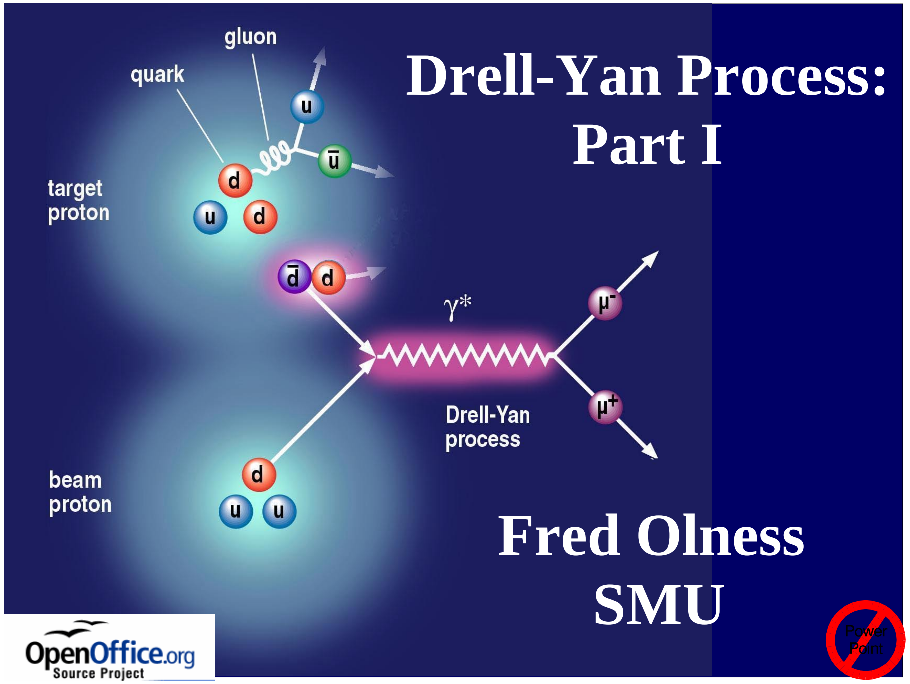


Historical Background

Part I: Drell-Yan Process

History:

Discovery of J/ψ, Upsilon, W/Z, and "New Physics" ???

Calculation of $q\ q \to \mu^+\mu^-$ in the Parton Model

Scaling form of the cross section

Rapidity, longitudinal momentum, and x_F

Comparison with data:

NLO QCD corrections essential (the K-factor)

σ(pd)/σ(pp) important for d-bar/ubar

W Rapidity Asymmetry important for slope of d/u at large x

Where are we going?

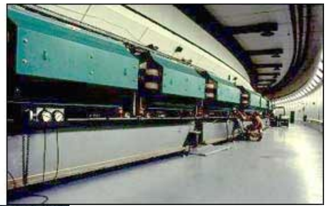
P_T Distribution

W-mass measurement

Resummation of soft gluons

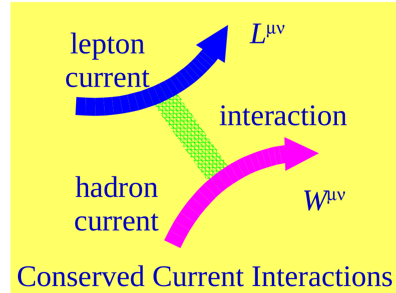
Our story begins in the late 1960's

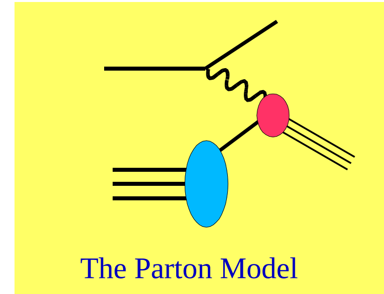
Brookhaven National Lab Alternating Gradient Synchrotron



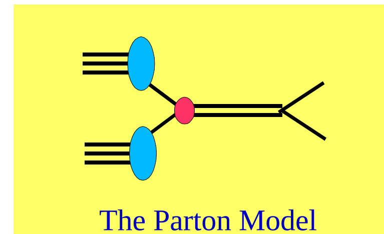
What is the explanation???

In DIS, we have two choices for an interpretation:





What about Drell-Yan???

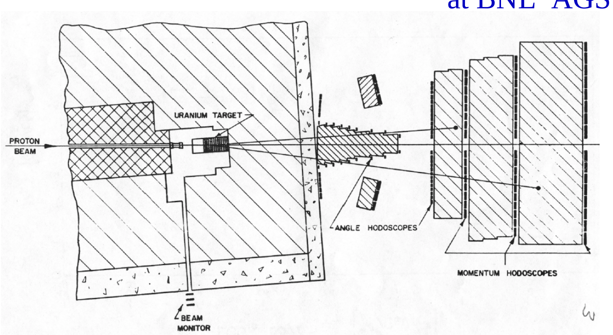


An Early Experiment:

The Goal: $p + N \rightarrow W + X$

They found: $p + N \rightarrow \mu^+ \mu^- + X$

at BNL AGS



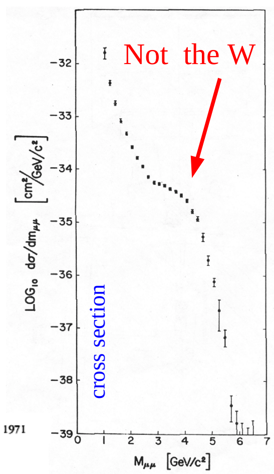
VOLUME 27, NUMBER 11 PHYSICAL REVIEW LETTERS 13 SEPTEMBER 1971

Production of Intermediate Bosons in Strong Interactions*

L. M. Lederman and B. G. Pope†
Columbia University, New York, New York 10533
(Received 14 June 1971)

Several searches for the weak intermediate boson (W) have been carried out using the reaction

$$p + Z \rightarrow W + \text{anything}, \quad (1)$$



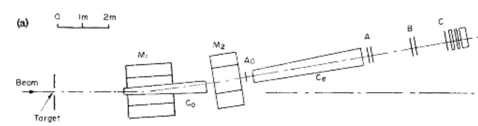
$M_{\mu\mu}$ GeV

Discovery of the J/Psi Particle

The Process: $p + Be \rightarrow e^+ e^- X$

very narrow width

$\Rightarrow$ long lifetime



at BNL AGS

VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECE

Experimental Observation of a Heavy Particle J†

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technolog
Cambridge, Massachusetts 02139

and

Y. Y. Lee
Brookhaven National Laboratory, Upton, New York 11973
(Received 12 November 1974)

We report the observation of a heavy particle J, with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + \mathrm{Be} \rightarrow e^+ + e^- + x$ by measuring the e^+e^- mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.

This experiment is part of a large program to ... daily with a thin Al foil. The beam spot

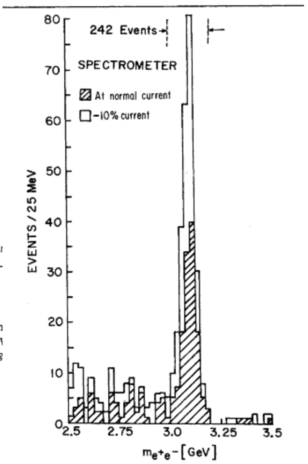
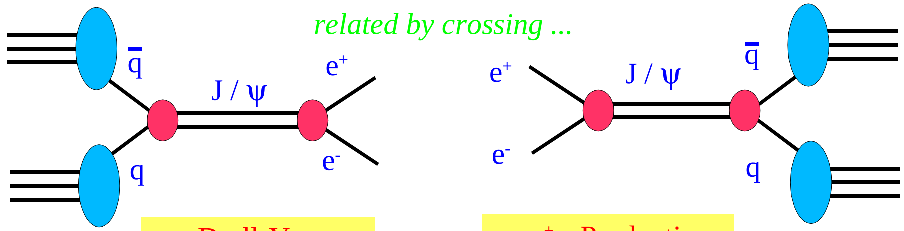


FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

The November Revolution

related by crossing ...



Drell-Yan
Brookhaven AGS

e^+e^- Production
SLAC SPEAR
Frascati ADONE

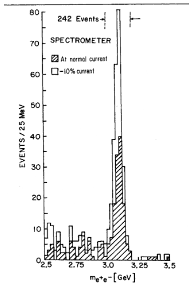
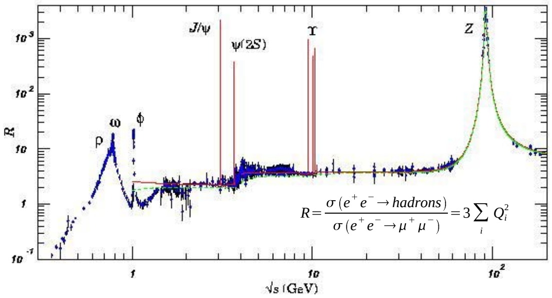
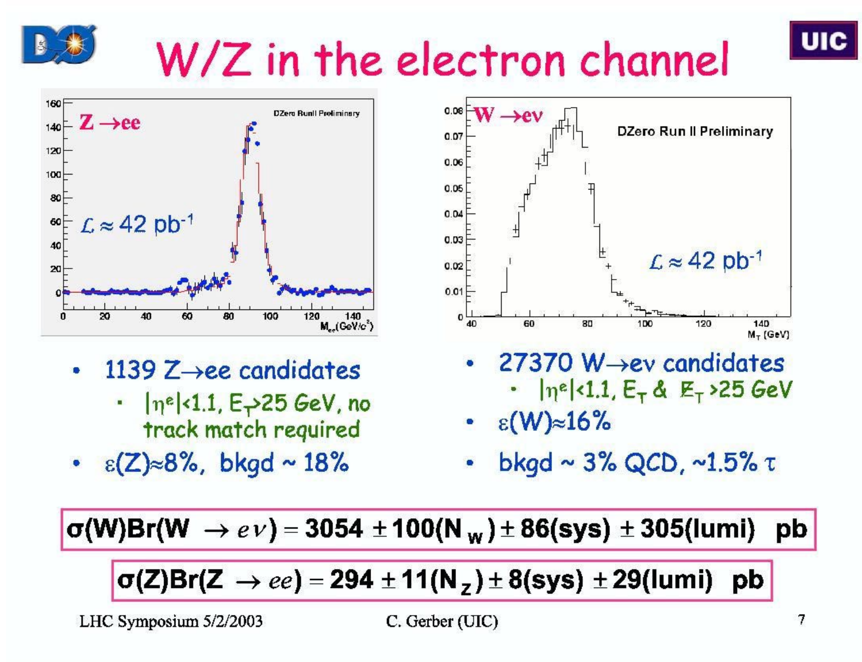


FIG. 2. Mass spectrum showing the existence of *J*. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.





More Discoveries with Drell-Yan

1974: The J/Psi (charm) discovery — $p+N \to J/\psi$

... 1976 Nobel Prize

1977: The Upsilon (bottom) discovery — $p+N \to \Upsilon$

1983: The W and Z discovery — $p + \bar{p} \to W/Z$

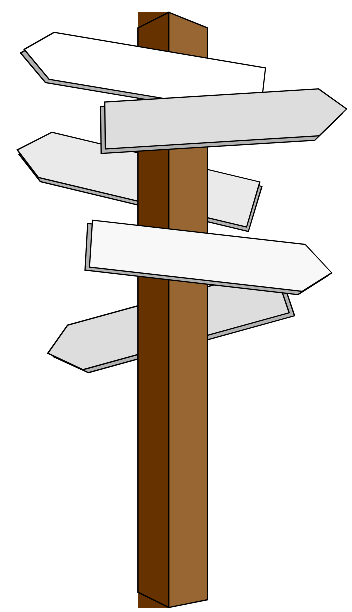
... 1984 Nobel Prize

The Future of Drell-Yan

Where do we find

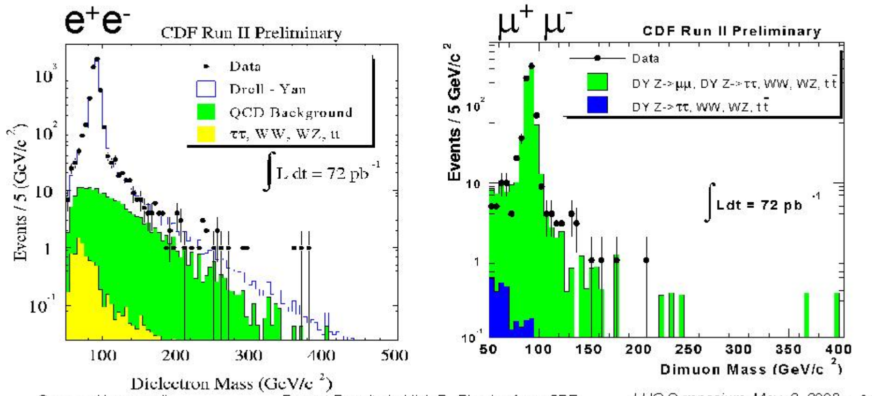
New Physics??

- New Higgs Bosons
- New W' or Z'
- SUSY
- ... unknown...



Search in Drell-Yan Spectrum

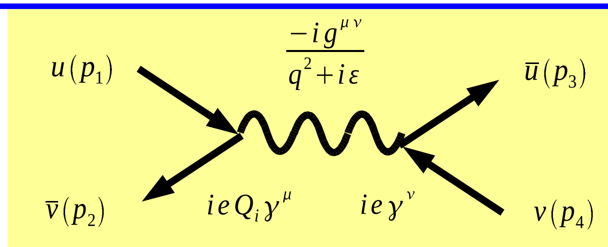
- High Mass Dileptons
  - electrons & muons used
- Sensitive to Z' and Randall-Sundrum Graviton
- No excess observed



First, we'll compute the partonic $\hat{\sigma}$ in the partonic CMS

Let's Calculate

Let's compute the Born process: $q+\bar{q}\rightarrow e^{+}+e^{-}$



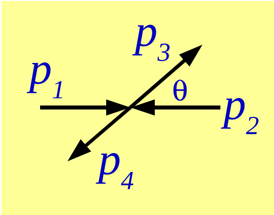
Gathering factors and contracting $g^{\mu\nu}$, we obtain:

$$-i M = i Q_i \frac{e^2}{q^2}\{\bar{v}(p_2)\gamma^\mu u(p_1)\}\ \{\bar{u}(p_3)\gamma_\mu v(p_4)\}$$

Squaring, and averaging over spin and color, ...

$$\overline{|M|^2} = \left(\frac{1}{2}\right)^2 3\left(\frac{1}{3}\right)^2 Q_i^2\ \frac{e^4}{q^4}\ Tr\left[\not{p}_2\gamma^\mu \not{p}_1\gamma^\nu\right]\ Tr\left[\not{p}_3\gamma_\mu \not{p}_4\gamma_\nu\right]$$

Let's work out some parton level kinematics



$$p_1 = \frac{\sqrt{\hat{s}}}{2}(1,0,0,+1)$$
$$p_2 = \frac{\sqrt{\hat{s}}}{2}(1,0,0,-1)$$
$$p_3 = \frac{\sqrt{\hat{s}}}{2}\left(1,+\sin(\theta),0,+\cos(\theta)\right)$$
$$p_4 = \frac{\sqrt{\hat{s}}}{2}\left(1,-\sin(\theta),0,-\cos(\theta)\right)$$

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$$

Defining the Mandelstam variables ...

$$\hat{s} = (p_1+p_2)^2 = (p_3+p_4)^2$$
$$\hat{t} = (p_1-p_3)^2 = (p_2-p_4)^2$$
$$\hat{u} = (p_1-p_4)^2 = (p_2-p_3)^2$$

$$\hat{t} = -\frac{\hat{s}}{2}\left(1-\cos(\theta)\right)$$
$$\hat{u} = -\frac{\hat{s}}{2}\left(1+\cos(\theta)\right)$$

... and put it together to find the cross section

$$d\hat{\sigma} \simeq \frac{1}{2\hat{s}}\overline{|M|^2}\, d\Gamma$$

In the partonic CMS system

$$d\Gamma = \frac{d^3 p_3}{(2\pi)^3 2E_3}\frac{d^3 p_4}{(2\pi)^3 2E_4}(2\pi)^4\,\delta(p_1+p_2-p_3-p_4) = \frac{d\cos(\theta)}{16\pi}$$

Recall,

$$\hat{t} = \frac{-\hat{s}}{2}\left(1-\cos(\theta)\right) \quad and \quad \hat{u} = \frac{-\hat{s}}{2}\left(1+\cos(\theta)\right)$$

so, the differential cross section is ...

$$\frac{d\hat{\sigma}}{d\cos(\theta)} = Q_i^2\,\alpha^2\,\frac{\pi}{6}\,\frac{1}{\hat{s}}\left(1+\cos^2(\theta)\right)$$

and the total cross section is ...

$$\hat{\sigma} = Q_i^2\,\alpha^2\,\frac{\pi}{6}\,\frac{1}{\hat{s}}\int_{-1}^{1} d\cos(\theta)\left(1+\cos^2(\theta)\right) = \frac{4\pi\alpha^2}{9\hat{s}}Q_i^2 \equiv \hat{\sigma}_0$$

We'll now compute the matrix element M

Manipulating the traces, we find ...

$$Tr\left[\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\right]\ Tr\left[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu\right]$$
$$= 4\left[p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - g^{\mu\nu}(p_1\cdot p_2)\right]\times 4\left[p_3^\mu p_4^\nu + p_4^\mu p_3^\nu - g^{\mu\nu}(p_3\cdot p_4)\right]$$
$$= 2^5\left[(p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3)\right]$$
$$= 2^3\left[\hat{t}^2 + \hat{u}^2\right]$$

Where we have used:

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$$

$$\hat{s} = 2(p_1\cdot p_2) = 2(p_3\cdot p_4)$$
$$\hat{t} = 2(p_1\cdot p_3) = 2(p_2\cdot p_4)$$
$$\hat{u} = 2(p_1\cdot p_4) = 2(p_2\cdot p_3)$$

Putting all the pieces together, we have:

$$\overline{|M|^2} = Q_i^2\,\alpha^2\,\frac{2^5\pi^2}{3}\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)$$

with

$$q^2 = (p_1+p_2)^2 = \hat{s}$$
$$\alpha = \frac{e^2}{4\pi}$$

Some Homework:

#1) Show:

$$\frac{d^3 p}{(2\pi)^3 2E} = \frac{d^4 p}{(2\pi)^4}(2\pi)\,\delta^+(p^2-m^2)$$

This relation is often useful as the RHS is manifestly Lorentz invariant

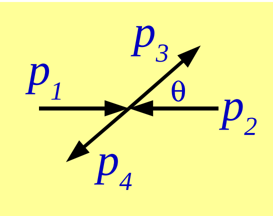
#2) Show that the 2-body phase space can be expressed as:

$$d\Gamma = \frac{d^3 p_3}{(2\pi)^3 2E_3}\frac{d^3 p_4}{(2\pi)^3 2E_4}(2\pi)^4\,\delta(p_1+p_2-p_3-p_4) = \frac{d\cos(\theta)}{16\pi}$$

Note, we are working with massless partons, and θ is in the partonic CMS frame

Some More Homework:

#3) Let's work out the general 2→2 kinematics for general masses.



a) Start with the incoming particles.
Show that these can be written in the general form:

$$p_1 = (E_1, 0, 0, +p) \qquad p_1^2 = m_1^2$$
$$p_2 = (E_2, 0, 0, -p) \qquad p_2^2 = m_2^2$$

... with the following definitions:

$$E_{1,2} = \frac{\hat{s} \pm m_1^2 \mp m_2^2}{2\sqrt{\hat{s}}} \qquad p = \frac{\Delta(\hat{s}, m_1^2, m_2^2)}{2\sqrt{\hat{s}}}$$

$$\Delta(a,b,c) = \sqrt{a^2+b^2+c^2-2(ab+bc+ca)}$$

Note that $\Delta(a,b,c)$ is symmetric with respect to its arguments, and involves the only invariants of the initial state: s, m_1^2, m_2^2.

b) Next, compute the general form for the final state particles, p_3 and p_4. Do this by first aligning p_3 and p_4 along the z-axis (as p_1 and p_2 are), and then rotate about the y-axis by angle θ.

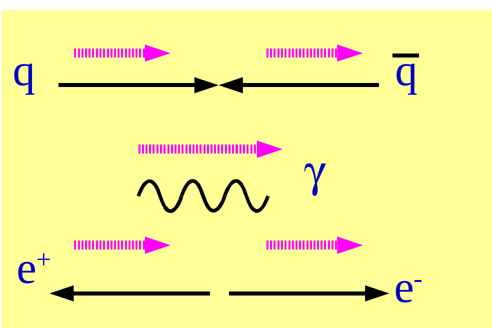
Next, we'll compute the hadronic CMS

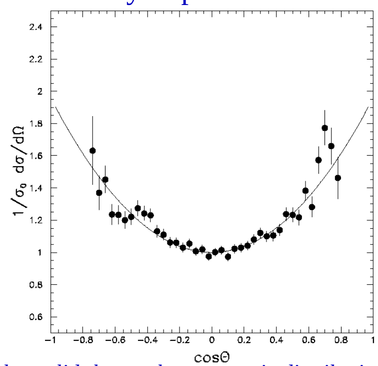
What does the angular dependence tell us?

Observe, the angular dependence: $q + \bar{q} \to e^+ + e^-$

$$\frac{d\hat{\sigma}}{d\cos(\theta)} = Q_i^2\, \alpha^2\, \frac{\pi}{6}\, \frac{1}{\hat{s}} \left(1+\cos^2(\theta)\right)$$

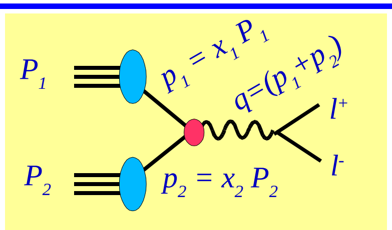
Characteristic of scattering of spin ½ constitutients by a spin 1 vector





Note, for the photon, the mirror image of the above is also valid; hence the symmetric distribution.
The W has V-A couplings, so we'll find: $(1+\cos\theta)^2$

Kinematics in the Hadronic Frame



$$P_1 = \frac{\sqrt{s}}{2}(1,0,0,+1) \qquad P_1^2 = 0$$
$$P_2 = \frac{\sqrt{s}}{2}(1,0,0,-1) \qquad P_2^2 = 0$$

$$s = (P_1+P_2)^2 = \frac{\hat{s}}{x_1 x_2} = \frac{\hat{s}}{\tau}$$

Therefore $\tau = x_1 x_2 = \frac{\hat{s}}{s} \equiv \frac{Q^2}{s}$

Fractional energy² between partonic and hadronic system

$$\frac{d\sigma}{dQ^2} = \sum_{q,\bar{q}} \int dx_1 \int dx_2 \left\{ q(x_1)\bar{q}(x_2) + \bar{q}(x_1) q(x_2) \right\} \hat{\sigma}_0\, \delta(Q^2 - \hat{s})$$

Hadronic cross section

Parton distribution functions

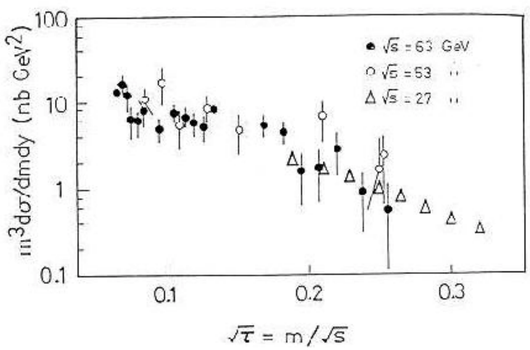
Partonic cross section

Scaling form of the Drell-Yan Cross Section

Using: $\hat{\sigma}_0 = \frac{4\pi\alpha^2}{9\hat{s}} Q_i^2$ and $\delta(Q^2 - \hat{s}) = \frac{1}{s\,x_1}\,\delta\!\left(x_2 - \frac{\tau}{x_1}\right)$

we can write the cross section in the scaling form:

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9} \sum_{q,\bar{q}} Q_i^2 \int_{\tau}^{1} \frac{dx_1}{x_1}\, \tau \left\{ q(x_1)\bar{q}(\tau/x_1) + \bar{q}(x_1) q(\tau/x_1) \right\}$$



Notice the RHS is a function of only τ, not Q.

This quantity should lie on a universal scaling curve.

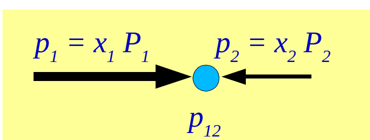
Cf., DIS case, & scattering of point-like constituents

So, we're ready to compare with data

(or so we think...)

Longitudinal Momentum Distributions

Partonic CMS has longitudinal momentum w.r.t. the hadron frame



$$p_{12} = (p_1 + p_2) = (E_{12}, 0, 0, p_L)$$

$$E_{12} = \frac{\sqrt{s}}{2}(x_1 + x_2)$$

$$p_L = \frac{\sqrt{s}}{2}(x_1 - x_2) \equiv \frac{\sqrt{s}}{2} x_F$$

x_F is a measure of the longitudinal momentum

The rapidity is defined as:

$$y = \frac{1}{2} \ln\left\{\frac{E_{12} + p_L}{E_{12} - p_L}\right\} = \frac{1}{2} \ln\left\{\frac{x_1}{x_2}\right\}$$

$$x_{1,2} = \sqrt{\tau}\, e^{\pm y}$$

$$d x_1 d x_2 = d\tau\, dy$$

$$d Q^2 dx_F = dy\, d\tau\; s\, \sqrt{x_F^2 + 4\tau}$$

$$\frac{d\sigma}{dQ^2\, dx_F} = \frac{4\pi\alpha^2}{9 Q^4} \frac{1}{\sqrt{x_F^2 + 4\tau}}\, \tau \sum_{q,\bar{q}} Q_i^2 \left\{ q(x_1)\bar{q}(\tau/x_1) + \bar{q}(x_1) q(\tau/x_1) \right\}$$

Let's compare data and theory

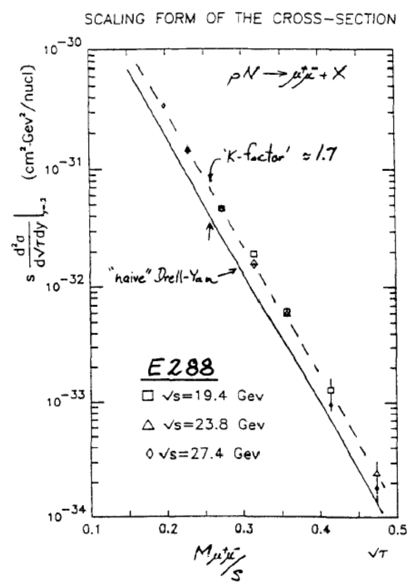


Table 1.2: Experimental K-factors.

Experiment		Interaction	Beam Momentum	$K = \sigma_{\text{meas.}}/\sigma_{\text{DY}}$
E288	[Kap 78]	$p\ Pt$	300/400 GeV	~ 1.7
WA39	[Cor 80]	$\pi^\pm\ W$	39.5 GeV	~ 2.5
E439	[Smi 81]	$p\ W$	400 GeV	1.6 ± 0.3
NA3	[Bad 83]	$(\bar{p}$ - $p)Pt$	150 GeV	2.3 ± 0.4
		$p\ Pt$	400 GeV	$3.1 \pm 0.5 \pm 0.3$
		$\pi^\pm\ Pt$	200 GeV	2.3 ± 0.5
		$\pi^-\ Pt$	150 GeV	2.49 ± 0.37
		$\pi^-\ Pt$	280 GeV	2.22 ± 0.33
NA10	[Bet 85]	$\pi^-\ W$	194 GeV	$\sim 2.77 \pm 0.12$
E326	[Gre 85]	$\pi^-\ W$	225 GeV	$2.70 \pm 0.08 \pm 0.40$
E537	[Ana 88]	$\bar{p}\ W$	125 GeV	$2.45 \pm 0.12 \pm 0.20$
E615	[Con 89]	$\pi^-\ W$	252 GeV	1.78 ± 0.06

J. C. Webb, Measurement of continuum dimuon production in 800-GeV/c proton nucleon collisions, arXiv:hep-ex/0301031.

Oooops,

we need the

QCD corrections

$$K = 1 + \frac{2\pi\alpha_s}{3}(\ldots) + \ldots = ?= e^{2\pi\alpha_s/3}$$

Drell-Yan can give us unique and detailed information about PDF's.

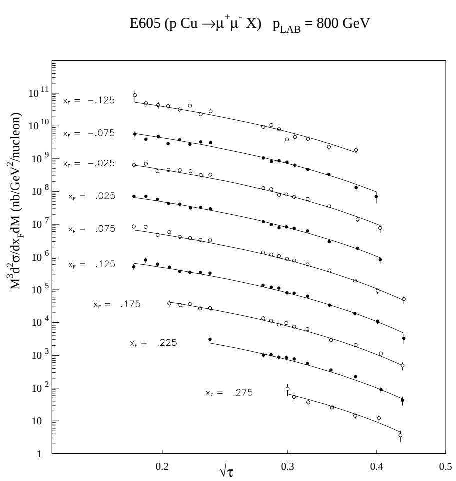
We'll now examine two examples:

1) Ratio of pp/pd cross section

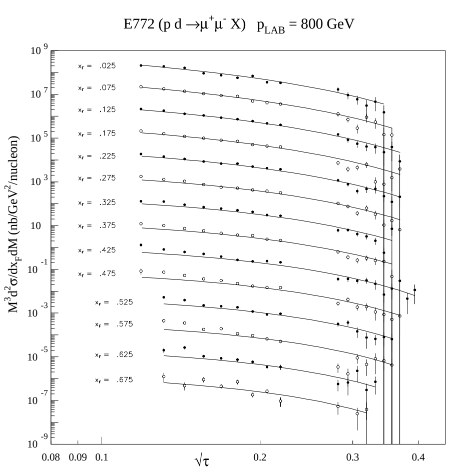
2) W Rapidity Asymmetry

Excellent agreement between data and theory

p + Cu at 800 GeV



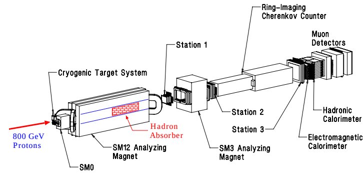
p + d at 800 GeV



pp & pN processes sensitive to anti-quark distributions

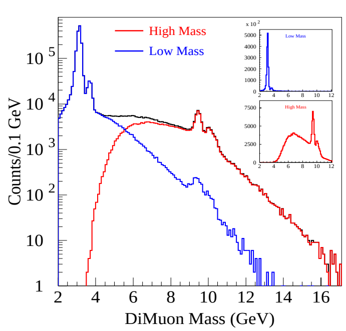
A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne,
Eur. Phys. J. C23, 73 (2002);
Eur. Phys. J. C14, 133 (2000);
Eur. Phys. J. C4, 463 (1998)

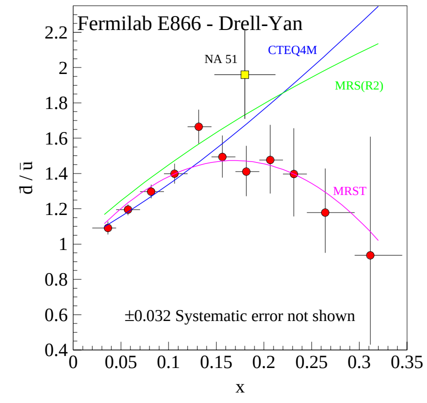
A measurement of $\bar{d}(x)/\bar{u}(x)$
Antiquark asymmetry in the Nucleon Sea
FNAL E866/NuSea



ACU, ANL, FNAL, GSU, IIT, LANL, LSU, NMSU, UNM, ORNL, TAMU, Valpo.

800 GeV $p+p$ and $p+d \to \mu^+\mu^- X$





Cross section ratio of pp *vs.* pd

Obtain the neutron PDF via isospin symmetry:

$$u \Leftrightarrow d$$
$$\bar{u} \Leftrightarrow \bar{d}$$

In the limit $x_1 >> x_2$:

$$\sigma^{pp} \propto \frac{4}{9} u(x_1)\bar{u}(x_2) + \frac{1}{9} d(x_1)\bar{d}(x_2)$$

$$\sigma^{pn} \propto \frac{4}{9} u(x_1)\bar{d}(x_2) + \frac{1}{9} d(x_1)\bar{u}(x_2)$$

For the ratio we have:

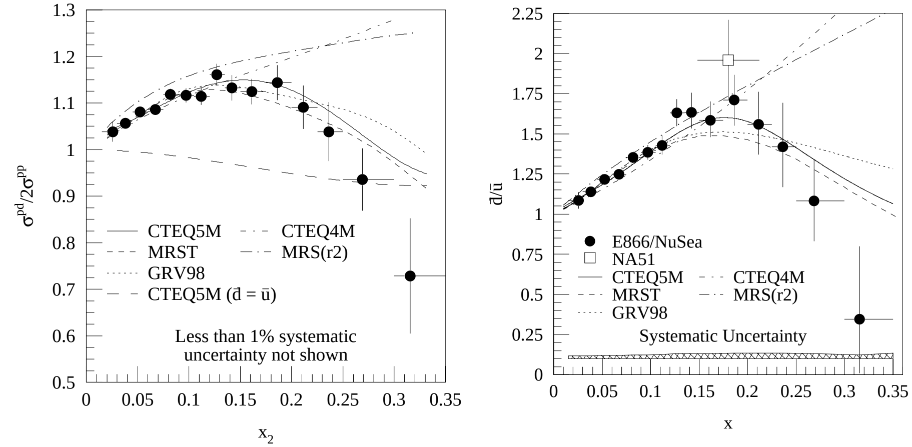
$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2} \frac{\left(1+\frac{1}{4}\frac{d_1}{u_1}\right)}{\left(1+\frac{1}{4}\frac{d_1}{u_1}\frac{\bar{d}_2}{\bar{u}_2}\right)} \left(1+\frac{\bar{d}_2}{\bar{u}_2}\right) \approx \frac{1}{2}\left(1+\frac{\bar{d}_2}{\bar{u}_2}\right)$$

As promised, this provides information about the sea-quark distributions

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2}\left(1+\frac{\bar{d}_2}{\bar{u}_2}\right)$$

EXERCISE: Verify the above.

E866 required significant changes in the hi-x sea distributions



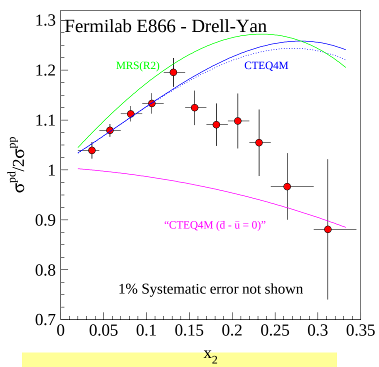
With increased flexibility in the parameterization of the sea-quark distributions, good fits are obtained

E.A. Hawker, et al. [FNAL E866/NuSea Collaboration], Measurement of the light antiquark flavor asymmetry in the nucleon sea, PRL 80, 3715 (1998)

H. L. Lai, et al.} [CTEQ Collaboration], Global {QCD} analysis of parton structure of the nucleon: CTEQ5 parton distributions, EPJ C12, 375 (2000)

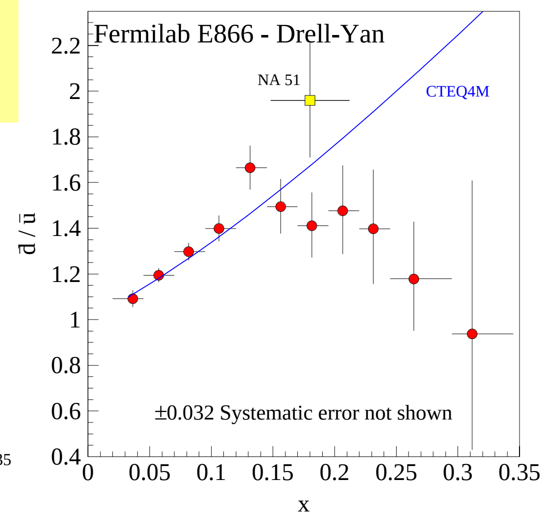
Does the theory match the data???

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2}\left(1+\frac{\bar{d}_2}{\bar{u}_2}\right)$$



Implies R<1 for large x:

$$\bar{d} \ll \bar{u}$$



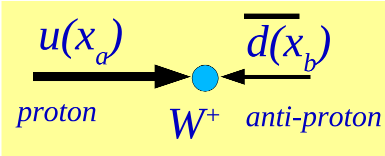
E.A. Hawker, et al. [FNAL E866/NuSea Collaboration], Measurement of the light antiquark flavor asymmetry in the nucleon sea, PRL 80, 3715 (1998)

Next ...

2) W Rapidity Asymmetry

Where do the W's and Z's come from ???

$$\frac{d\sigma}{dy}(W^{\pm}) = \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}\sum_{q\bar{q}}|V_{q\bar{q}}|^2\left[q(x_a)\,\bar{q}(x_b) + q(x_b)\,\bar{q}(x_a)\right]$$



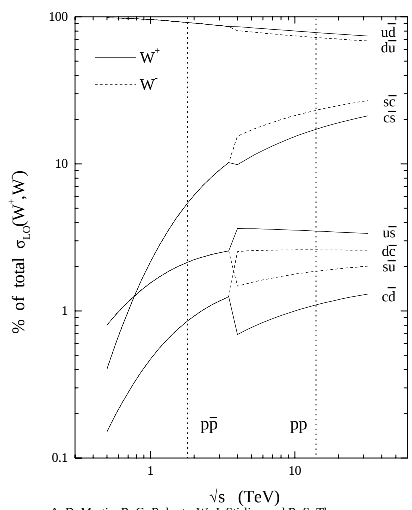
For anti-proton:

$$u(x) \Leftrightarrow \bar{u}(x) \qquad d(x) \Leftrightarrow \bar{d}(x)$$

Therefore

$$\frac{d\sigma}{dy}(W^{+}) \approx \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}\left[u(x_a)\,d(x_b)\right]$$
$$\frac{d\sigma}{dy}(W^{-}) \approx \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}\left[d(x_a)\,u(x_b)\right]$$

flavour decomposition of W cross sections

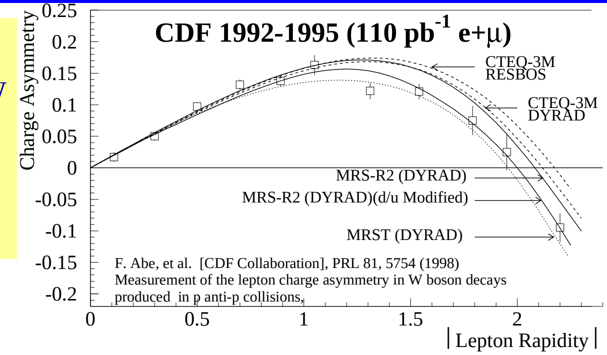


A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C23, 73 (2002); Eur. Phys. J. C4, 463 (1998)

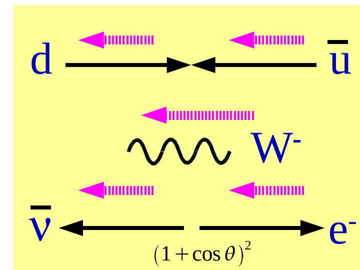
Charged Lepton Asymmetry

Unfortunately, we don't measure the W directly since W→eν.

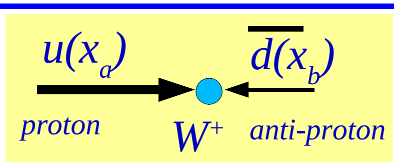
Still the lepton contains important information



$$A(y) = \frac{\frac{d\sigma}{dy}(l^{+}) - \frac{d\sigma}{dy}(l^{-})}{\frac{d\sigma}{dy}(l^{+}) + \frac{d\sigma}{dy}(l^{-})}$$



A bit of calculation



$$A(y) = \frac{\frac{d\sigma}{dy}(W^{+}) - \frac{d\sigma}{dy}(W^{-})}{\frac{d\sigma}{dy}(W^{+}) + \frac{d\sigma}{dy}(W^{-})}$$

With the previous approximation,

$$A \approx \frac{u(x_a)d(x_b) - d(x_a)u(x_b)}{u(x_a)d(x_b) + d(x_a)u(x_b)} = \frac{R_{du}(x_b) - R_{du}(x_a)}{R_{du}(x_b) + R_{du}(x_a)}$$

where $R_{du}(x) = \frac{d(x)}{u(x)}$

We can make Taylor expansions:

$$x_{1,2} = x_0 e^{\pm y} \simeq x_0(1 \pm y)$$

$$R_{du}(x_{1,2}) \approx R_{du}(x_0) \pm y\, x_0\, R'_{du}(\sqrt{\tau})$$

Thus, the asymmetry is:

$$A(y) = -y\, x_0 \frac{R'_{du}(x_0)}{R_{du}(x_0)}$$

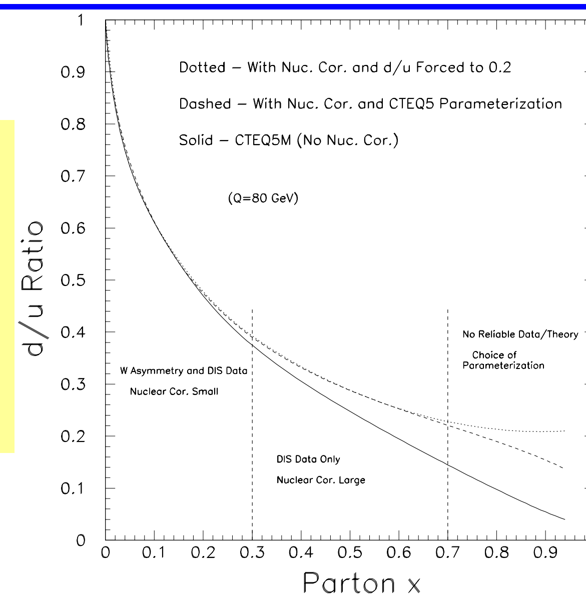
EXERCISE: Verify the above.

d/u Ratio at High-x

The form of the d/u ratio at large x as a function of

1) Parameterization

2) Nuclear Corrections



S. Kuhlmann, et al., Large-x parton distributions, PL B476, 291 (2000)

End of Part I: Where have we been???

History:

Discovery of J/ψ, Upsilon, W/Z, and "New Physics" ???

Calculation of $q\,q \to \mu^+\mu^-$ in the Parton Model

Scaling form of the cross section

Rapidity, longitudinal momentum, and x_F

Comparison with data:

NLO QCD corrections essential (the K-factor)

σ(pd)/σ(pp) important for d-bar/ubar

W Rapidity Asymmetry important for slope of d/u at large x

Where are we going?

P_T Distribution

W-mass measurement

Resummation of soft gluons

Part II: W Boson Production as an example

Finding the W Boson Mass:

The Jacobian Peak, and the W Boson P_T

Multiple Soft Gluon Emissions

Single Hard Gluon Emission

Road map of Resummation

Summing 2 logs per loop: multi-scale problem (Q,q_T)

Correlated Gluon Emission

Non-Perturbative physics at small q_T.

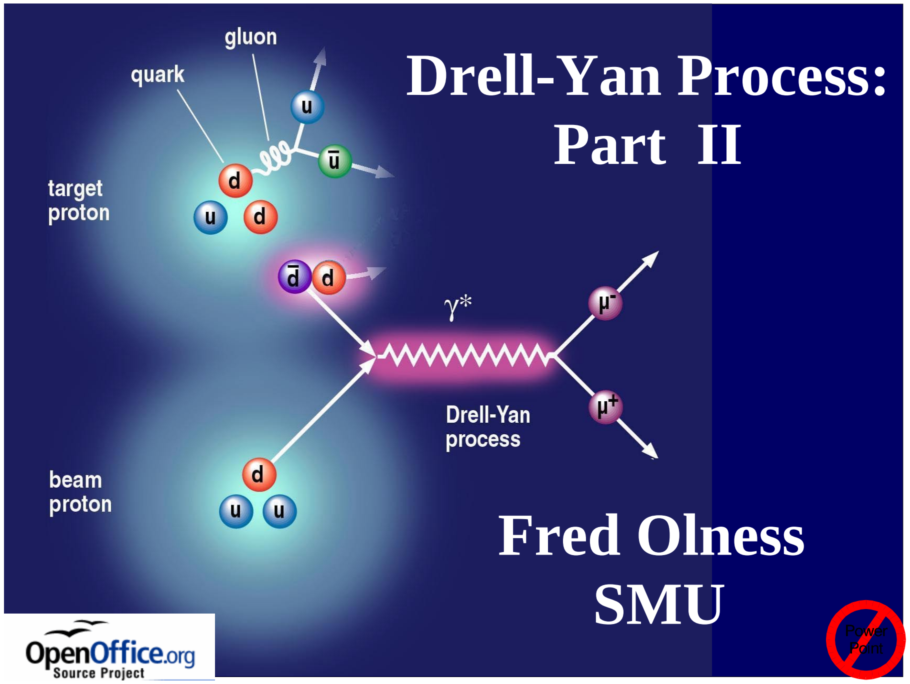
Transverse Mass Distribution:

Improvement over P_T distribution

What can we expect in future?

Tevatron Run II

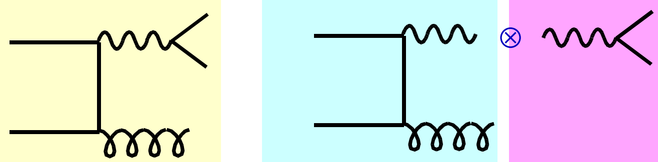
LHC



Side Note: From $pp \to \gamma/Z/W$, we can obtain $pp \to \gamma/Z/W \to l^+l^-$

Schematically:

$$d\sigma(q\bar{q} \to l^+ l^- g) = d\sigma(q\bar{q} \to \gamma^* g) \times d\sigma(\gamma^* \to l^+ l^-)$$



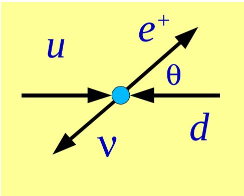
For example:

$$\frac{d\sigma}{dQ^2\, d\hat{t}}(q\bar{q} \to l^+ l^- g) = \frac{d\sigma}{d\hat{t}}(q\bar{q} \to \gamma^* g) \times \frac{\alpha}{3\pi Q^2}$$

Part II: W Boson Production as an example

How do we measure the W-boson mass?

$$u + \bar{d} \rightarrow W^{+} \rightarrow e^{+} \nu$$



- Can't measure W directly
- Can't measure ν directly
- Can't measure longitudinal momentum

We can measure the P_T of the lepton

How can we use this to extract the W-Mass???

The Jacobian Peak

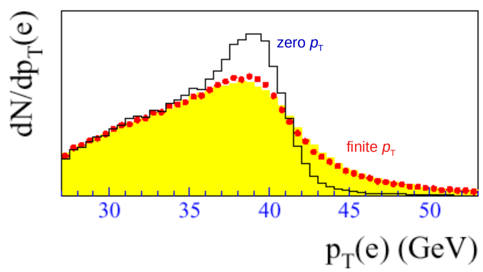
Now that we've got the picture, here's the math ... *(in the W CMS frame)*

$$p_T^2 = \frac{\hat{s}}{4}\sin^2\theta \qquad \cos\theta = \sqrt{1 - \frac{4p_T^2}{\hat{s}}} \qquad \frac{d\cos\theta}{dp_T^2} = \frac{2}{\hat{s}}\frac{1}{\cos\theta}$$

So we discover the P_T distribution has a singularity at cosθ=0, or θ=π/2

$$\frac{d\sigma}{dp_T^2} = \frac{d\sigma}{d\cos\theta} \times \frac{d\cos\theta}{dp_T^2} \approx \frac{d\sigma}{d\cos\theta} \times \frac{1}{\cos\theta}$$

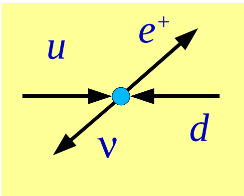
singularity!!!



BUT !!!

Measuring the Jacobian peak is complicated if the W boson has finite P_T.

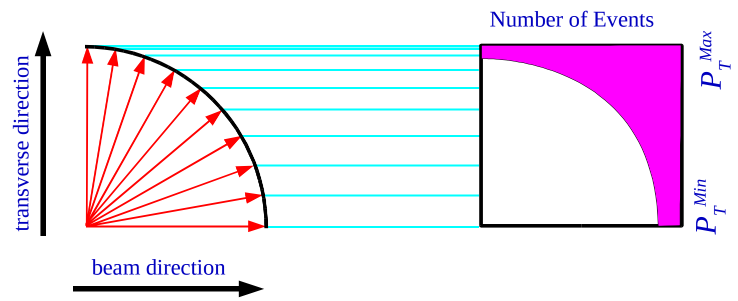
The Jacobian Peak



Suppose lepton distribution is uniform in θ

The dependence is actually $(1+\cos\theta)^2$, but we'll take care of that later

What is the distribution in P_T?

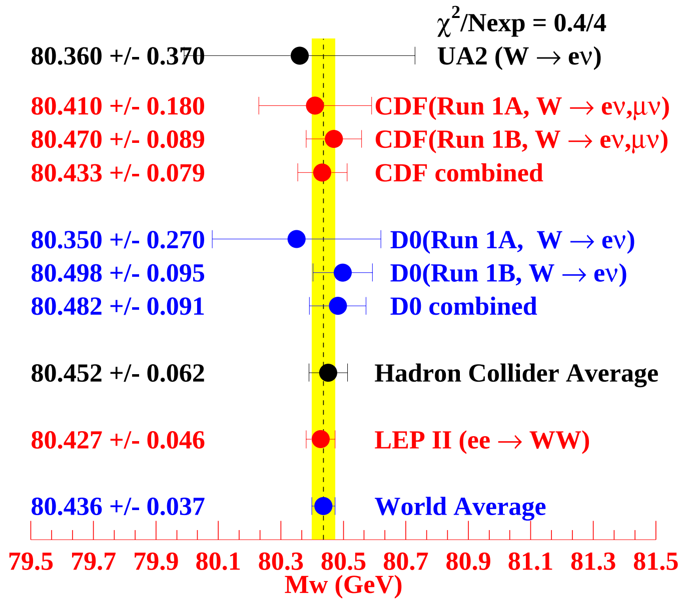


We find a peak at $P_T^{max} \approx M_W/2$

1) The W-mass is important fundamental quantity of the Standard Model

2) P_T Distribution is important for measuring the W-mass

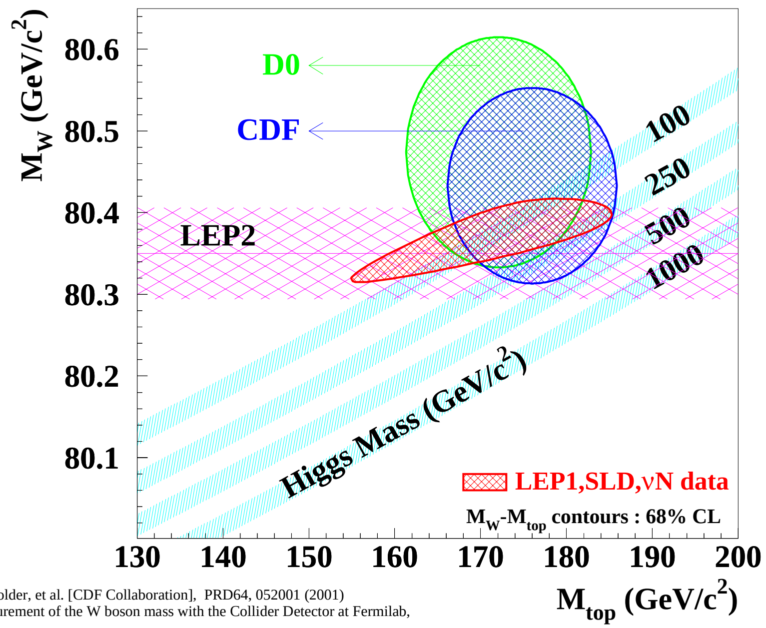
The W-Mass is an important fundamental quantity



What gives the W

P_T ???

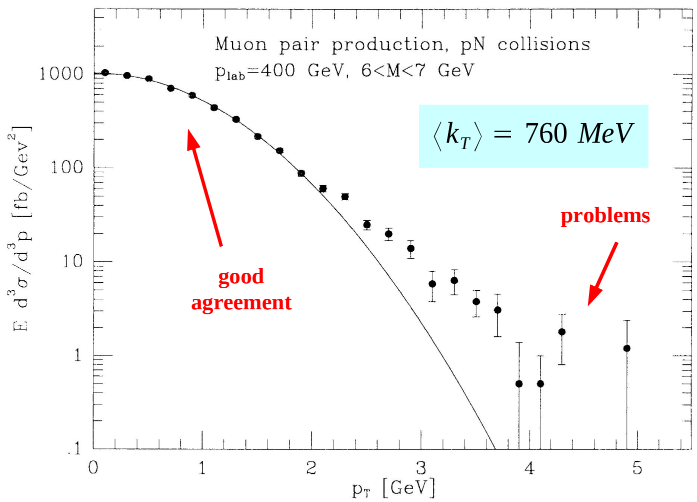
The W-Mass is an important fundamental quantity



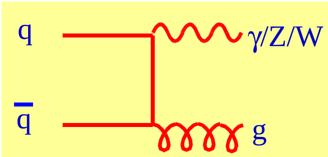
T.Affolder, et al. [CDF Collaboration], PRD64, 052001 (2001)
Measurement of the W boson mass with the Collider Detector at Fermilab,

What about the intrinsic k_T of the partons?

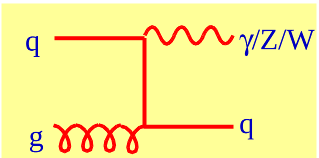
Assume a Gaussian form:
$$\frac{d^2\sigma}{d^2p_T} \approx \sigma_0\, e^{-p_T^2}$$



For high P_T, we need a hard parton emission



annihilation



Compton

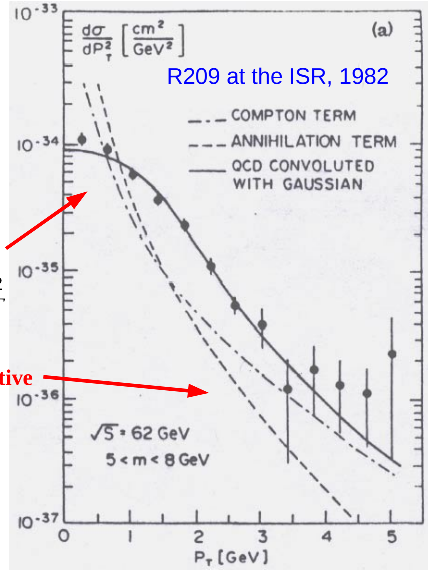
Combination of Gaussian & QCD corrections

Gaussian

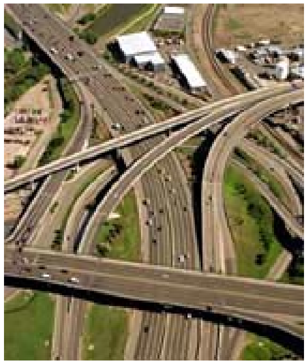
$e^{-p_T^2}$

Perturbative

$\frac{1}{p_T^2}$

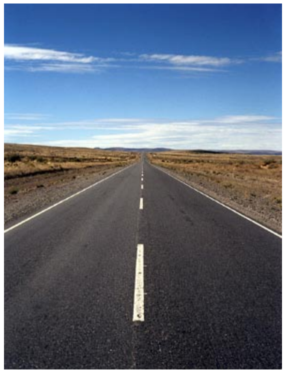


Road map for Resummation



BEFORE

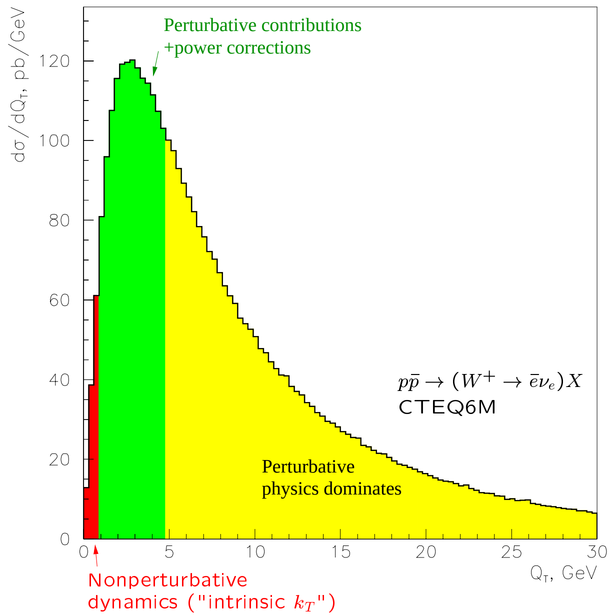
$\Longrightarrow$



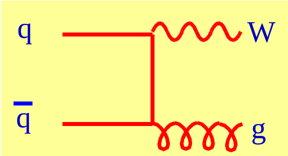
AFTER

The complete P_T spectrum for the W boson

The full P_T spectrum for the W-boson showing the different theoretical regions



NLO P_T distribution for the W boson



In the limit $P_T \to 0$

$$\frac{d\sigma}{d\tau\, dy\, dp_T^2} \approx \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \frac{4\alpha_s}{3\pi} \frac{\ln s/p_T^2}{p_T^2}$$

singular

$$\int_0^s \frac{d\sigma}{d\tau\, dy\, dp_T^2} dp_T^2 = \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} + O(\alpha_s)$$

finite

$$\int_0^{p_T^2} \frac{d\sigma}{d\tau\, dy\, dp_T^2} dp_T^2 = \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \left\{1 - \int_{p_T^2}^s \frac{4\alpha_s}{3\pi} \frac{\ln s/p_T^2}{p_T^2} d\,p_T^2\right\}$$

$$= \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \left\{1 - \frac{2\alpha_s}{3\pi} \ln^2 \frac{s}{p_T^2}\right\}$$

effect of gluon emission

$$= \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \exp\left\{\frac{-2\alpha_s}{3\pi} \ln^2 \frac{s}{p_T^2}\right\}$$

assume this exponentiates

Parisi & Petronzio, NP B154, 427 (1979)
Dokshitzer, D'yakanov, Troyan, Phy. Rep. 58, 271 (1980)

Resummation of soft gluons: Step #1

Differentiating the previous expression for $d^2\sigma/d\tau\, dy$

Sudakov Form Factor

$$\frac{d\sigma}{d\tau\, dy\, dp_T^2} \approx \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \frac{4\alpha_s}{3\pi} \frac{\ln s/p_T^2}{p_T^2} \times \exp\left\{-\frac{2\alpha_s}{3\pi}\ln^2\frac{s}{p_T^2}\right\}$$

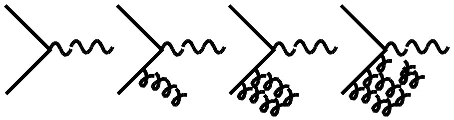
finite at $p_T=0$

We just resummed (exponentiated) an infinite series of soft gluon emissions

$$e^{-\alpha_s L^2} \approx 1 - \alpha_s L^2 + \frac{(\alpha_s L^2)^2}{2!} - \frac{(\alpha_s L^2)^3}{3!} + \ldots$$

$L = \ln \frac{s}{p_T^2}$

Soft gluon emissions treated as uncorrelated



I've skipped over some details ..

Parisi & Petronzio, NP B154, 427 (1979)
Dokshitzer, D'yakanov, Troyan, Phy. Rep. 58, 271 (1980)

Curci, Greco, Srivastava, PRL 43, 834 (1979); NP B159, 451 (1979)
Jeff Owens, 2000 CTEQ Summer School Lectures

1) We summed only the leading logarithmic singularity

we resum these terms

we miss these terms

$$\frac{d\sigma}{dq_T^2} \sim \frac{\alpha_s L}{q_T^2}\left\{1 + \alpha_s^1 L^2 + \alpha_s^2 L^4 + \ldots\right\}$$
$$\frac{\alpha_s L}{q_T^2}\left\{\quad + \alpha_s^1 L^1 + \alpha_s^2 L^3 + \alpha_s^3 L^5 + \ldots\right\}$$

$L = \ln \frac{s}{p_T^2}$

The terms we are missing are suppressed by $\alpha_s L$, not α_s!

If (somehow) we could sum the sub-leading log ...

$$\frac{d\sigma}{dq_T^2} \sim \frac{\alpha_s L}{q_T^2} e^{\alpha_s (L^2 + L)}$$

we resum these terms

we miss these terms

$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2}\left\{\alpha_s^1(L^1+1) + \alpha_s^2(L^3+L^2) + \alpha_s^3(L^5+L^4) + \ldots\right\}$$
$$\frac{1}{q_T^2}\left\{\quad + \alpha_s^2(L^1+1) + \alpha_s^3(L^3+L^2) + \alpha_s^4(L^5+L^4) + \ldots\right\}$$

Now, the terms we are missing are suppressed only by α_s!

We skipped over a few details ...

1) We summed only the leading logarithmic singularity, $\alpha_s L^2$.

We'll need to do better to ensure convergence of perturbation series

2) We assumed exponentiation; proof of this is non-trivial.

The existence of two scales $(Q,p_T) \equiv (Q,q_T)$ yields 2 logs per loop

3) Gluon emission was assumed to be uncorrelated.

This leads to too strong a suppression at $P_T=0$.

Will need to impose momentum conservation for P_T.

4) In the limit $P_T \to 0$, terms of order $\alpha_S(\mu=P_T) \to \infty$;

Must handle this Non-Perturbative region.

2) We assumed exponentiation; proof is non-trivial

Review where the logs come from

Review one-scale problem *(Q)*

resummation via RGE

Review two-scale problem *(Q,q_T)*

resummation via RGE+ Gauge Invariance

Where do the Logs come from?

Drelly-Yan at 2 Loops:

$$
\begin{aligned}
H^{(2),S+V}_{q\bar{q}}(z) = {} & \left[\frac{\alpha_s}{4\pi}\right]^2 \delta(1-z)\Bigg\{C_A C_F\left[\left(\tfrac{193}{3}-24\zeta(3)\right)\ln\left(\frac{Q^2}{M^2}\right)-11\ln^2\left(\frac{Q^2}{M^2}\right)-\tfrac{12}{5}\zeta(2)^2+\tfrac{592}{9}\zeta(2)+28\zeta(3)-\tfrac{1535}{12}\right] \\
& +C_F^2\left[\left(18-32\zeta(2)\right)\ln^2\left(\frac{Q^2}{M^2}\right)+\left(24\zeta(2)-176\zeta(3)-93\right)\ln\left(\frac{Q^2}{M^2}\right) \right.\\
& \left. +\tfrac{8}{5}\zeta(2)^2-70\zeta(2)-60\zeta(3)+\tfrac{511}{4}\right] \\
& +n_f C_F\left[2\ln^2\left(\frac{Q^2}{M^2}\right)-\tfrac{34}{3}\ln\left(\frac{Q^2}{M^2}\right)+8\zeta(3)-\tfrac{112}{9}\zeta(2)+\tfrac{127}{6}\right]\Bigg\} \\
& +C_A C_F\left[-\tfrac{44}{3}\mathcal{D}_0(z)\ln^2\left(\frac{Q^2}{M^2}\right)+\left\{\left[\tfrac{536}{9}-16\zeta(2)\right]\mathcal{D}_0(z)-\tfrac{176}{3}\mathcal{D}_1(z)\right\}\ln\left(\frac{Q^2}{M^2}\right) \right.\\
& \left. -\tfrac{176}{3}\mathcal{D}_2(z)+\left[\tfrac{1072}{9}-32\zeta(2)\right]\mathcal{D}_1(z)+\left[56\zeta(3)+\tfrac{176}{3}\zeta(2)-\tfrac{1616}{27}\right]\mathcal{D}_0(z)\right] \\
& +C_F^2\left[\left[64\mathcal{D}_1(z)+48\mathcal{D}_0(z)\right]\ln^2\left(\frac{Q^2}{M^2}\right)+\left\{192\mathcal{D}_2(z)-96\mathcal{D}_1(z)-\left[128+64\zeta(2)\right]\mathcal{D}_0(z)\right\}\ln\left(\frac{Q^2}{M^2}\right) \right.\\
& \left. +128\mathcal{D}_3(z)-\left(128\zeta(2)+256\right)\mathcal{D}_1(z)+256\zeta(3)\mathcal{D}_0(z)\right] \\
& +n_f C_F\left[\tfrac{8}{3}\mathcal{D}_0(z)\ln^2\left(\frac{Q^2}{M^2}\right)+\left[\tfrac{32}{3}\mathcal{D}_1(z)-\tfrac{80}{9}\mathcal{D}_0(z)\right]\ln\left(\frac{Q^2}{M^2}\right)+\tfrac{32}{3}\mathcal{D}_2(z)-\tfrac{160}{9}\mathcal{D}_1(z)-\left[\tfrac{224}{27}-\tfrac{32}{3}\zeta(2)\right]\mathcal{D}_0(z)\right].
\end{aligned}
\tag{7.14}
$$

Two mass scales: $\{Q^2, M^2\}$. Logarithms!!!

Total Cross Section: $\sigma(e^+e^-)$ at 3 Loops

$$
\begin{aligned}
\sigma(Q^2) = \sigma_0 \Bigg\{ & 1+\frac{\alpha_s(Q^2)}{4\pi}(3C_F)+\left[\frac{\alpha_s(Q^2)}{4\pi}\right]^2\left[-C_F^2\left(\frac{3}{2}\right)+C_F C_A\left(\frac{123}{2}-44\zeta(3)\right)-C_F T n_f\left(-22+16\zeta(3)\right)\right] \\
& +\left[\frac{\alpha_s(Q^2)}{4\pi}\right]^3\Bigg[C_F^3\left(-\frac{69}{2}\right)+C_F^2 C_A\left(-127-572\zeta(3)+880\zeta(5)\right) \\
& +C_F C_A^2\left(\frac{90445}{54}-\frac{10948}{9}\zeta(3)+\frac{440}{3}\zeta(5)\right) \\
& +C_F^2 T n_f\left(-29+304\zeta(3)-320\zeta(5)\right)+C_F C_A T n_f\left(-\frac{31040}{27}+\frac{7168}{9}\zeta(3)+\frac{160}{3}\zeta(5)\right) \\
& +C_F T^2 n_f^2\left(\frac{4832}{27}-\frac{1216}{9}\zeta(3)\right)-C_F\pi^2\left(\frac{11}{3}C_A-\frac{4}{3}Tn_f\right)^2+\frac{\left(\sum_f Q_f\right)^2}{\left(N\sum_f Q_f^2\right)}\frac{D}{16}\left(\frac{176}{3}-128\zeta(3)\right)\Bigg]\Bigg\}.
\end{aligned}
\tag{5.1}
$$

One mass scale: Q^2. No logarithms!!!

Renormalization Group Equation

More Differential Quantities $\Rightarrow$ More Mass Scales $\Rightarrow$ More Logs!!!

$$\frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right) \qquad \frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right) \;\; and \;\; \ln\left(\frac{q_T^2}{\mu^2}\right)$$

How do we resum logs? Use the Renormalization Group Equation

For a physical observable R:

$$\mu\frac{dR}{d\mu} = 0$$

Using the chain rule:

$$\left\{\mu^2\frac{\partial}{\partial\mu^2}+\underbrace{\left[\mu^2\frac{\partial\alpha_s(\mu^2)}{\partial\mu^2}\right]}_{\beta(\alpha_s(\mu))}\frac{\partial}{\partial\alpha_s(\mu^2)}\right\}R(\mu^2,\alpha_s(\mu^2)) = 0$$

Solution $\Rightarrow$ $\ln\left(\dfrac{Q^2}{\mu^2}\right) = \displaystyle\int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)}\frac{dx}{\beta(x)}$

Renormalization Group Equation: *OVER SIMPLIFIED!*

$$\left\{\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s(\mu)) \frac{\partial}{\partial \alpha_s(\mu^2)}\right\} R(\mu^2, \alpha_s(\mu^2)) = 0$$

If we expand R in powers of α_s, and we know β, we then know μ dependence of R.

$$R(\mu, Q, \alpha_s(\mu^2)) = R_0 + \alpha_s(\mu^2)\, R_1\left[\ln\left(Q^2/\mu^2\right) + c_1\right] + \alpha_s^2(\mu^2)\, R_2\left[\ln^2\left(Q^2/\mu^2\right) + \ln\left(Q^2/\mu^2\right) + c_2\right] + O(\alpha_s^3(\mu^2))$$

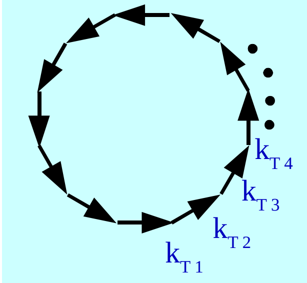
Since μ is arbitrary, choose μ=Q.

$$R(Q, Q, \alpha_s(Q^2)) = R_0 + \alpha_s(Q^2)\, R_1\left[0 + c_1\right] + \alpha_s^2(Q^2)\, R_2\left[0 + 0 + c_2\right] + \ldots$$

We just summed the logs

3) We assumed gluon emission was uncorrelated

$$\frac{d\sigma}{d\tau\, dy\, dp_T^2} \approx \frac{\ln s/p_T^2}{p_T^2} \times \exp\left\{-\frac{2\alpha_s}{3\pi} \ln^2 \frac{s}{p_T^2}\right\}$$



This leads to too strong a suppression at P_T=0.
Need to impose momentum conservation for P_T.

A particle can receive finite k_T kicks, yet still have P_T=0

A convenient way to impose transverse momentum conservation is in impact parameter space (b-space) via the following relation:

$$\delta^{(2)}\left(\sum_{i=1}^{n} \vec{k}_{iT} - \vec{p}_T\right) = \frac{1}{(2\pi)^2} \int d^2b\, e^{-i\vec{b}\cdot\vec{p}_T} \prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{iT}}$$

Two-Scale Problems

For R(μ,Q,α_s), we could resum ln(Q^2/μ^2) by taking Q=μ.
What about R(μ,Q,q_T,α_s); how do we resum ln(Q^2/μ^2) and ln(q_T^2/μ^2).
Are we stuck? Can't have $\mu^2=Q^2$ and $\mu^2=q_T^2$ at the same time!

Solution: Use Gauge Invariance; cast in similar form to RGE

Use axial-gauge with axial vector ξ.
This enters the cross section in the form: ($\xi \bullet$p).

$$\sigma\left(x, \frac{Q^2}{\mu^2}, \frac{(p\cdot\xi)^2}{\mu^2}, \ldots\right)$$

$\frac{d\sigma}{d\mu^2} = 0$ RGE allows us to vary μ to resum logs

$\frac{d\sigma}{d(p\cdot\xi)^2} = 0$ Gauge invariance allows us to vary ($\xi \bullet$p) to resum logs

It is covenient to transform to impact parameter space (b-space) to implement this mechanism

The details will fill multiple lectures:
See Sterman TASI 1995; Soper CTEQ 1995

4) We encounter Non-Perturbative Physics

$$S(b, Q) = \int_{\sim 1/b^2}^{\sim Q^2} \frac{d\mu^2}{\mu^2}\left\{A(\alpha_s(\mu^2)) \ln\left(\frac{Q^2}{\mu^2}\right) + B(\alpha_s(\mu^2))\right\}$$

as b $\rightarrow\infty$, α_S(~1/b) $\rightarrow\infty$. **PROBLEM!!!**

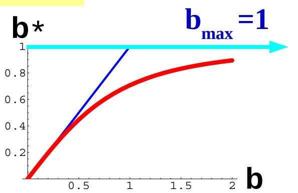
Solution: Use a Non-Perturbative Sudakov form factor (S_{NP}) in the region of large b (small q_T)

$$\widetilde{\sigma}(b) \sim e^{S(b)} \rightarrow e^{S(b_*)} * e^{S_{NP}(b)}$$

with

$$b_* = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$

Note, as b $\rightarrow\infty$, $b_* \rightarrow b_{max}$.



A Brief *(but incomplete)* History of Non-Perturbative Corrections

Original CSS: $S_{NP}^{CSS}(b) = h_1(b,\xi_a)+h_2(b,\xi_b)+h_3(b)\ln Q^2$

J. Collins and D. Soper, *Nucl.Phys.* **B193** 381 (1981);
erratum: **B213** 545 (1983); J. Collins, D. Soper, and G. Sterman, *Nucl. Phys.* **B250** 199 (1985).

Davies, Webber, and Stirling (DWS): $S_{NP}^{DWS}(b) = b^2\left[g_1+g_2\ln(b_{max}Q^2)\right]$

C. Davies and W.J. Stirling, Nucl. Phys. B244 337 (1984);
C. Davies, B. Webber, and W.J. Stirling, Nucl. Phys. B256 413 (1985).

Ladinsky and Yuan (LY): $S_{NP}^{LY}(b) = g_1 b\left[b+g_3\ln(100\,\xi_a\xi_b)\right]+g_2 b^2\ln(b_{max}Q)$

G.A. Ladinsky and C.P. Yuan, Phys. Rev. D50 4239 (1994);
F. Landry, R. Brock, G.A. Ladinsky, and C.P.Yuan, Phys. Rev. D63 013004 (2001).

"BLNY": $S_{NP}^{BLNY}(b) = b^2\left[g_1+g_1 g_3\ln(100\,\xi_a\xi_b)+g_2\ln(b_{max}Q)\right]$

F. Landry, "Inclusion of Tevatron Z Data into Global Non-Perturbative QCD Fitting", Ph.D. Thesis, Michigan State University, 2001.
F. Landry, R. Brock, P. Nadolsky, and C.P.Yuan, PRD67, 073016 (2003)

"q_T resummation": $\widetilde{F}^{NP}(q_T) = 1-e^{-\tilde{a}\,q_T^2}$ *(not in b-space)*

R.K. Ellis, Sinisa Veseli, Nucl.Phys. B511 (1998) 649-669
R.K. Ellis, D.A. Ross, S. Veseli, Nucl.Phys. B503 (1997) 309-338

Functional Extrapolation:

J. Qui, X. Zhang, PRD63, 114011 (2001); E. Berger, J. Qiu, PRD67, 034023 (2003)

Analytical Continuation:

A. Kulesza, G. Sterman, W. vogelsang, PRD66, 014011 (2002)

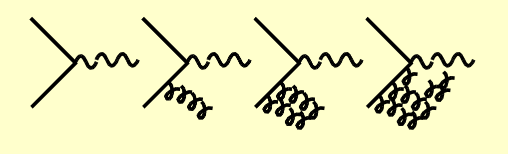
What do we get for the cross section

$$\frac{d\sigma}{dy\,dQ^2\,dq_T^2} = \frac{1}{(2\pi)^2}\int_0^\infty d^2b\; e^{i b\cdot q_T}\,\widetilde{W}(b,Q)\; e^{-S(b_*,Q)+S_{NP}(b,Q)}$$

with

$$-S(b,Q) = -\int_{\sim 1/b^2}^{\sim Q^2}\frac{d\mu^2}{\mu^2}\left\{A\ln\left(\frac{Q^2}{\mu^2}\right) + B\right\}$$

where we have resummed the soft gluon contributions



I've left out A LOT of material

Recap: Where have we been???

1) We now summed the two leading logarithmic singularities, $\alpha_s(L^2+L)$.

2) We still assumed exponentiation; but sketched ingredients of proof.

The existence of two scales $(Q,p_T)\equiv(Q,q_T)$ yields 2 logs per loop

Use Renormalization Group + Gauge Invariance

Transformation to b-space

3) Gluon emission was assumed to be uncorrelated.

Impose momentum conservation for P_T. *(In b-space)*

4) Introduced Non-Perturbative function for small q_T (large b) region.

Putting it all together: $\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$

Let's expand out the resummed expression:

$$\frac{d\sigma}{dq_T^2} \sim \frac{\alpha_s L}{q_T^2}\, e^{\alpha_s(L^2+L)} \sim \frac{1}{q_T^2}\left\{\alpha_s L + \alpha_s^2(L^3+L^2)+\ldots\right\}$$

Compare the above with the perturbative and asymptotic results:

$$d\sigma_{resum} \sim \left\{\alpha_s L + \alpha_s^2(L^3+L^2+0+0) + \alpha_s^3(L^5+L^4)+\ldots\right\}$$
$$d\sigma_{pert} \sim \left\{\alpha_s L + \alpha_s^2(L^3+L^2+L^1+1) + \alpha_s^3(0+0)\right\}$$
$$d\sigma_{asym} \sim \left\{\alpha_s L + \alpha_s^2(L^3+L^2+0+0) + \alpha_s^3(0+0)\right\}$$

Note that σ_{ASYM} removes overlap between σ_{RESUM} and σ_{PERT}.

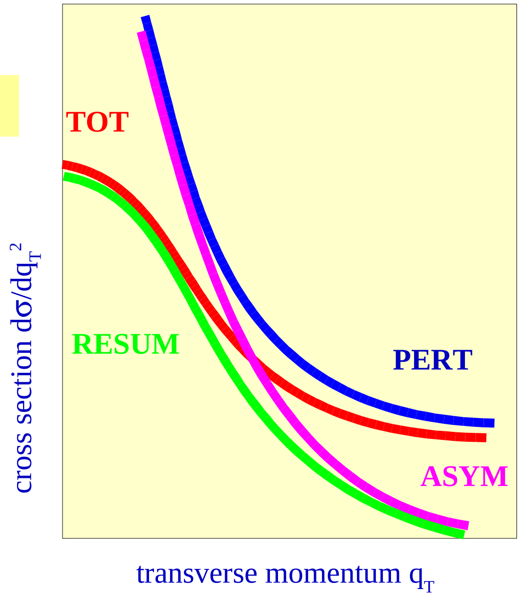
We expect:

σ_{RESUM} is a good representation for $q_T \sim 0$
σ_{PERT} is a good representation for $q_T \sim M_W$

Putting it all together: $\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$

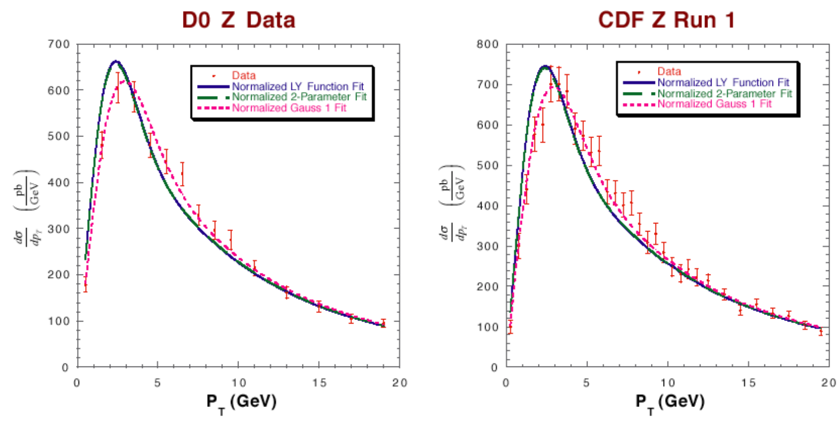
$\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$

σ_{RESUM} for $q_T \sim 0$
σ_{PERT} for $q_T \sim M_W$



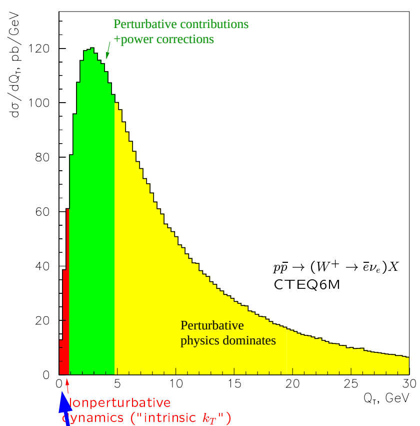
Let's compare with some real results

We'll look at Z data where we can measure both leptons for $Z \to e^+e^-$



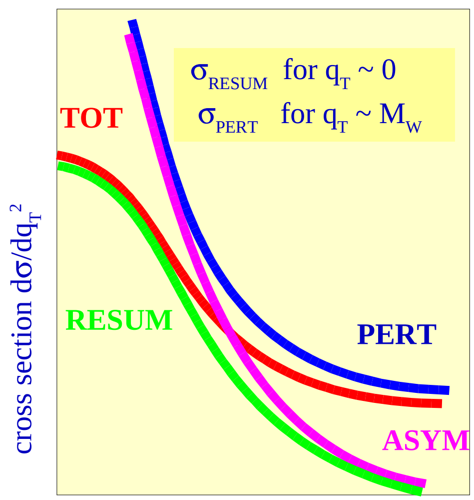
different $S_{NP}(b,Q)$ functions yield difference at small q_T.

Putting it all together: $\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$



Extra power of q_T

$d\sigma/dq_T^1$



σ_{RESUM} for $q_T \sim 0$
σ_{PERT} for $q_T \sim M_W$

transverse momentum q_T

$\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$

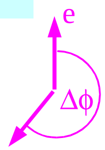
Let's return to the measurement of M_W

Transverse Mass Distribution

We can measure $d\sigma/dp_T$ and look for the Jacobian peak.
However, there is another variable that is relatively insensitive to $p_T(W)$.

Transverse Mass	$M_T^2(e,\nu) = \left(\lvert\vec{p}_{eT}\rvert + \lvert\vec{p}_{\nu T}\rvert\right)^2 - \left(\vec{p}_{eT} + \vec{p}_{\nu T}\right)^2$
Invariant Mass	$M^2(e,\nu) = \left(\lvert\vec{p}_{e}\rvert + \lvert\vec{p}_{\nu}\rvert\right)^2 - \left(\vec{p}_{e} + \vec{p}_{\nu}\right)^2$

In the limit of vanishing longitudinal momentum, $M_T \sim M$.
M_T is invariant under longitudinal boosts.



M_T can also be expressed as: $M_T^2(e,\nu) = 2\lvert\vec{p}_{eT}\rvert\lvert\vec{p}_{\nu T}\rvert\left(1-\cos\Delta\phi_{e\nu}\right)$

For small values of P_T^W, M_T is invariant to leading order.

Exercise:

a) Verify the above definitions of M_T are ≡.

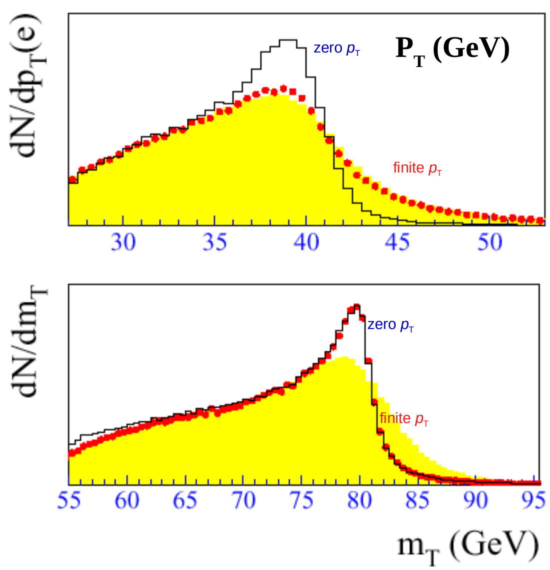
b) For $p_{Te} = +p^* + p_T^W/2$ and $p_{T\nu} = -p^* + p_T^W/2$; verify M_T is invariant to leading order in p_T^W.

The Future:

Tevatron Run II ... *happening now*

LHC ... *happening soon*

Compare P_T and Transverse Mass Distribution



M_T distribution is much less sensitive to P_T of W

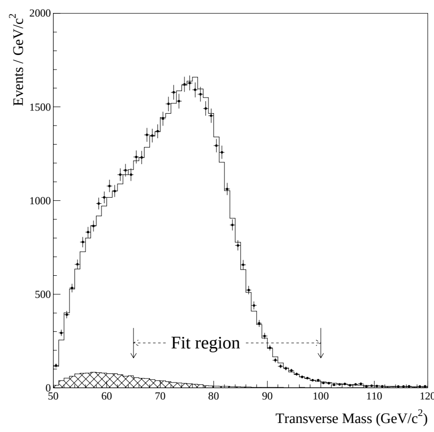
Still, we need P_T distribution of W to extract mass and width with precision

PDF and $p_T(W)$ uncertainties will need to be controlled: currently uncertainty: ~10-15 & 5-10 MeV/c^2

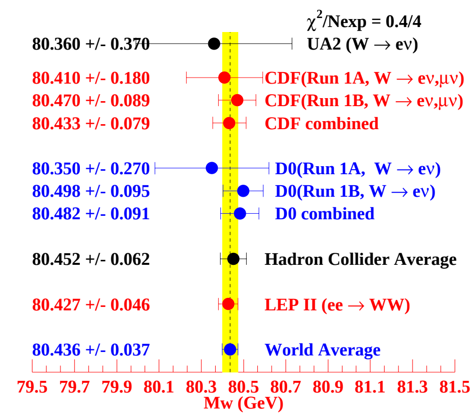
Statistical precision in Run II will be miniscule…placing an enormous burden on control of modeling uncertainties.

Transverse Mass Distribution and M_W Measurement

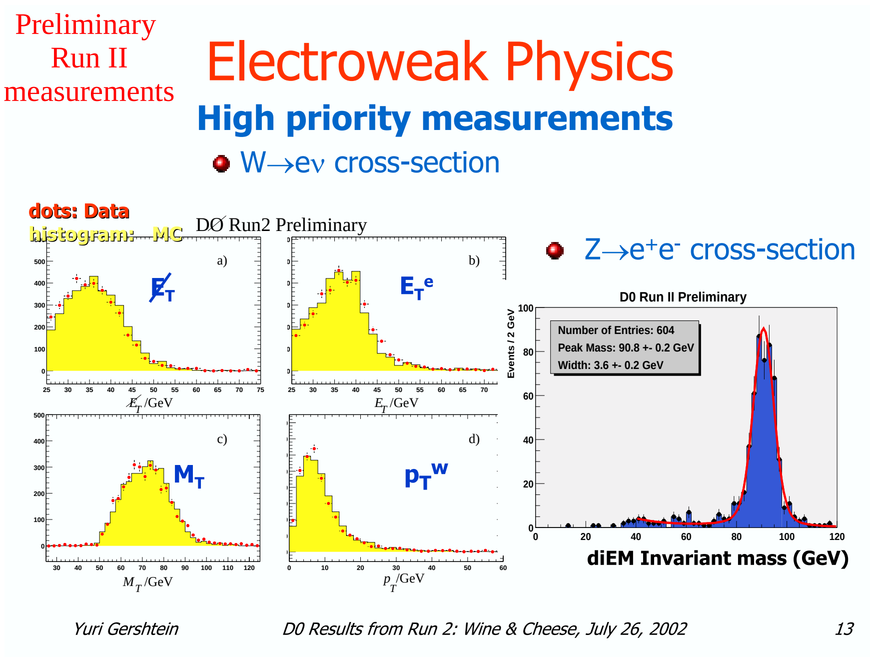
Transverse Mass Distribution from CDF



Combined World Measurements of M_W



T.Affolder, et al. [CDF Collaboration], PRD64, 052001 (2001)
Measurement of the W boson mass with the Collider Detector at Fermilab,



Part II: Drell-Yan Process: Where have we been???

Finding the W Boson Mass:

- The Jacobian Peak, and the W Boson P_T
- Multiple Soft Gluon Emissions
- Single Hard Gluon Emission

Road map of Resummation

- Summing 2 logs per loop: multi-scale problem (Q,q_T)
- Correlated Gluon Emission
- Non-Perturbative physics at small q_T.

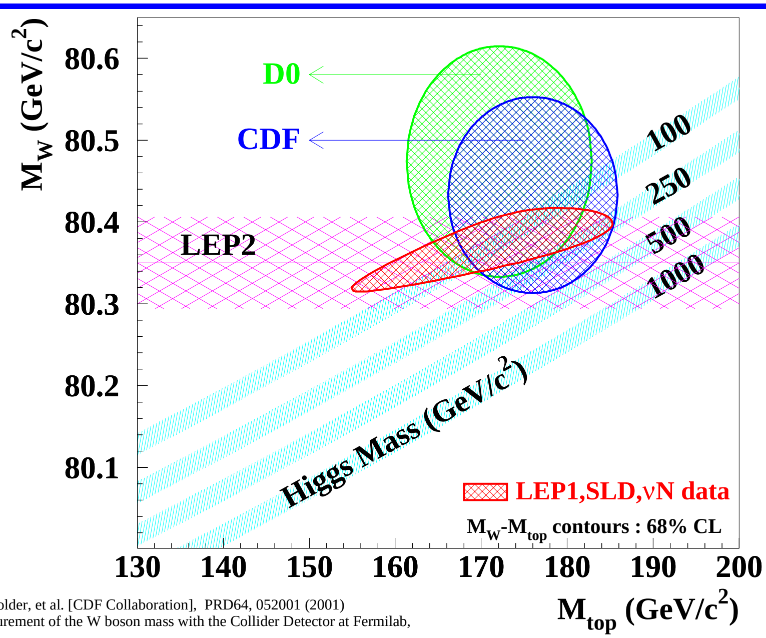
Transverse Mass Distribution:

- Improvement over P_T distribution

What can we expect in future?

- Tevatron Run II
- LHC

The W-Mass is an important fundamental quantity



T.Affolder, et al. [CDF Collaboration], PRD64, 052001 (2001)
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Thanks to ...

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and the many web pages where I borrowed my figures ...

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CTEQ Pedagogical Page:

CTEQ Lectures:
C.P. Yuan, 2002
Chip Brock, 2001
Jeff Owens, 2000

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