Introduction

QED: accelerated charges radiate.

QCD identical: accelerated colours radiate.
gluons also charged.
$\Rightarrow$ cascade of partons.
$=$ parton shower.

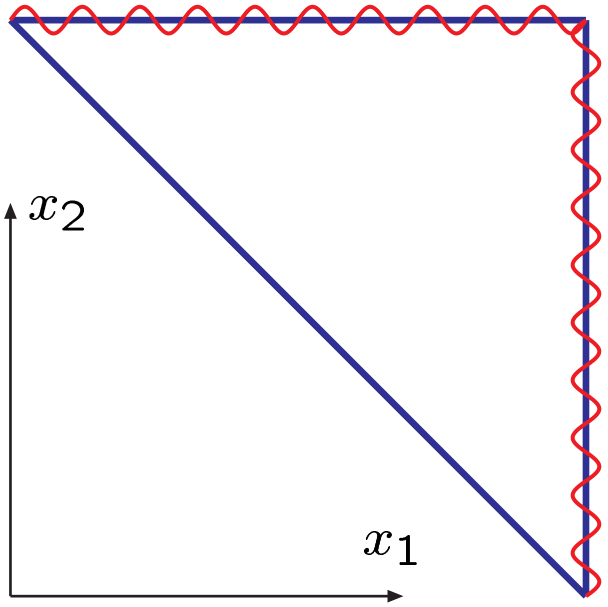
1) e^+e^- annihilation to jets.
2) Soft and collinear emission.
3) Sudakov form factors.
4) Angular ordering.
5) Initial-state radiation.
6) Hard scattering.
7) Heavy quarks.
8) The Colour Dipole Model.

e^+e^- annihilation to jets

Three-jet cross section:

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0\, C_F \frac{\alpha_s}{2\pi}\, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Singular as $x_{1,2} \to 1$:



Rewrite in terms of quark-gluon opening angle θ and gluon energy fraction x_3:

$$\frac{d\sigma}{d\cos\theta\, dx_3} = \sigma_0\, C_F \frac{\alpha_s}{2\pi} \left\{ \frac{2}{\sin^2\theta}\, \frac{1+(1-x_3)^2}{x_3} - x_3 \right\}$$

Singular as $\sin\theta \to 0$ or $x_3 \to 0$.

This formula is a cheat; it defines θ in a special frame, not the lab. Check the equivalent in the lab, and show that in the soft and collinear limits it is frame-independent.

can separate into two independent jets:

$$\begin{aligned}
\frac{2\,d\cos\theta}{\sin^2\theta} &= \frac{d\cos\theta}{1-\cos\theta}+\frac{d\cos\theta}{1+\cos\theta} \\
&= \frac{d\cos\theta}{1-\cos\theta}+\frac{d\cos\bar\theta}{1-\cos\bar\theta} \\
&\approx \frac{d\theta^2}{\theta^2}+\frac{d\bar\theta^2}{\bar\theta^2}
\end{aligned}$$

$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi}\frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}$$

jets evolve independently

Exactly same form for anything $\propto \theta^2$, eg
transverse momentum: $k_\perp^2 = z^2(1-z)^2\,\theta^2\,E^2$
invariant mass: $q^2 = z(1-z)\,\theta^2\,E^2$

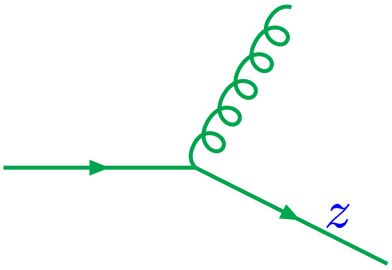
$$\frac{d\theta^2}{\theta^2} = \frac{dk_\perp^2}{k_\perp^2} = \frac{dq^2}{q^2}$$

Collinear limit

Universal:

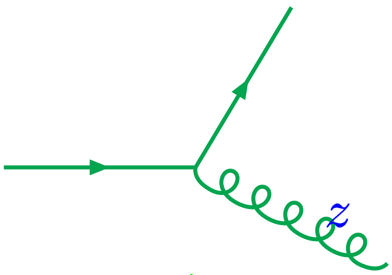
$$d\sigma = \sigma_0 \, \frac{\alpha_s}{2\pi} \, \frac{d\theta^2}{\theta^2} \, dz \, P(z,\phi) d\phi$$

$P(z,\phi) =$
Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting kernel: dependent on flavour: and spin



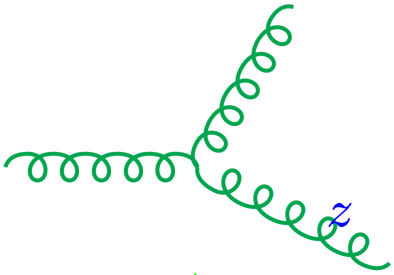
$q \to qg$

$C_F \frac{1+z^2}{1-z}$



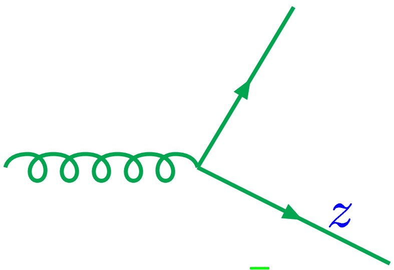
$q \to gq$

$C_F \frac{1+(1-z)^2}{z}$



$g \to gg$

$C_A \frac{z^4+1+(1-z)^4}{z(1-z)}$



$g \to q\bar{q}$

$T_R \left(z^2 + (1-z)^2\right)$

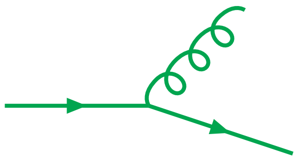
See e.g. Ellis, Stirling and Webber for spin-dependence.

Resolvable partons

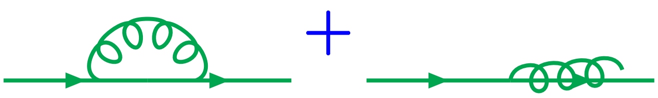
What is a parton?
Collinear parton pair $\longleftrightarrow$ single parton.

Introduce resolution criterion, eg $k_\perp > Q_0$.

Virtual corrections must be combined with unresolvable emission



Resolvable emission
finite



Virtual+Unresolvable
finite

Unitarity: resolved+unresolved=1

Sudakov Form Factor

Probability(emission between $q^2 + dq^2$ and q^2)

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\, P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

Define probability(no emission between Q^2 and q^2) to be $\Delta(Q^2, q^2)$. Then evolution equation:

$$\frac{d\Delta(Q^2,q^2)}{dq^2} = \Delta(Q^2,q^2)\frac{d\mathcal{P}}{dq^2}$$

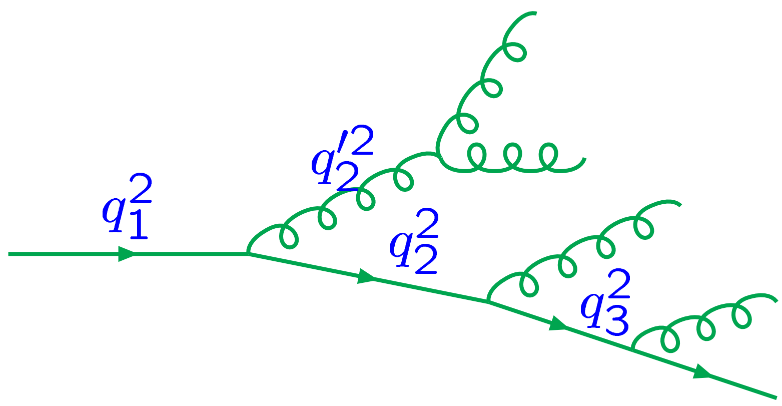
$$\Rightarrow \Delta(Q^2,q^2) = \exp - \int_{q^2}^{Q^2}\frac{dk^2}{k^2}\bar{P}(k^2).$$

c.f. radioactive decay
atom has probability λ per unit time to decay.
Probability(no decay before T) $= \exp - \int^T dt\, \lambda$

$\Delta(Q^2, Q_0^2) \equiv \Delta(Q^2)$ Sudakov form factor.
$=$ Probability(emitting no resolvable radiation)

$$\Delta_q(Q^2) \sim \exp - C_F\frac{\alpha_s}{2\pi}\log^2\frac{Q^2}{Q_0^2}$$

Multiple emission



$q_1^2 > q_2^2 > q_3^2 > \dots$
$q_1^2 > q_2'^2 \dots$

But starting condition? $q_1^2 <???$
Process dependent.

Monte Carlo implementation

Can generate branching according to

$$d\mathcal{P} = \frac{dq^2}{q^2}\bar{P}(q^2)\,\Delta(Q^2,q^2)$$

by choosing $0 < \rho < 1$ uniformly:
If $\rho < \Delta(Q^2)$, no resolvable radiation, evolution stops. Otherwise, solve

$$\rho = \Delta(Q^2,q^2)$$

for $q^2 =$ emission scale.

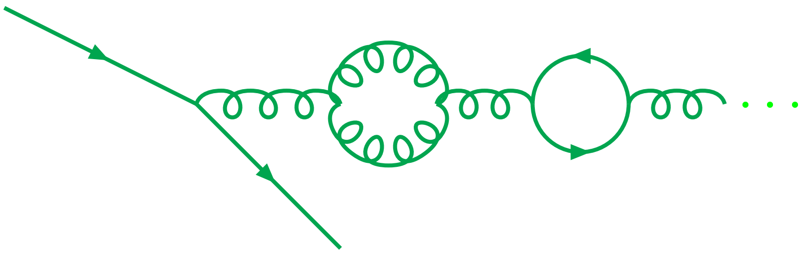
Considerable freedom:
Evolution scale: $q^2/k_\perp^2/\theta^2$?
z: energy? lightcone-momentum? $(= \frac{E+p_z}{E+p_z})$
massless partons become massive. How?
Upper limit for q^2?

All formally free choices, can be numerically very important.

Write a parton shower algorithm for $e^+e^- \to \mu^+\mu^- + n\gamma$. Try some of these choices. Which are important?
Add photon decays $\gamma \to e^+e^-$, and their showers, $e \to e\gamma$.

Running coupling

Effect of summing up higher orders:



absorbed by replacing α_s by $\alpha_s(k_\perp^2)$.
Much faster parton multiplication – phase space fills with soft gluons.

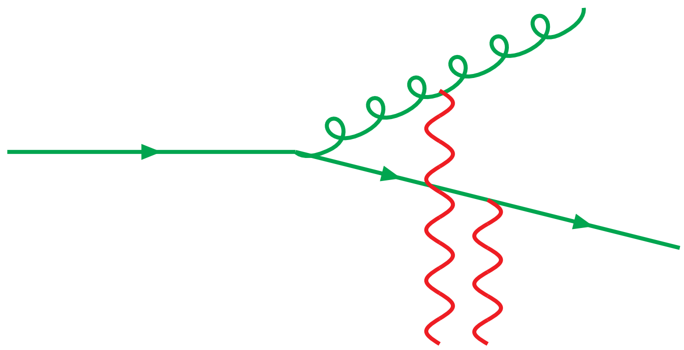
Must then avoid Landau pole: $k_\perp^2 \gg \Lambda^2$.
Q_0 now becomes physical parameter!

$\rightarrow$ power corrections, even at parton level!

Add running coupling to your muon shower. Replace α_{em} by α_s. Study the thrust distribution with fixed and running α as a function of Q_0.

Soft limit

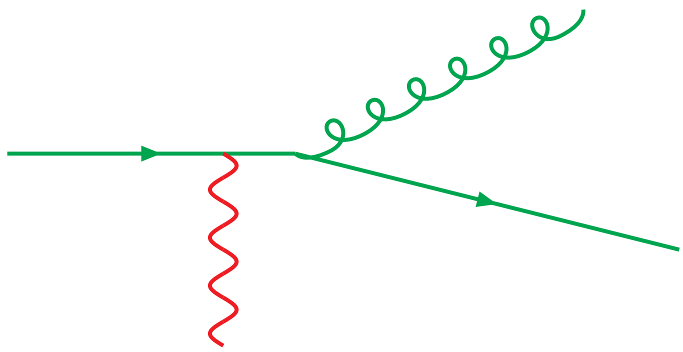
also universal. But at amplitude level. . .



soft gluon comes from everywhere in event.
⇒ Quantum interference.
Spoils independent evolution picture?

Angular ordering

No:



outside angular ordered cones, soft gluons sum coherently only see colour charge of whole jet.

Soft gluon effects fully incorporated by using θ^2 as evolution variable: angular ordering.

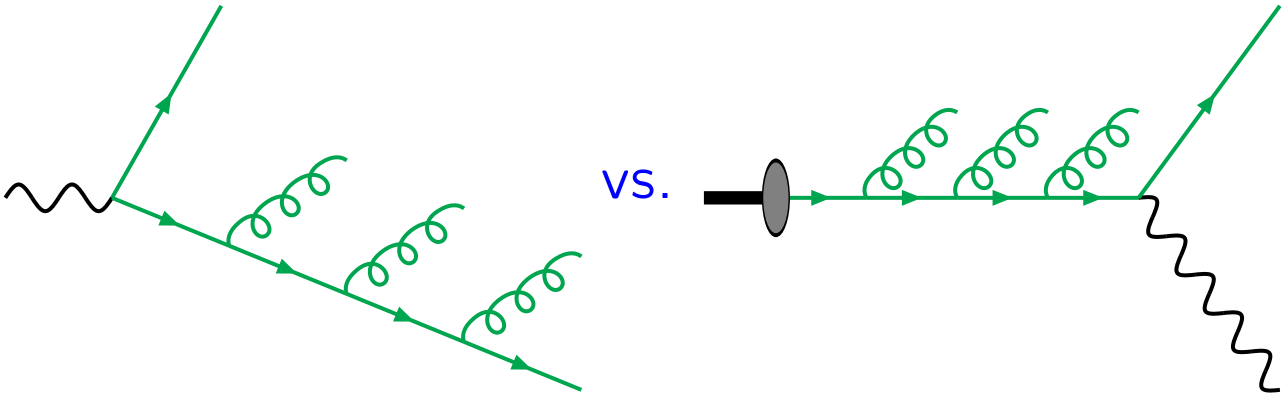
First gluon emission not necessarily hardest.

n.b. not same as using q^2 and vetoing non-ordered angles!

Initial state radiation

In principle identical to final state
(at least for not too small x)

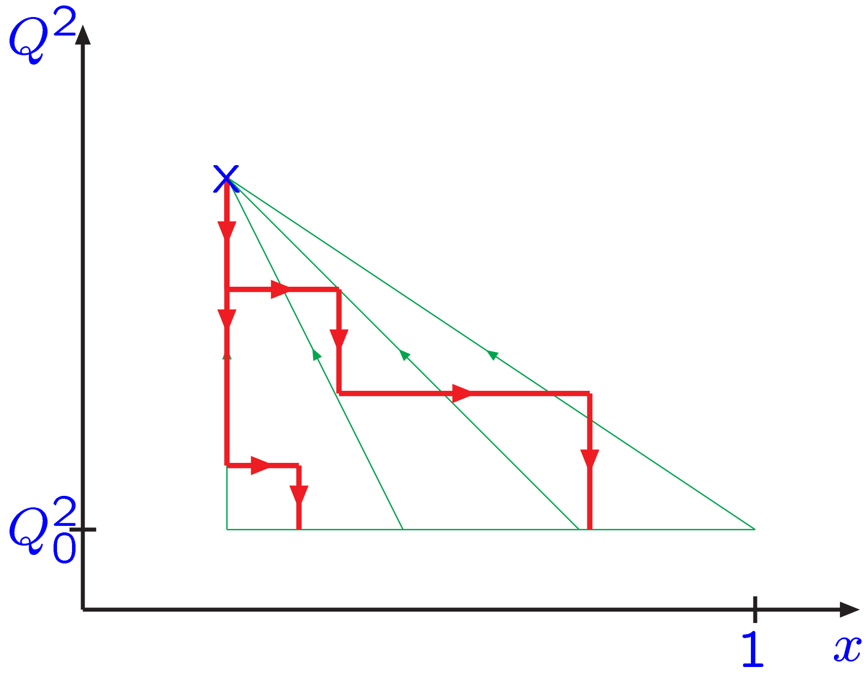
In practice different because both
ends of evolution fixed:



Use approach based on evolution equations. . .

Backward evolution

DGLAP evolution: pdfs at (x, Q^2) as a function of pdfs at $(> x, Q_0^2)$:



evolution paths sum over all possible events

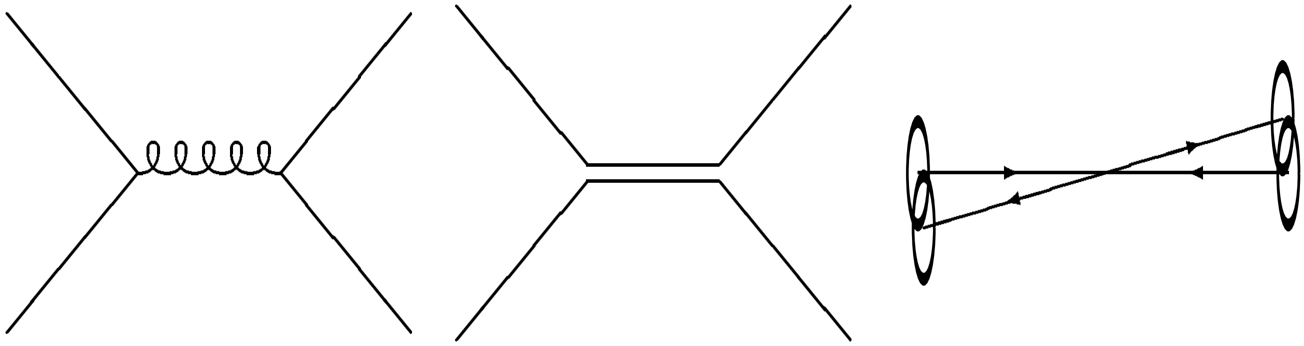
formulated as backward evolution: start from hard scattering, working down in q^2, up in x towards incoming hadron.

Algorithm identical to final-state, but with $\Delta_i(Q^2, q^2)$ replaced by $\Delta_i(Q^2, q^2)/f_i(x, q^2)$.

See Ellis, Stirling and Webber for proof.

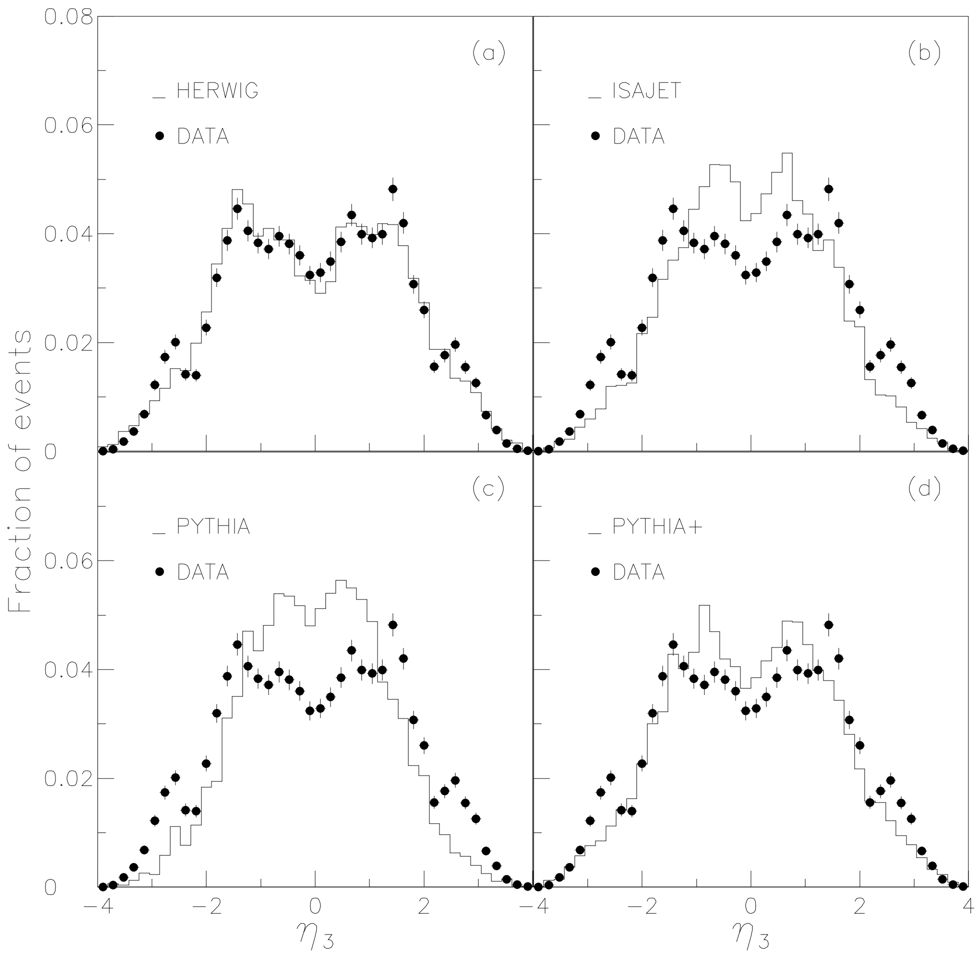
Hard scattering

Sets up initial conditions for parton showers.
Colour coherence important here too.



Emission from each parton confined to cone stretching to its colour partner.

Essential to fit Tevatron data...

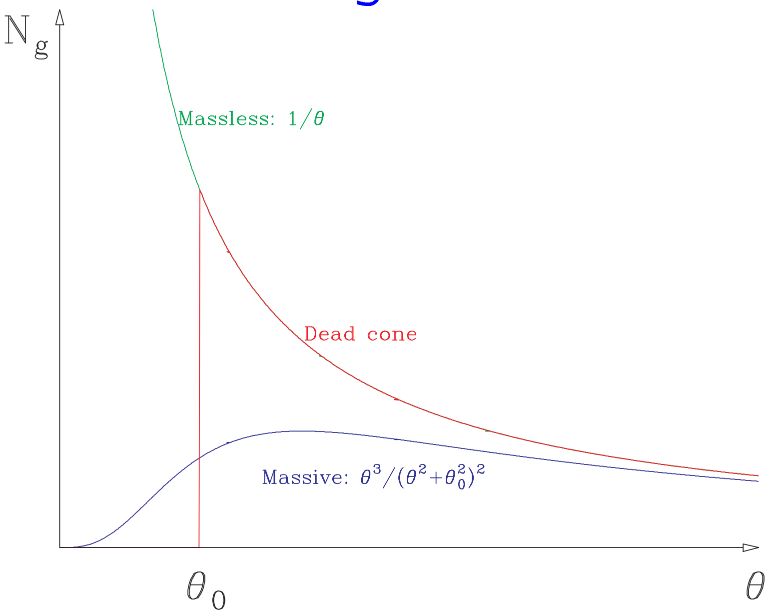


Rapidity distribution of the third-hardest jet in multi-jet events, in comparison with CDF data.

HERWIG has complete treatment of colour coherence, PYTHIA+ partial.

Heavy Quarks

look like light quarks at large angles
but sterile at small angles:



implemented as energy-dependent cutoff:

$$\theta > \theta_0 = \frac{m_q}{E_q}.$$

The 'dead cone'.
Too extreme?

Add gluon emission to your $q\bar{q} \to t\bar{t} \to b\bar{b}\ell\nu\ell'\nu'$ event generator. Why does this disagree with the calculation of Orr, Stelzer and Stirling???

The Colour Dipole Model

Conventional parton showers: start from collinear limit, modify to incorporate soft gluon coherence.

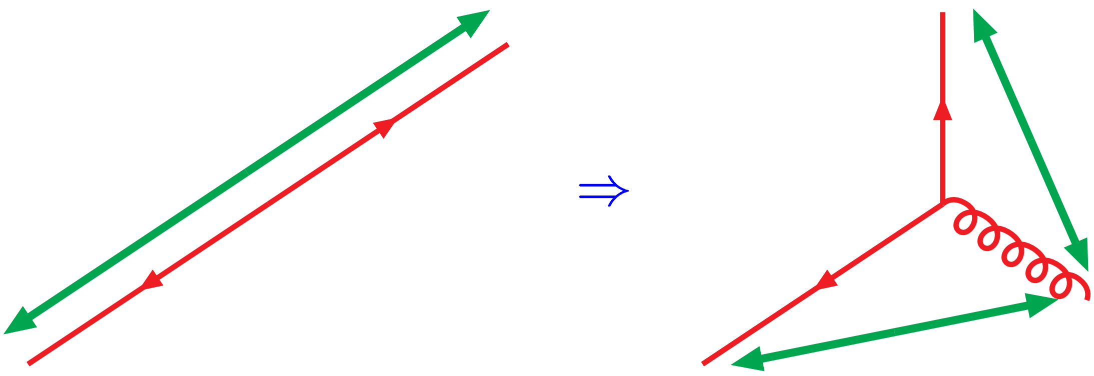
Colour Dipole Model: start from soft limit.

Emission of soft gluons from colour-anticolour dipole universal:

$$d\sigma \approx \sigma_0 \, \tfrac{1}{2} C_A \frac{\alpha_s(k_\perp)}{2\pi} \frac{dk_\perp^2}{k_\perp^2} \, dy,$$

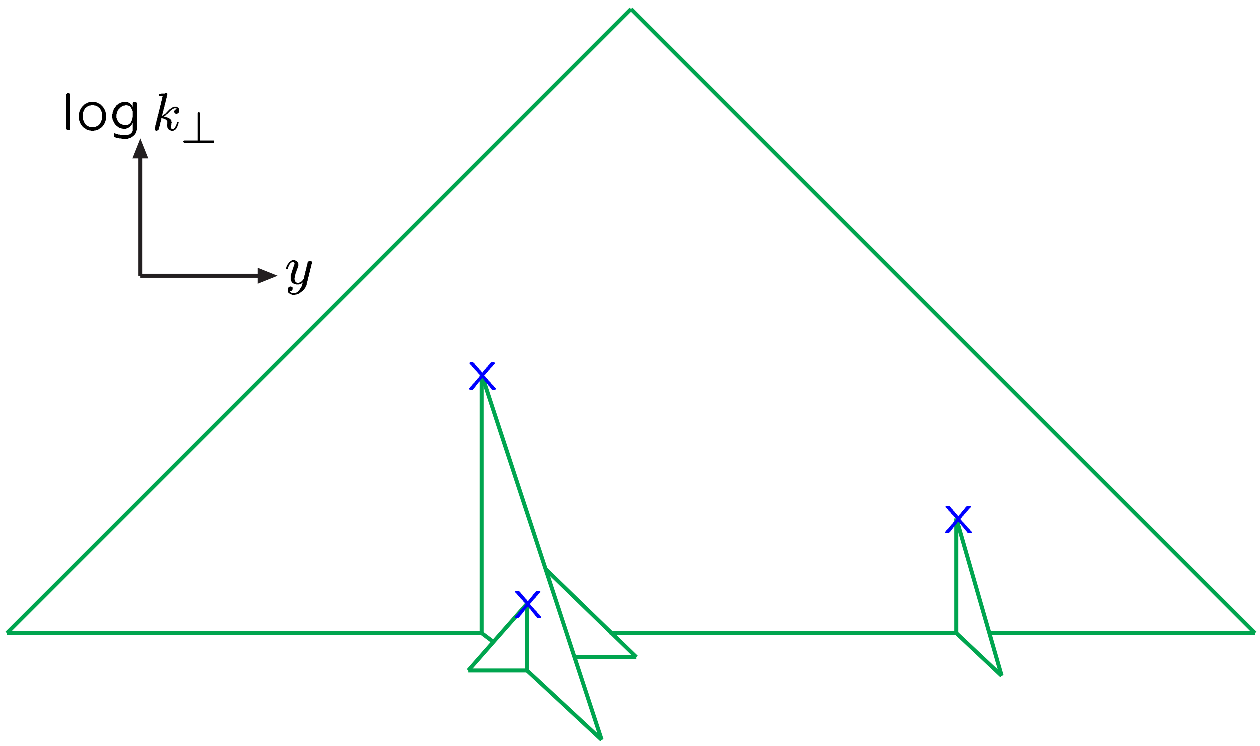
y = rapidity = $\log \tan \theta/2$

After emitting a gluon, colour dipole is split:



Subsequent dipoles continue to cascade.
c.f. parton shower: one parton $\rightarrow$ two
CDM: one dipole $\rightarrow$ two = two partons $\rightarrow$ three

Represented in 'origami diagram':

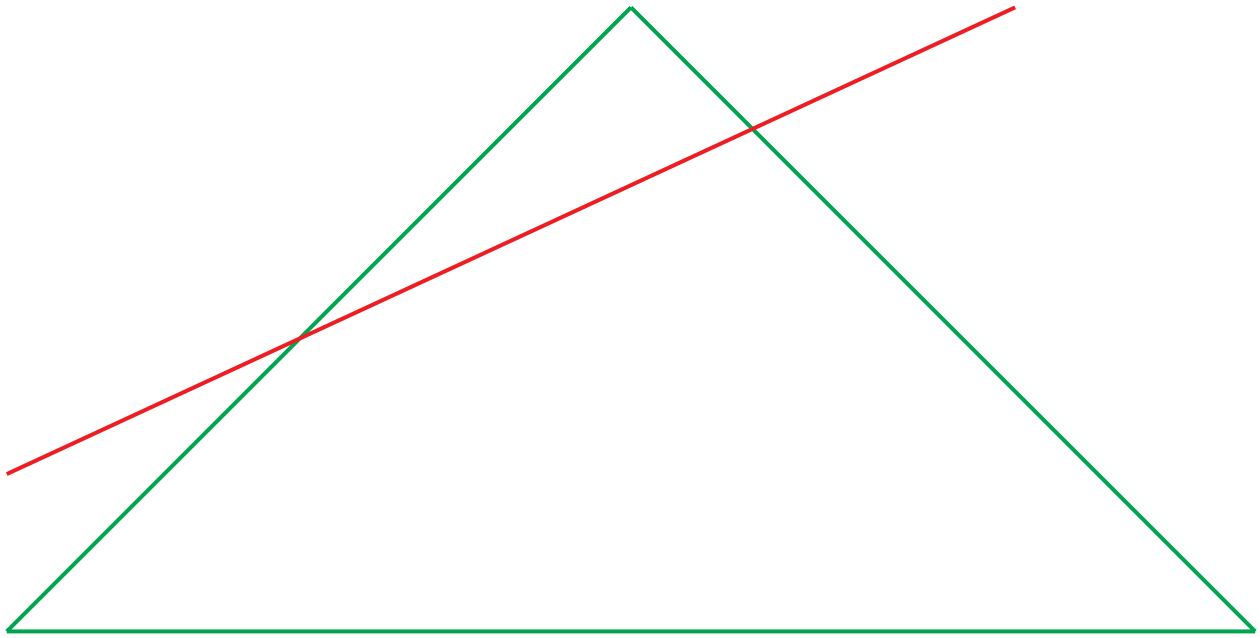


Similar to angular-ordered parton shower for e^+e^- annihilation.

Initial-state radiation in the CDM

There is none!

Hadron remnant forms colour dipole with scattered quark. Treated like any other dipole.
Except remnant is an extended object: phase space suppression.



Biggest difference relative to angular-ordered.
Much more radiation at small x.
Much better fit to HERA data.
Theoretically well-founded?

The Programs

ISAJET: q^2 ordering; no coherence; huge range of hard processes.

PYTHIA/LEPTO: q^2 ordering; veto of non-ordered final state emission; partial implementation of angular ordering in initial state; big range of hard processes.

HERWIG: complete implementation of colour coherence; NLO evolution for large x; smaller range of hard processes.

ARIADNE: complete implementation of colour dipole model; best fit to HERA data; interfaced to PYTHIA/LEPTO for hard processes.

Summary

Accelerated colour charges radiate gluons.
Gluons are also charged $\rightarrow$ cascade.

Probabilistic language derived from factorization theorems of full gauge theory.
Colour coherence is a fact of life: do not trust those who ignore it!

Modern parton shower models are very sophisticated implementations of perturbative QCD, but would be useless without hadronization models. . .