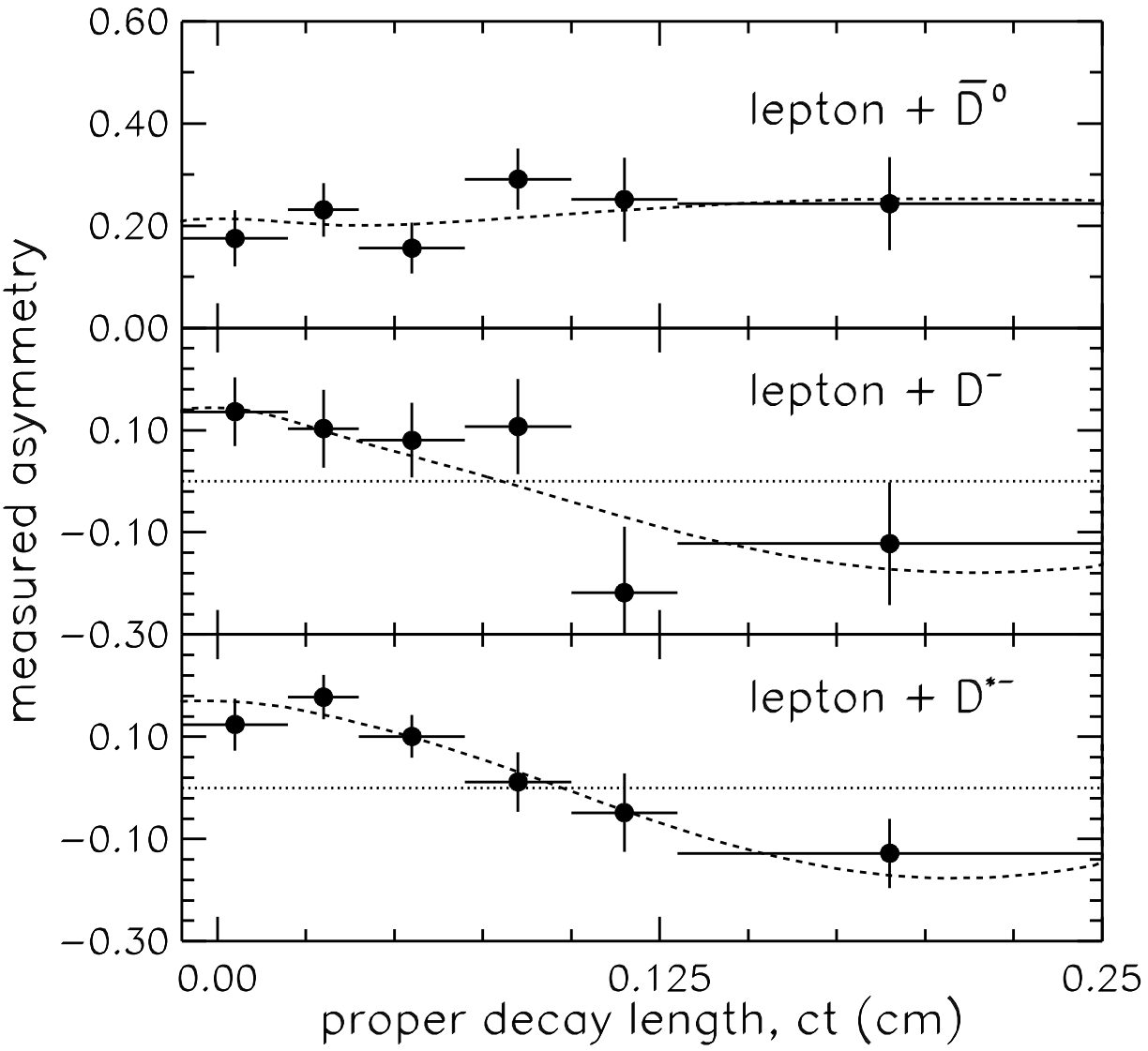


Same Side Tagging

- Measurement of $B^0\bar{B}^0$ oscillations in $B^0 \to \ell D^{(*)}\nu X$ events using SST

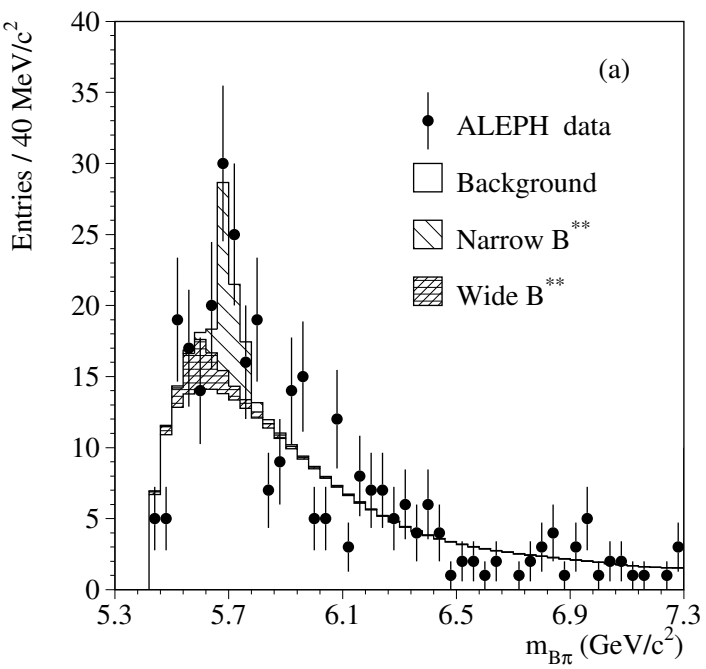
$$\Delta m_d = (0.471^{+0.078}_{-0.068} \pm 0.034)\ \mathrm{ps}^{-1}$$

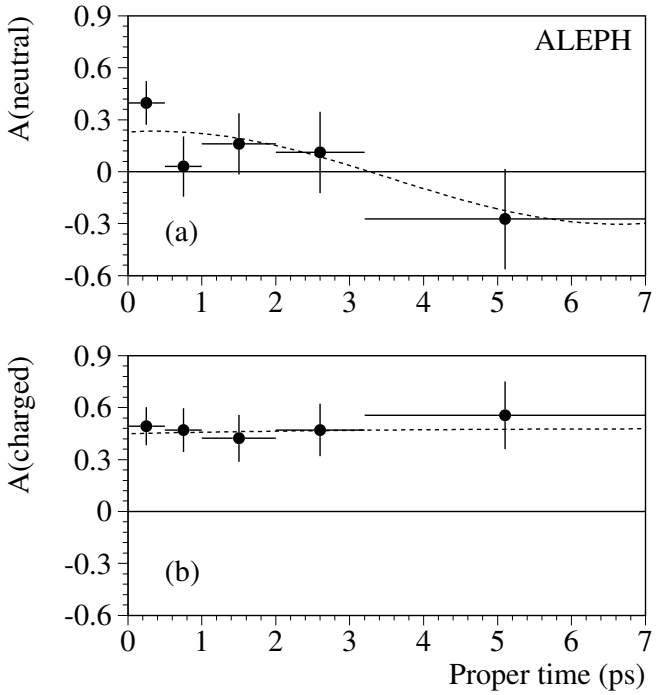
- Same side tag: $\varepsilon\mathcal{D}^2$: $\sim (2.4 \pm 0.9)\%$ (CDF)
  $\sim (14 \pm 4)\%$ (ALEPH) $\sim (10 \pm 2)\%$ (OPAL)



More Flavour Tags

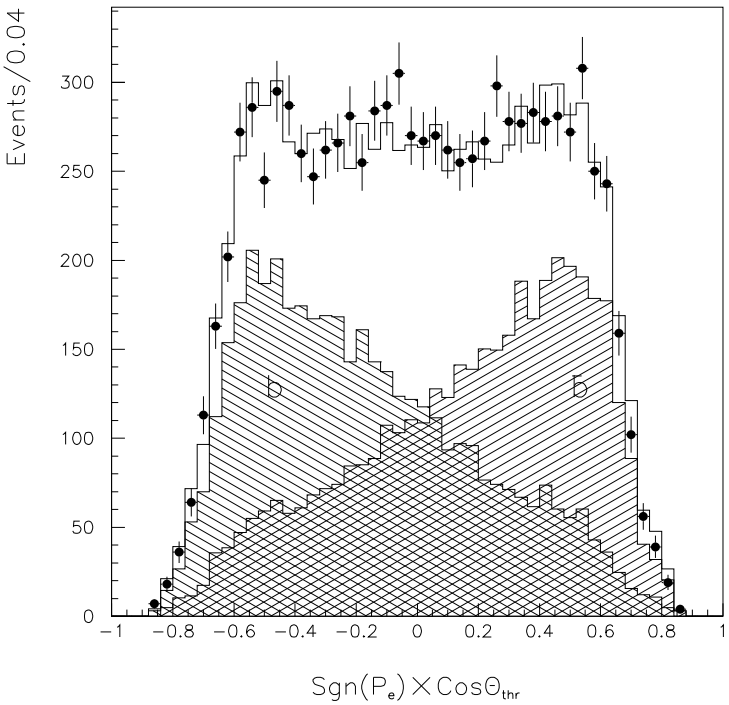
- ALEPH: Same side tagging with B^{**}
  - 238 B^+ and 166 B^0; $\varepsilon\mathcal{D}^2 \sim (9.1 \pm 1.3)\%$





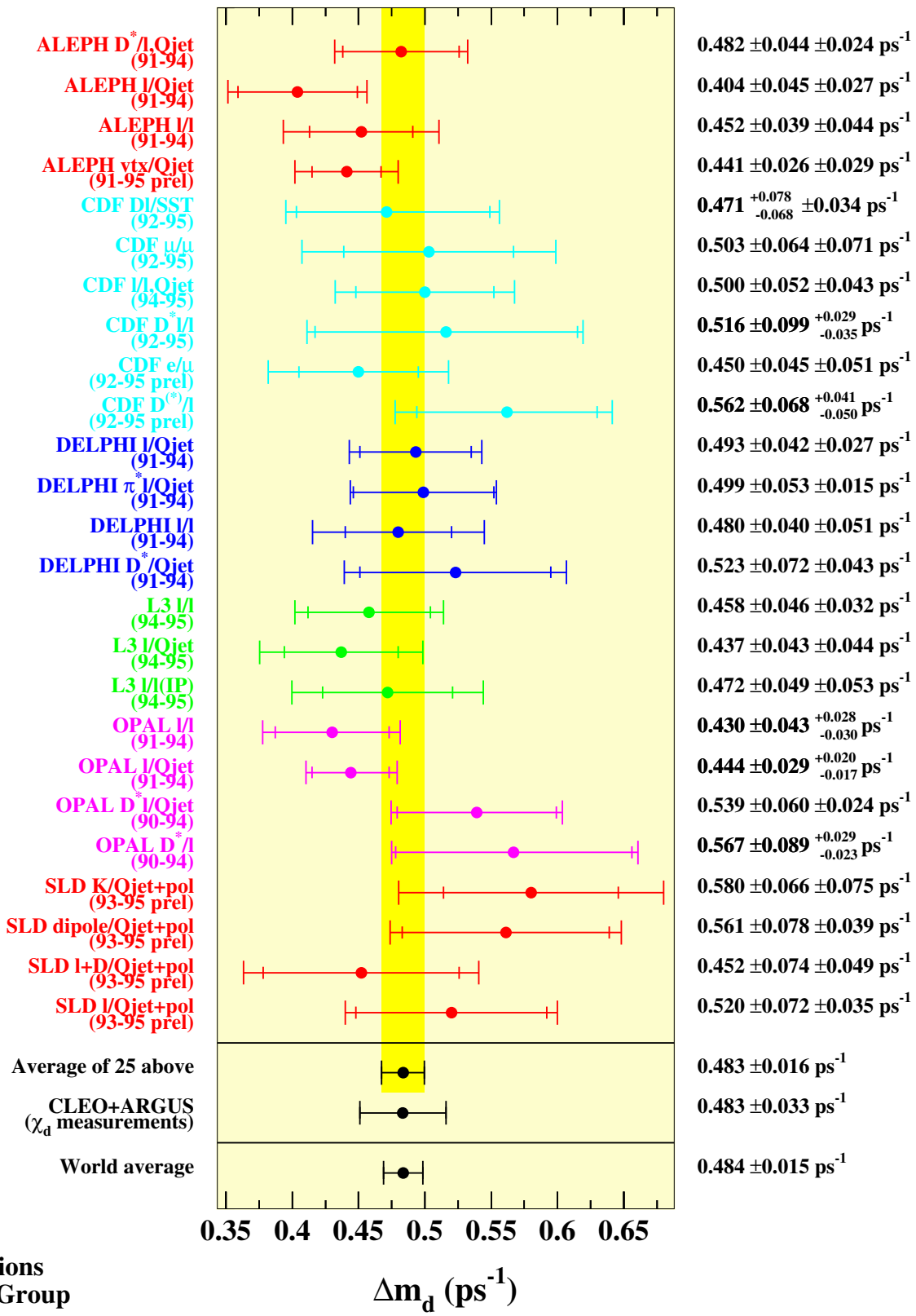
- SLD: Tagging with polarized e beam
  - For beam polarization $\langle\mathcal{P}_e\rangle = 73\%$:

    $\mathcal{D} = 48\%$, $\varepsilon\mathcal{D}^2 \sim 20\%$



B^0 Oscillations

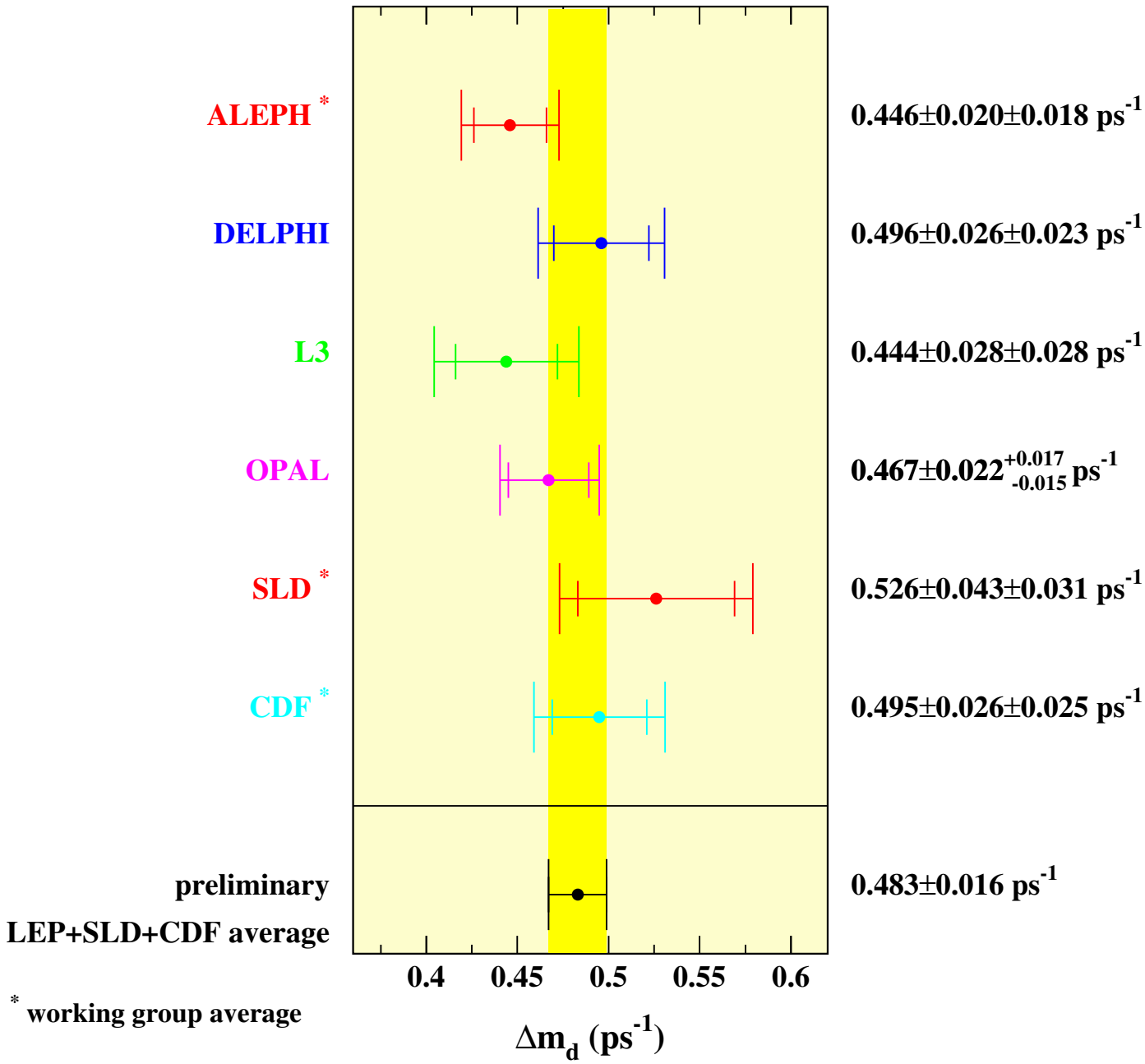
Summary of measurements from all experiments:



B Oscillations
Working Group

B^0 Oscillations

Comparison by experiment:



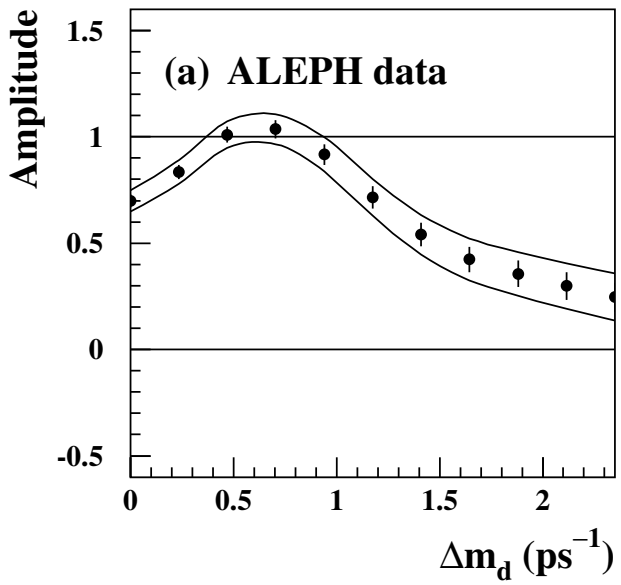
* working group average

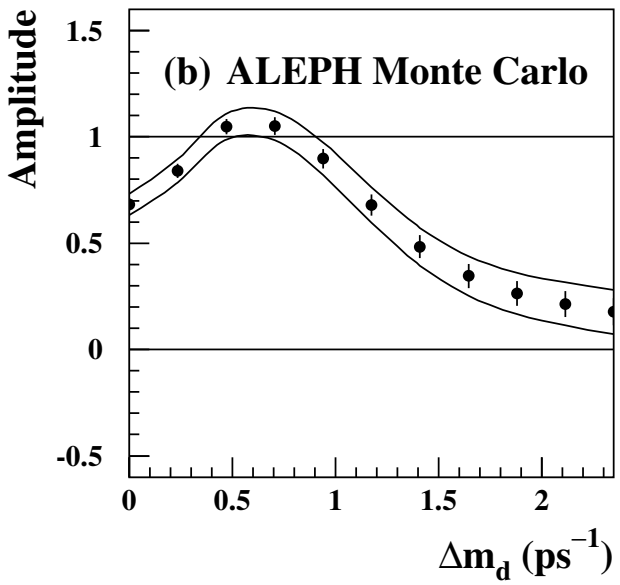
B_S^0 Oscillations

- $B_S^0 \bar{B}_S^0$ oscillations not yet observed
- Use amplitude method to derive limit on Δm_S
- Effective mixing rate is given by:

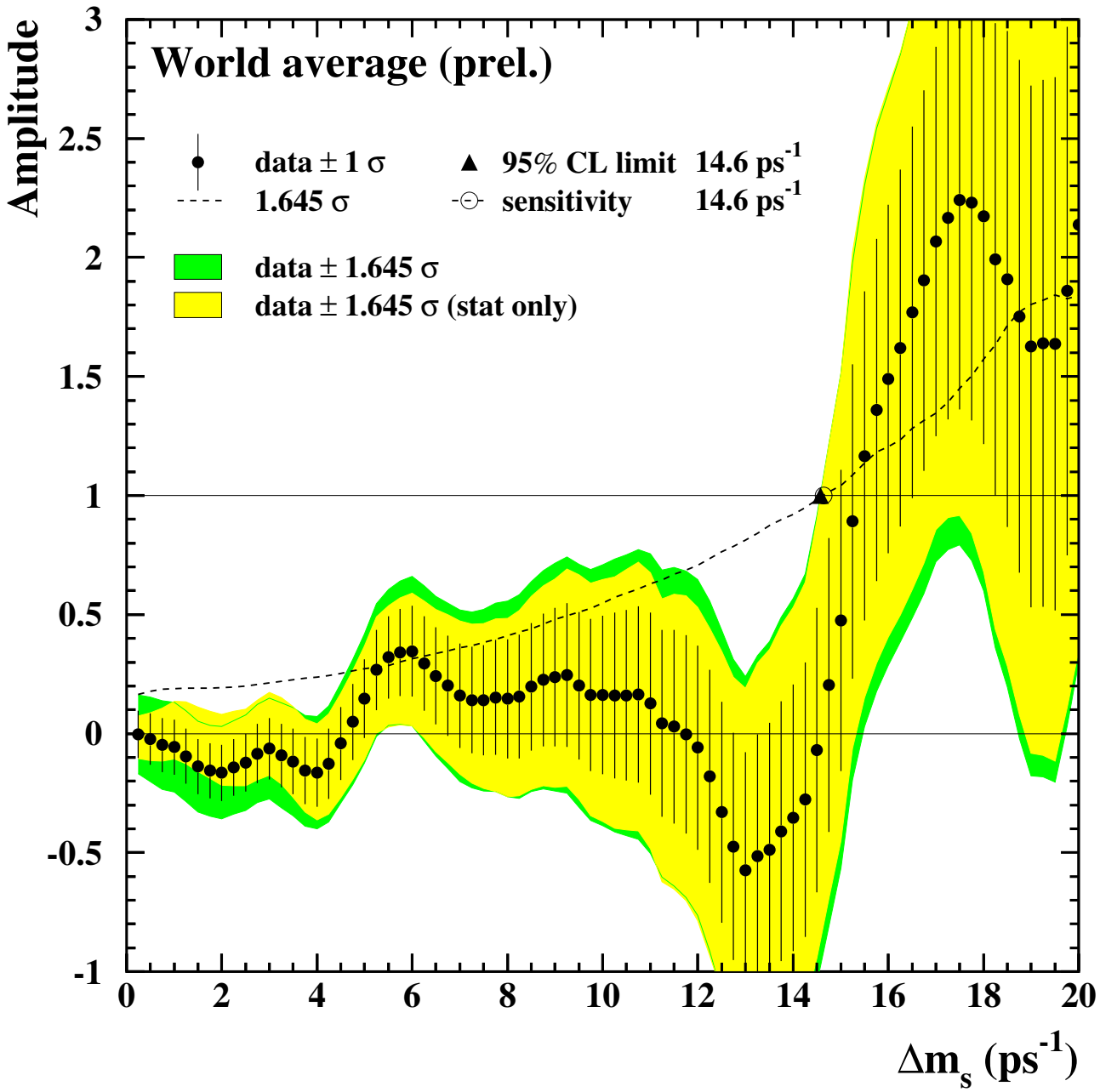
$$\mathcal{P}_{\text{mix}}(t) = 1/2 \cdot \Gamma\, e^{-\Gamma t} \cdot (1 - \mathcal{D}\,\mathcal{A} \cos \Delta m\, t)$$
$$\mathcal{P}_{\text{unmix}}(t) = 1/2 \cdot \Gamma\, e^{-\Gamma t} \cdot (1 + \mathcal{D}\,\mathcal{A} \cos \Delta m\, t)$$

- Effective dilution $\mathcal{D}$: $\mathcal{D}_{\text{eff}} = \mathcal{D}_{\text{tag}} \cdot e^{-(\Delta m_S\, \sigma_t)^2/2}$
- $\mathcal{A}$ = amplitude:
  - $\mathcal{A} = 1$ by definition for true signal
  - Fit for Δm_S directly with $\mathcal{A} = 1$, or
  - Fit for $\mathcal{A}$ with Δm_S fixed (amplitude scan)
- Limit on Δm_S set by $1.645 \cdot \sigma(\mathcal{A}) = 1$ (sensitivity for 95% C.L.)
- ALEPH: Amplitude fit verified on Δm_d





B_S^0 Oscillations

- Amplitude scan of Δm_S
- World average limit:

$$\underline{\Delta m_S > 14.6 \text{ ps}^{-1}} \quad (95\% \text{ C.L.})$$