The Color Glass Condensate:

An effective theory of QCD at high energies

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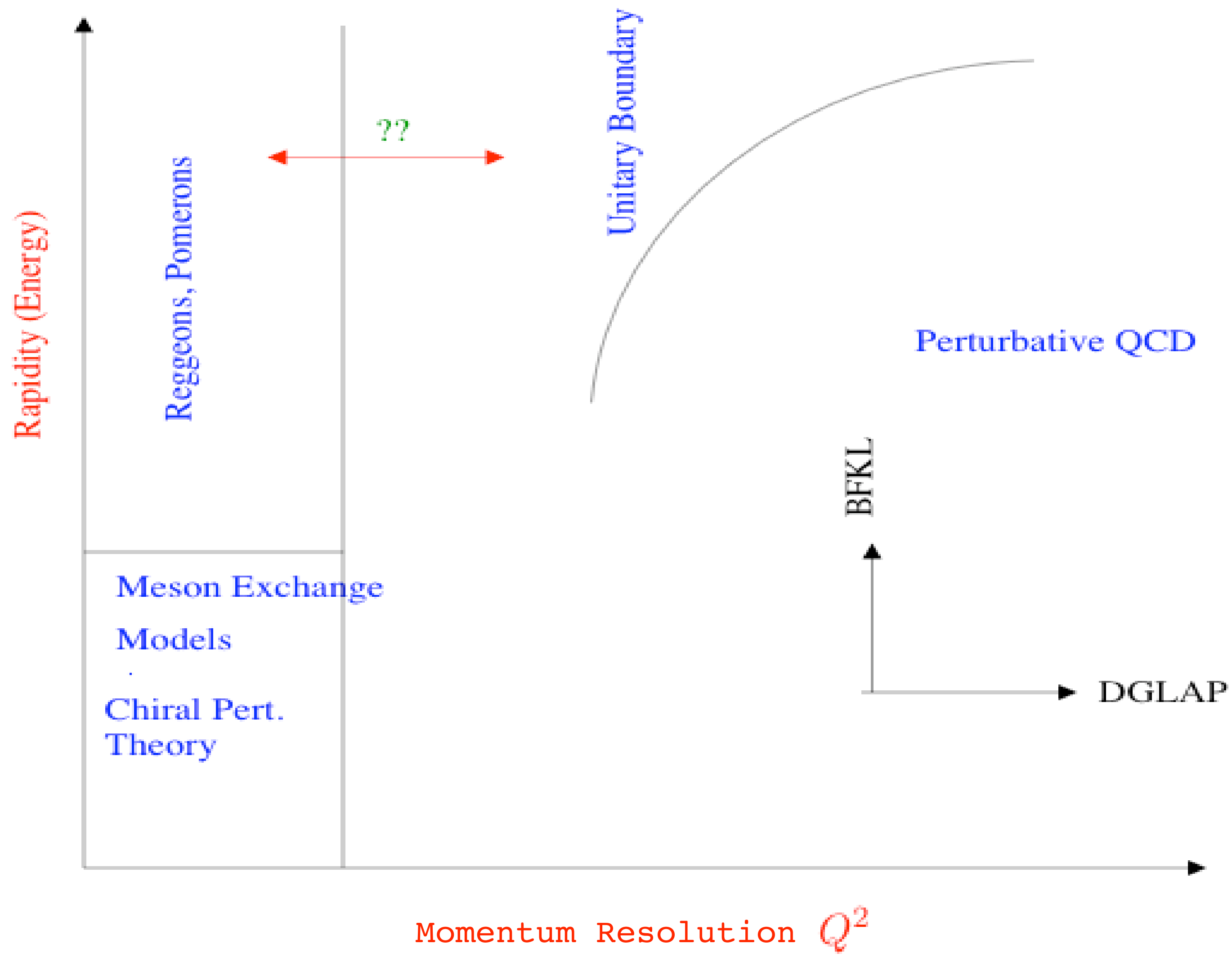
Recent Reviews on the CGC:

- L. McLerran, hep-ph/0311028
- E. Iancu & R. Venugopalan, hep-ph/0303204
- A. H. Mueller, hep-ph/9911289

Road map of the strong interactions

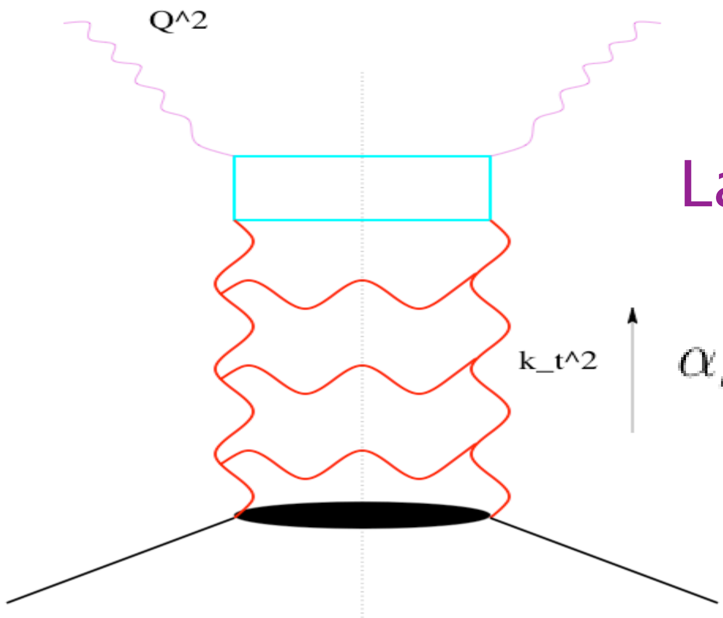


QCD evolution equations at small x

a) The DGLAP equation (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

If x_Bj << 1, gluon bremsstrahlung is dominant in QCD evolution:

P_gg > P_qg > P_qq



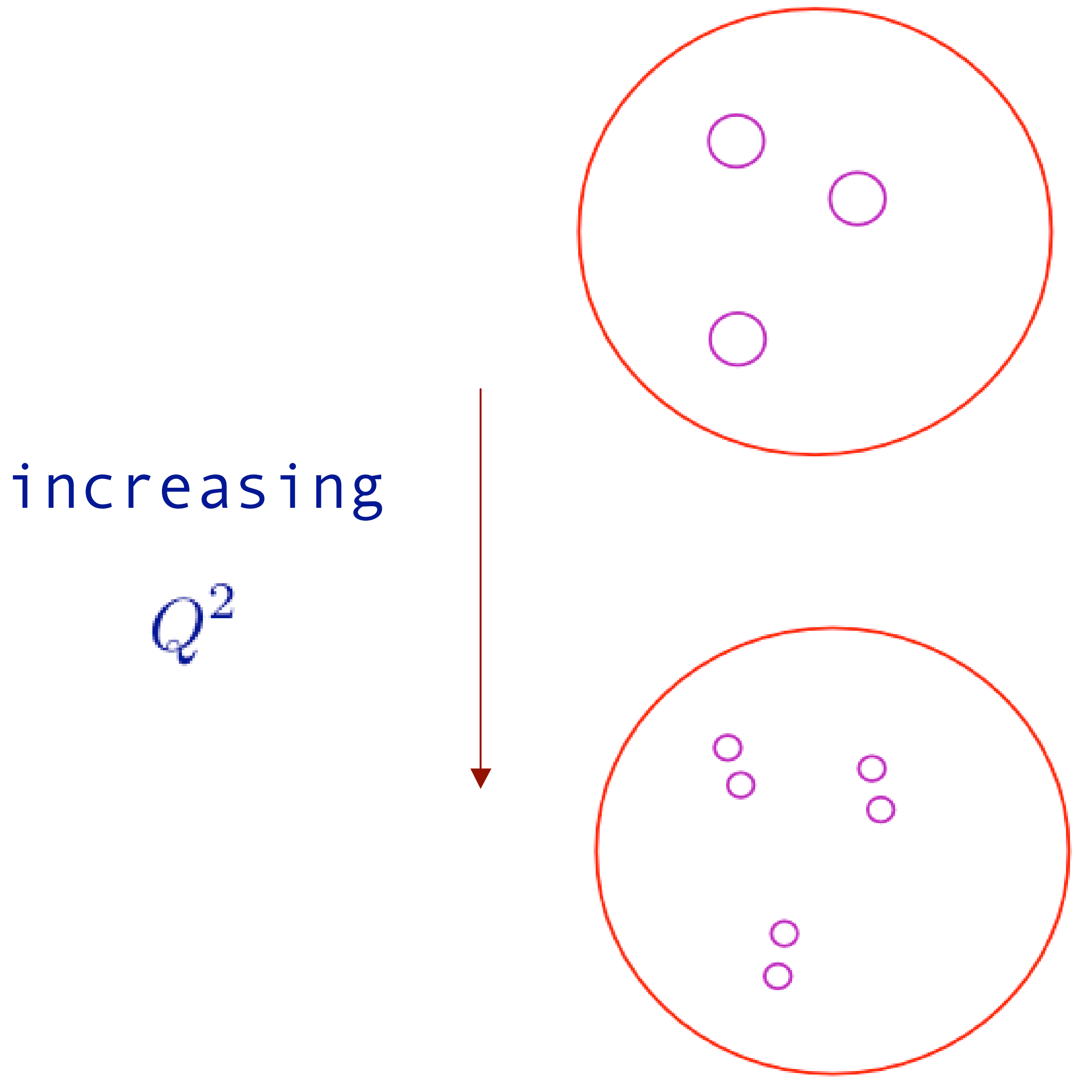
Large Logs from Bremsstrahlung

$$\alpha_S \int \frac{dx}{x} \frac{d^2 k_\perp}{k_\perp^2} \rightarrow \alpha_S^p \ln^m(1/x) \ln^n(Q^2)$$

For $Q^2 >> \Lambda^2_{\rm QCD}$ and x~1, resum $\alpha_S \ln(Q^2)$

At small x, sum double logs

# of gluons grows rapidly...



But… the phase space density decreases
-the proton becomes more dilute

Thus far, our discussion has focused on the Bjorken limit in QCD:

$$Q^2 \to \infty \, ; \, s \to \infty \, ; \, x_{\rm Bj} \approx \frac{Q^2}{s} = \text{fixed}$$

Asymptotic freedom, Factorization Theorems, machinery of precision physics in QCD...

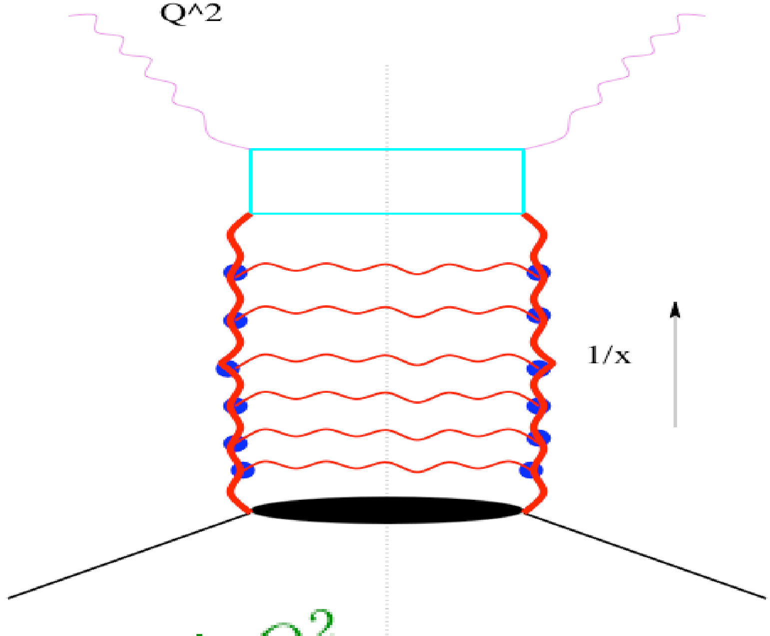
Other interesting limit-is the Regge limit of QCD:

$$x_{\rm Bj} \to 0 \, ; \, s \to \infty \, ; \, Q^2 (>> \Lambda_{\rm QCD}^2) = \text{fixed}$$

Physics of strong fields in QCD, multi-particle production- possibly discover novel universal properties of theory in this limit

QCD evolution equations at small x

b) The BFKL equation (Balitsky-Fadin-Kuraev-Lipatov)

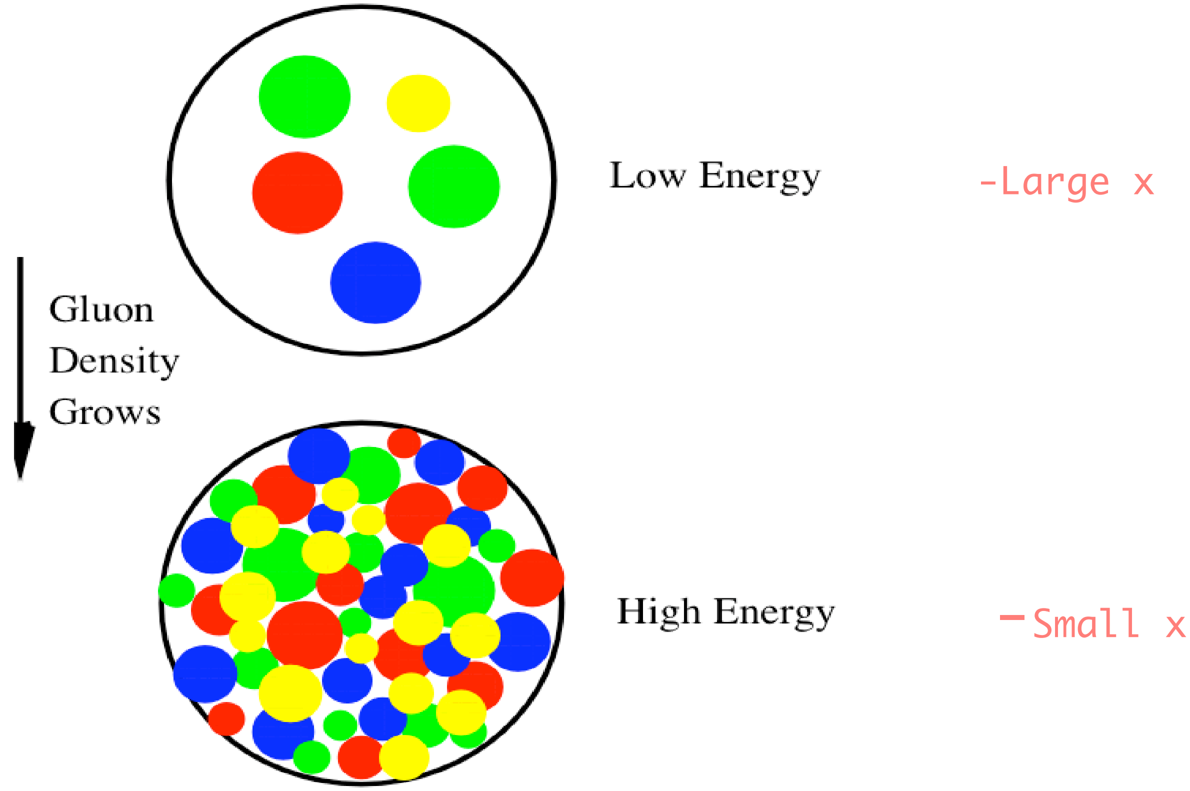


Evolution in x, not Q^2

Re-sums $\alpha_S \ln(1/x)$

# of gluons grows even more rapidly

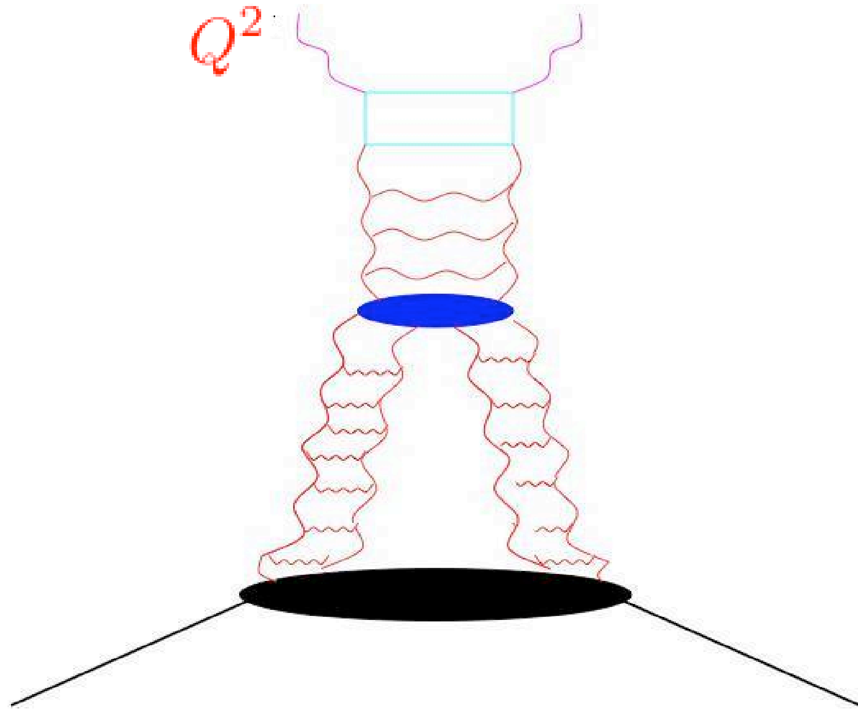
$$xG(x,Q^2) \propto \frac{1}{x^\lambda}\,;\ \lambda \sim 0.5$$



Phase space density grows rapidly-BFKL evolution breaks down when phase space density f~ 1

Gluon density saturates at f= $\frac{1}{\alpha_S}$

Gluon recombination-higher twist effects



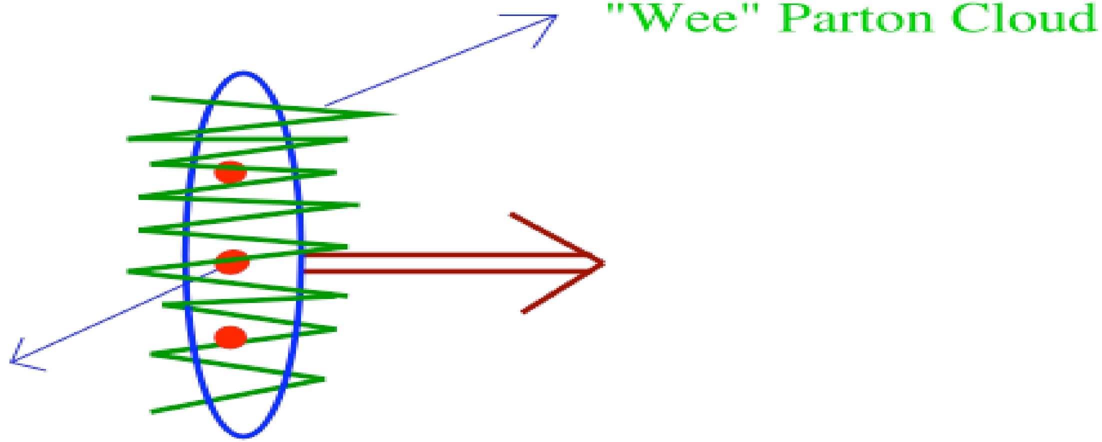
Gribov, Levin, Ryskin
Mueller, Qiu
Blaizot, Mueller

Recombination effects compete with DGLAP Bremsstrahlung effects when

$$\alpha_S x G(x, Q^2) \sim R^2 Q^2$$

Saturation of the gluon density for $Q \equiv Q_s(x)$

A hadron at high energies



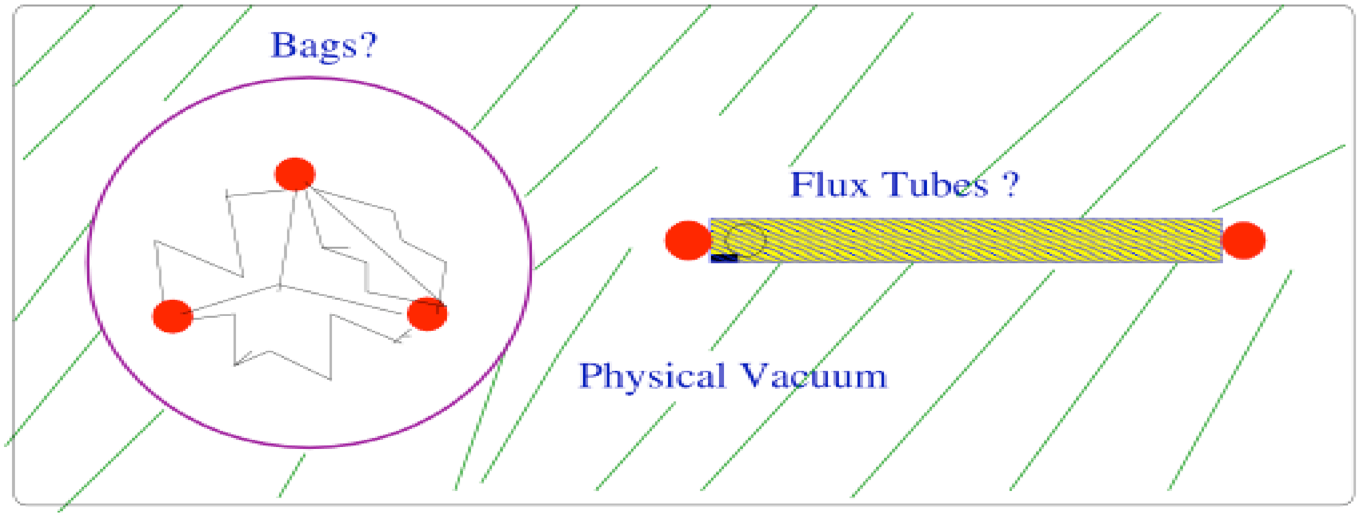
$$|h> = |qqq> + |qqqg> + \cdots + |qqq \cdots gggq\bar{q}>$$

Each wee parton carries only a small fraction

$x = k^+ / P^+$ of the momentum P^+ of the hadron

What is the behavior of wee partons in a high energy hadron?

Unlike QED, the QCD light cone vacuum is very complicated:
–various topological objects, Instantons, Monopoles, Skyrmions, ...
Hadrons may be bags or flux tubes or solitons:
Complex phenomena - Chiral symmetry breaking, Confinement,...



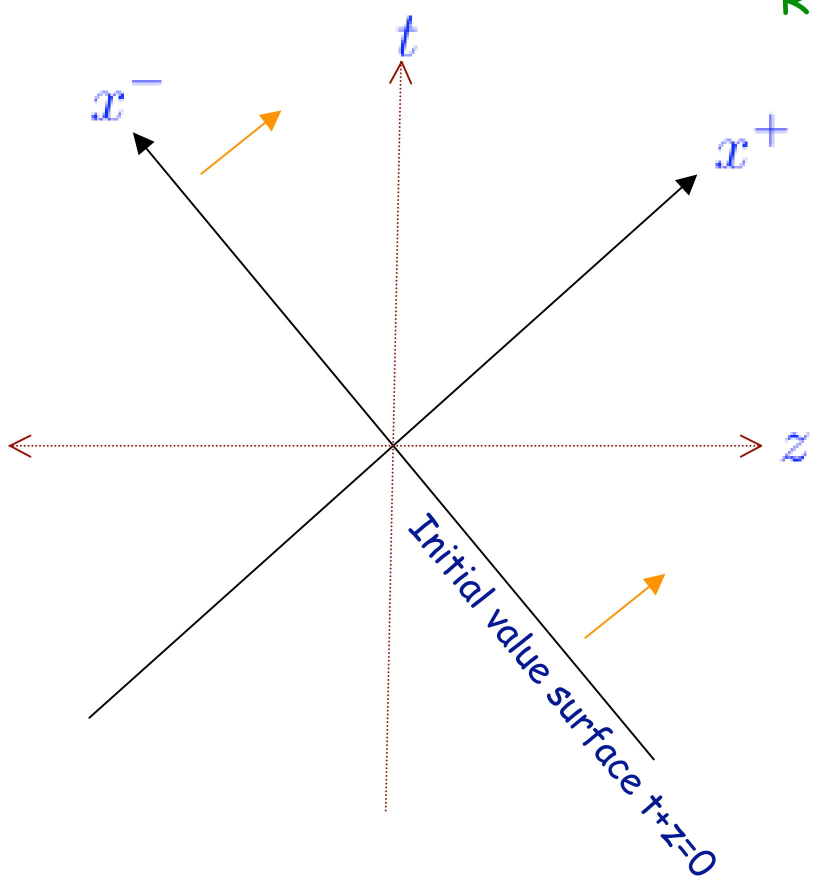
- ❑ Given this, how does one describe the structure of hadrons in high energy scattering?

How does one construct a Lorentz invariant wave fn for a hadron?

Partial answer: formulate the theory on the light cone

Life on the light cone

RV, nucl-th/9808023



Quantize theory on light like surface: $x^+ = 0$

Quantum field theories quantized on light like surfaces have remarkable properties

Weinberg, 1966
Susskind, 1968

Q.F.T on the light cone

↓ isomorphism

Two dimensional quantum mechanics

Light cone dispersion relation:

$$P^- = \frac{(P_t^2 + M^2)}{2P^+}$$

Energy (P^-), Momenta (P_t), Mass (M)

Light cone pert. theory = Rayleigh-Schrodinger pert. theory

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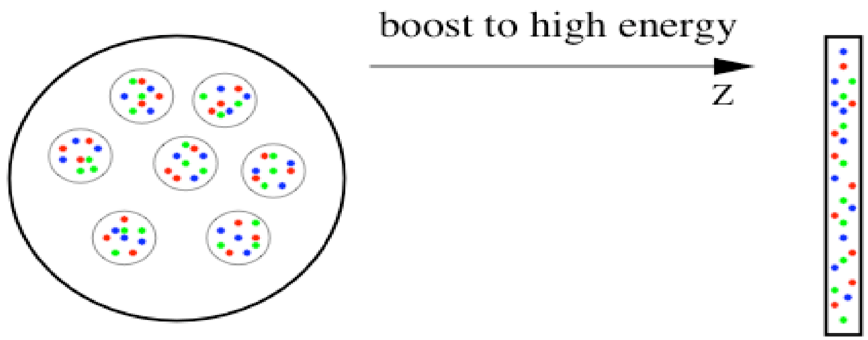
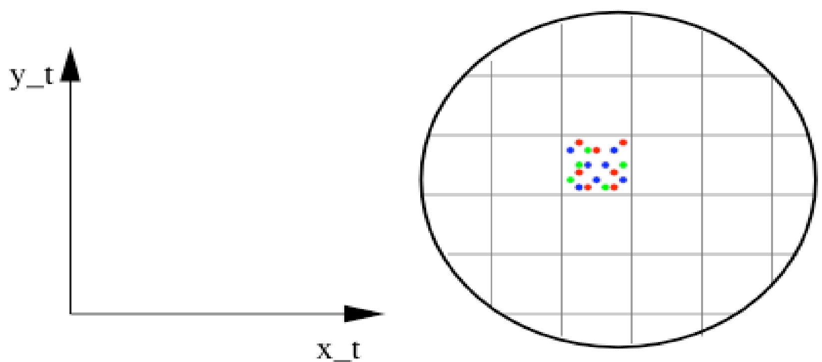
The MV model

McLerran, RV; Kovchegov
Jalilian-Marian,Kovner,McLerran,Weigert

Consider large nucleus in the IMF frame: $P^+ \to \infty$



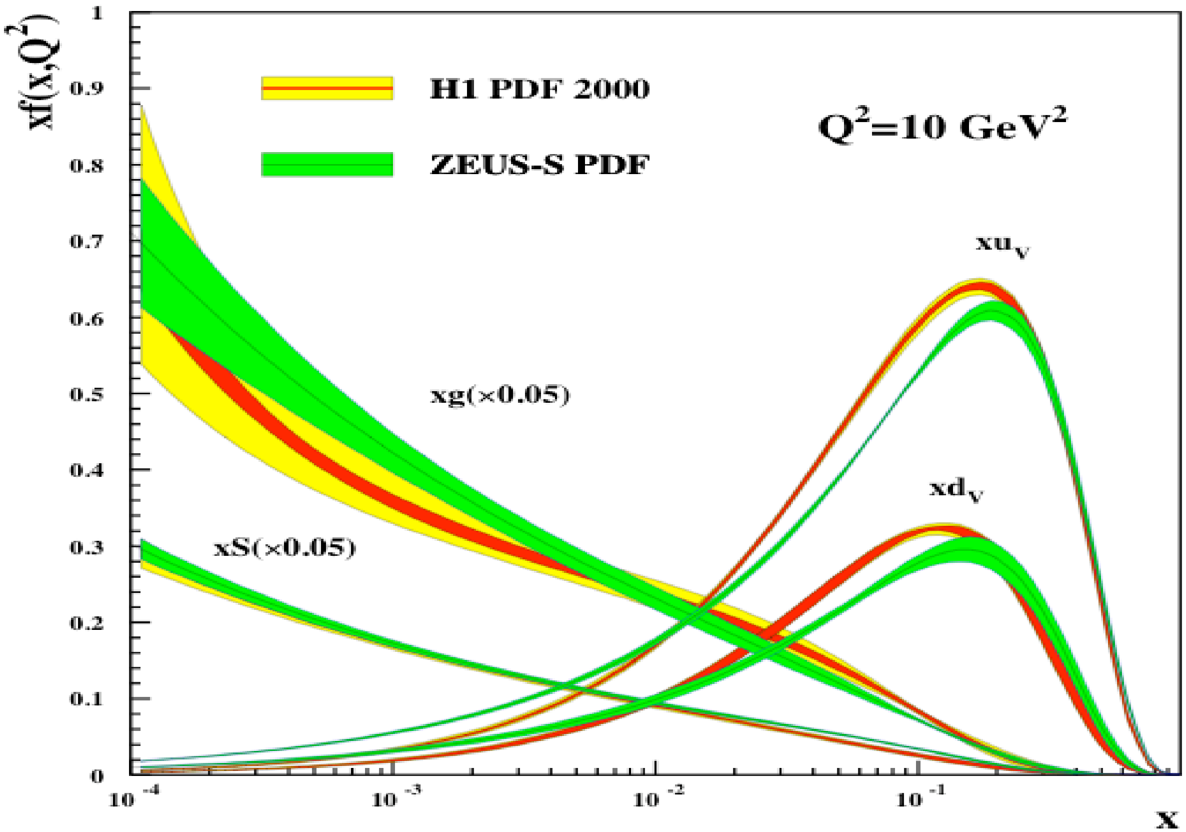


L^2 << 1 fm^2

One large component of the current-others suppressed by $\frac{1}{P^+}$

Wee partons see a large density of valence color charges at small transverse resolutions.

Born-Oppenheimer: separation of large x and small x modes

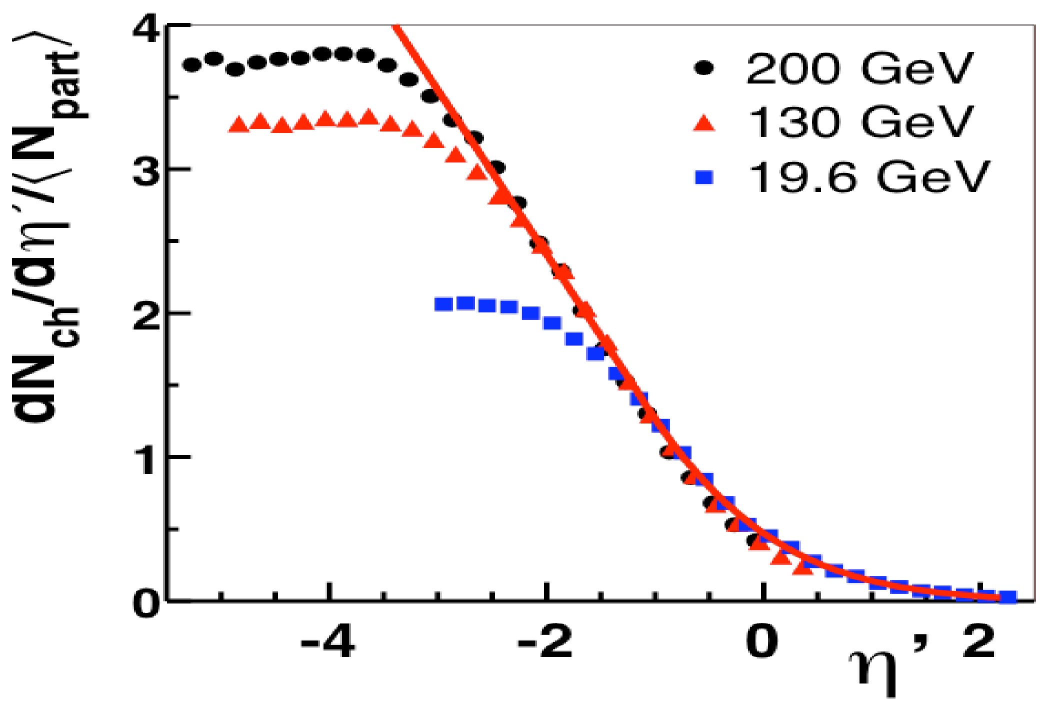


$$\tau_{\rm wee} \sim \frac{1}{k^-} = \frac{2k^+}{k_\perp^2} \equiv \frac{2x\,P^+}{k_\perp^2}$$

$$\tau_{\rm valence} = \frac{2P^+}{k_\perp^2} >> \tau_{\rm wee} \text{ for } x << 1$$

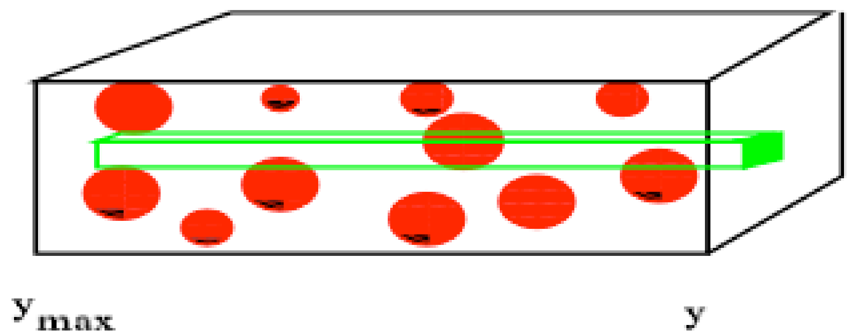
Valence partons are static over wee parton life times

Limiting fragmentation



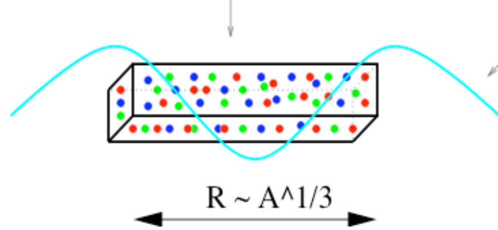
Suggestive that valence partons are
recoil-less sources-unaffected by Bremsstrahlung
of wee partons

Random sources



$$\lambda_{\text{wee}} \sim \frac{1}{k^+} \equiv \frac{1}{xP^+} >> \lambda_{\text{valence}} \equiv \frac{Rm_p}{P^+}$$

$$=> x << A^{-1/3};$$



$$< \rho^a >= 0 ; < \rho^a(x_\perp)\rho^b(y_\perp) >= \mu_A^2 \delta^{ab} \delta^{(2)}(x_\perp - y_\perp)$$

Gaussian random sources

The effective action

Scale separating sources and fields

Generating functional:

$$\mathcal{Z}[j] = \int [d\rho]\, W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho]}} \right\}$$

Gauge invariant weight functional describing distribution of the sources

$$S[A,\rho] = \frac{-1}{4} \int d^4x\, F_{\mu\nu}^2 + \frac{i}{N_c} \int d^2x_\perp dx^- \delta(x^-) \mathrm{Tr}\left(\rho(x_\perp)\, U_{-\infty,\infty}[A^-]\right)$$

where $U_{-\infty,\infty}[A^-] = \mathcal{P} \exp\left(ig \int dx^+ A^{-,a} T^a\right)$

To lowest order, $= -J^+ A^-$ with $J^+ = g\, \rho(x_\perp)\, \delta(x^-)$

For a large nucleus,

$$W[\rho] = \exp\left(-\int d^2x_\perp \frac{\rho^a \rho^a}{2\,\mu_A^2}\right)$$

where, for valence quark sources, one has $\mu_A^2 = \dfrac{g^2 A}{2\pi R_A^2} \propto A^{1/3}\,\mathrm{fm}^{-2}$

For A >>1, $\mu_A^2 >> \Lambda^2_{\mathrm{QCD}}$ and $\alpha_S(\mu_A^2) << 1$

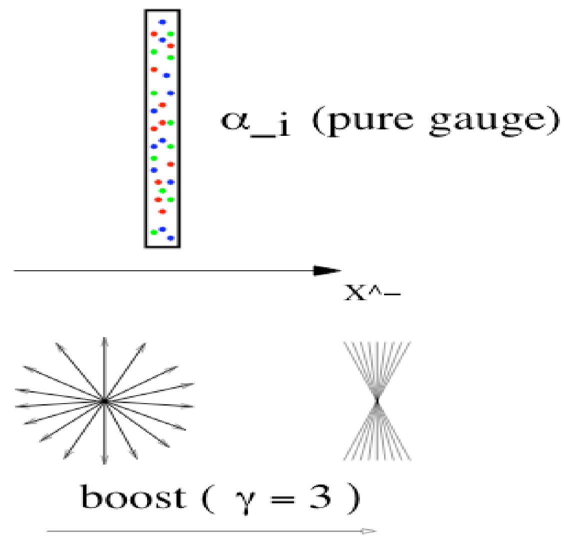
Effective action describes a weakly coupled albeit non-perturbative system

The classical field of the nucleus at high energies

Saddle point of effective action-> Yang-Mills equations

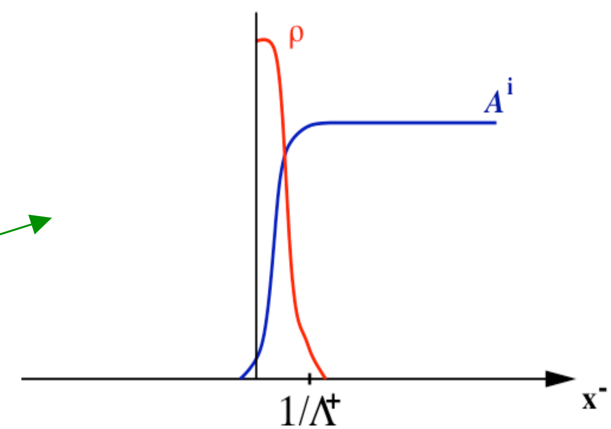
$$D_\mu F^{\mu\nu} = \delta^{\nu +}\delta(x^-)\rho^a(x_\perp)$$

Solutions are non-Abelian Weizsäcker-Williams fields

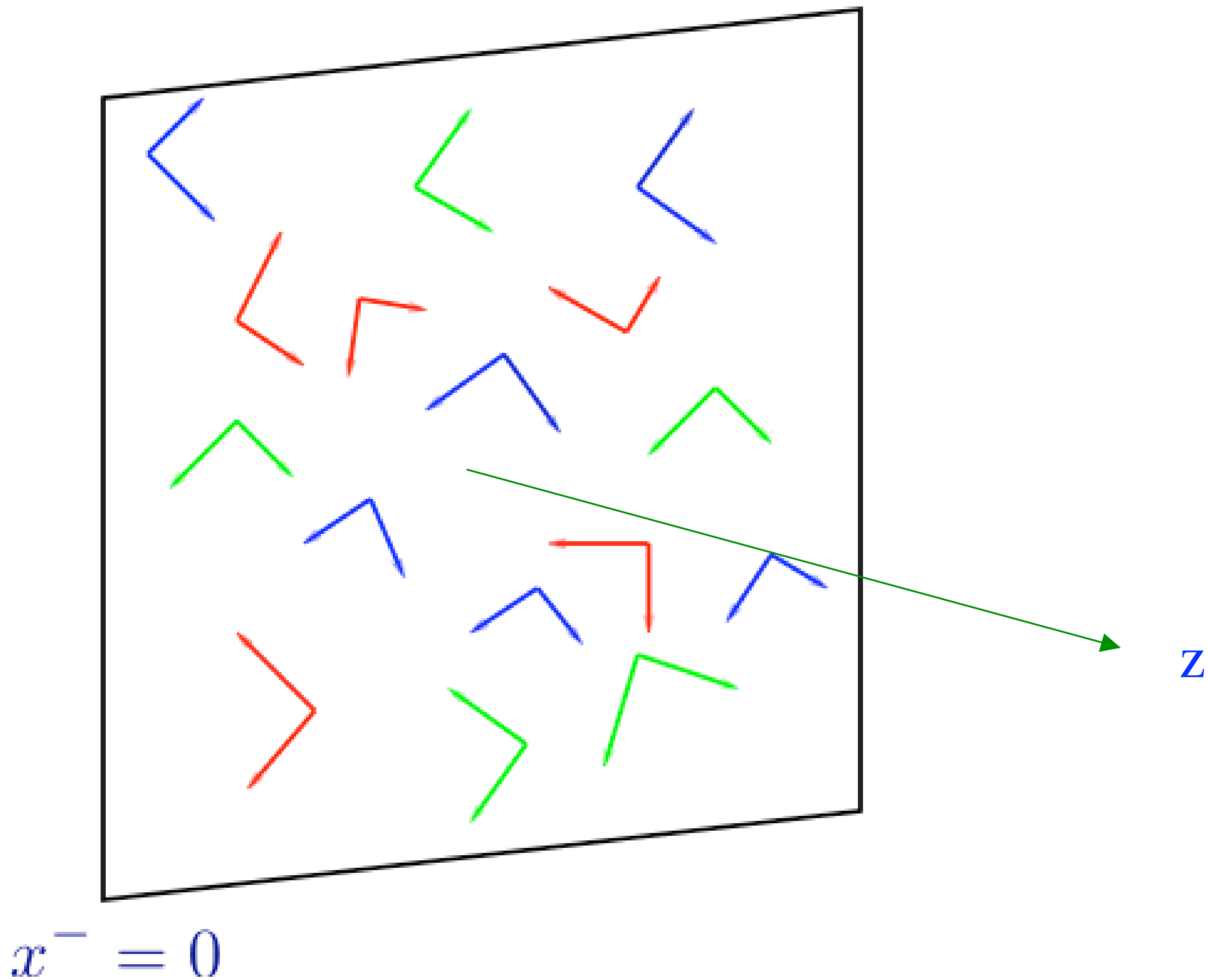


$$A^+ = A^- = 0 \; ;$$
$$F^{ij} = 0 => A^i = \theta(x^-)\alpha^i \, ,$$
$$\text{where } \alpha^i = \frac{-1}{ig} U \nabla^i U^\dagger$$
$$\text{and } \nabla \cdot \alpha = g\rho$$

Careful solution requires smearing in x^-



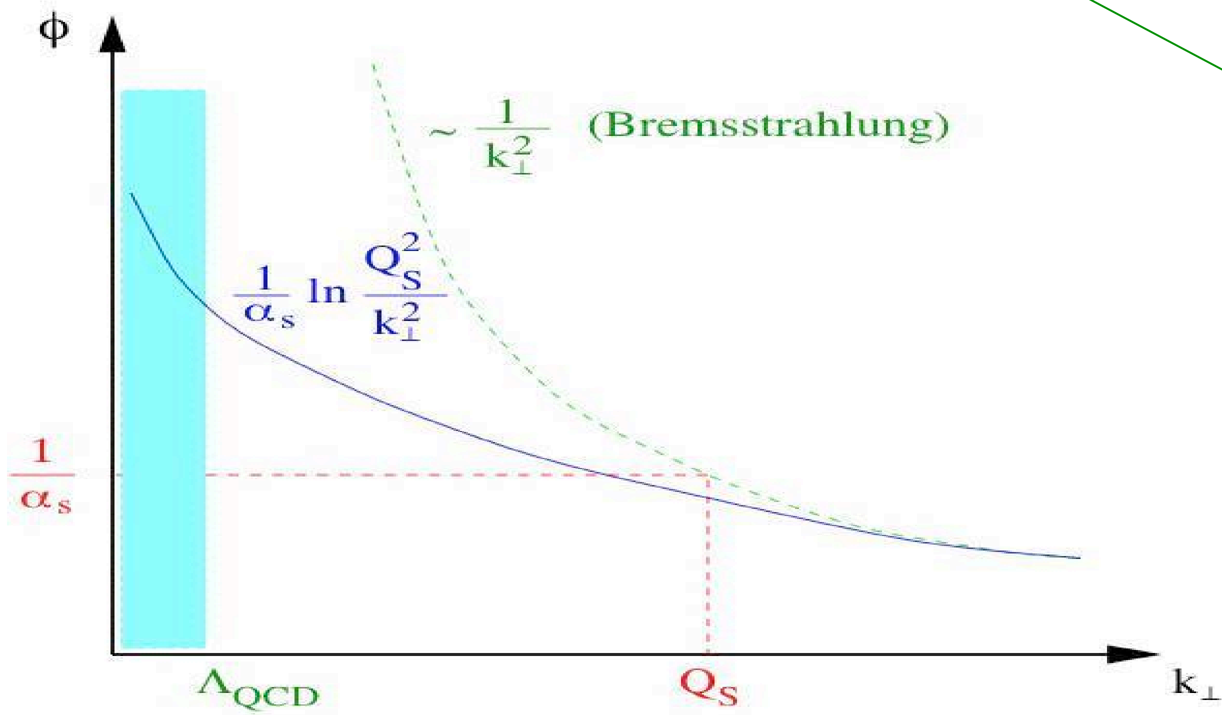
Random Electric & Magnetic fields in the plane of the fast moving nucleus



Average over ρ^a to compute gluon distribution $< AA >_\rho$

$$< AA >_\rho = \int [d\rho] A_{\mathrm{cl.}}(\rho) A_{\mathrm{cl.}}(\rho) W_{\Lambda^+}[\rho]$$

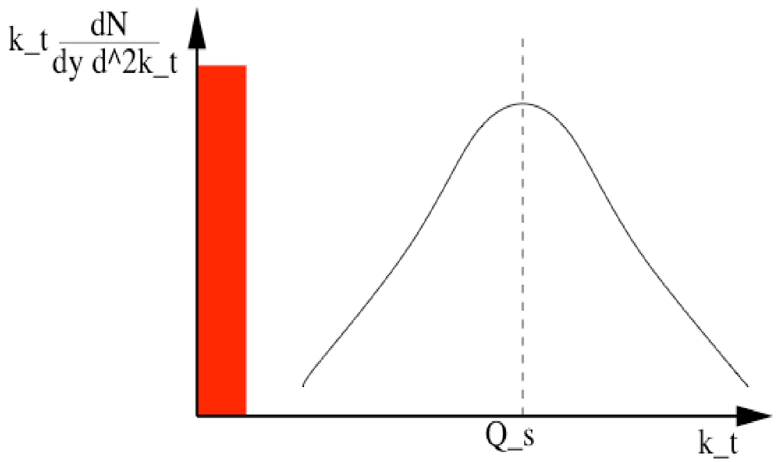
Gaussian in MV



ϕ = gluon phase space density = $\dfrac{(2\pi)^3}{2(N_c^2-1)} \dfrac{dN}{\pi R^2 d^2k_\perp dy}$

$Q_s^2 \approx \alpha_S N_c \mu_A^2 \ln\left(\frac{Q_s^2}{\Lambda^2}\right) \sim A^{1/3} \ln A \approx A^{1/3}$ for A >> 1

THE COLOR GLASS CONDENSATE



✓ *Typical momentum of gluons is* Q_s

✓ *Bosons with large occupation #* $\sim \frac{1}{\alpha_S}$ *- form a condensate*

✓ *Gluons are colored*

✓ *Random sources evolving on time scales much larger than natural time scales-very similar to spin glasses*

Hadron/nucleus at high energies is a Color Glass Condensate

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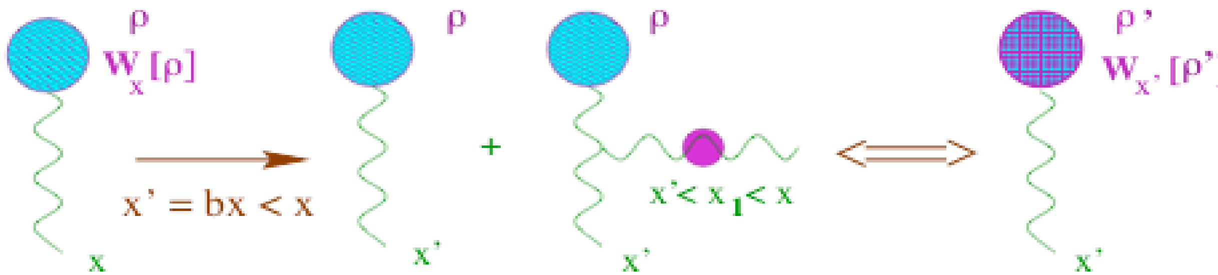
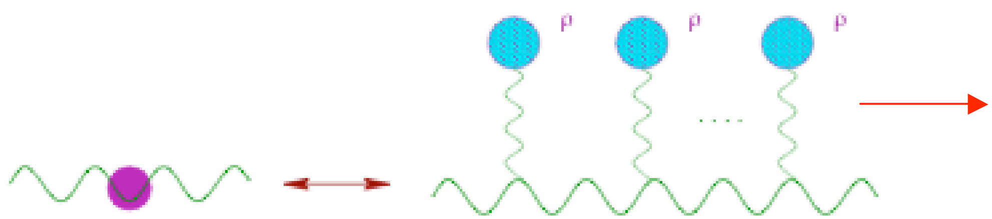
Constructive EFT: Wilson RG at small x



Color charge grows due to inclusion of fields into hard source with decreasing x: $\rho' = \rho + \delta\rho => W_x[\rho] \rightarrow W_{x'}[\rho']$



Because of strong fields $A \sim 1/g$
All insertions are O(1)

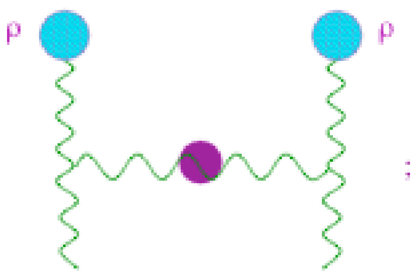
$W_x[\rho]$ *obeys a non-linear Wilson renormalization group equation-the JIMWLK equation*

(Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner)

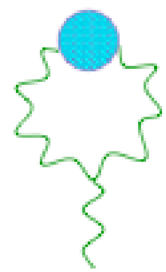
At each step in the evolution, compute 1-point and 2-point functions in the background field

$$\sigma^a(x)[\rho] = <\delta\rho^a_Y(x)>_\rho \; ; \; \chi^{ab}(x,y)[\rho] = <\delta\rho^a_Y(x)\delta\rho^b_Y(y)>_\rho$$

χ =



; σ =



$$\sigma^a(x) = \frac{1}{2}\int d^2y \frac{\delta\chi^{ab}(x,y)}{\delta\rho^b_Y(y)}$$

The JIMWLK (functional RG) equation:

$$\boxed{\frac{\partial W_x[\rho]}{\partial \ln(1/x)} = \frac{1}{2}\int_{x_\perp, y_\perp} \frac{\delta}{\delta\rho^a_x(x_\perp)} \chi^{ab}(x_\perp, y_\perp)[\rho] \frac{\delta}{\delta\rho^b_x(y_\perp)} W_x[\rho]}$$

$\Rightarrow$ An infinite hierarchy of ordinary differential equations for the correlators $< A_1 A_2 \cdots A_n >_y$

Correlation Functions

Change of variables: $\rho^a \rightarrow \alpha^a\,;\ \nabla^2\alpha = \rho$

$$< O[\alpha] >_Y = \int [d\alpha] O[\alpha] W_Y[\alpha]$$

Iancu, McLerran; Weigert

Brownian motion in functional space: Fokker-Planck equation!

"diffusion coefficient"

$$=> \frac{\partial}{\partial Y} < O[\alpha] >_Y = < \frac{1}{2} \int_{x,y} \frac{\delta}{\delta \alpha_Y^a(x)} \chi_{x,y}^{ab} \frac{\delta}{\delta \alpha_Y^b(y)} O[\alpha] >_Y$$

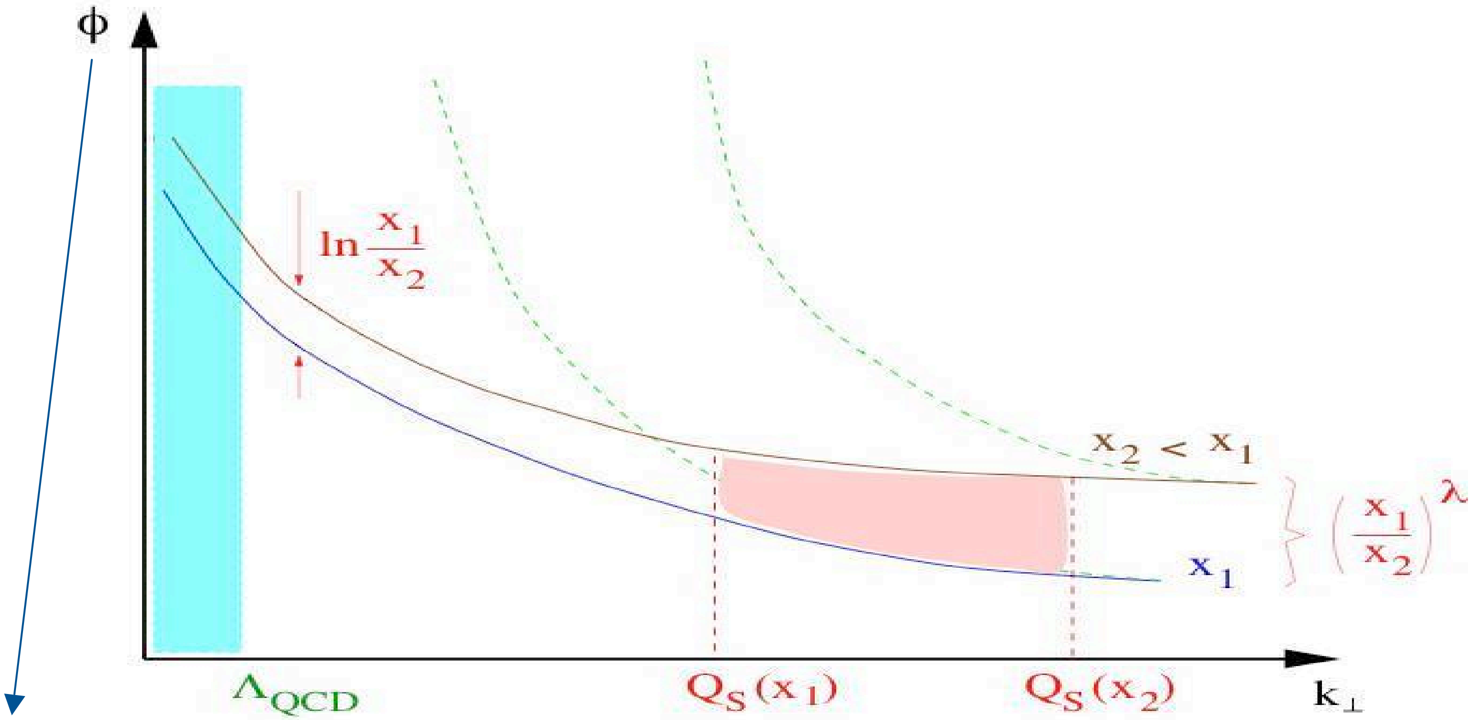
"time"

Consider the 2-point function: $< \alpha(x_\perp) \alpha(y_\perp) >_Y$

Can solve JIMWLK in the weak field limit: $g\,\alpha << 1$

Recover the BFKL equation in this low density limit

Can also solve JIMWLK in the strong field limit: $g\,\alpha \sim 1$ Iancu,McLerran



Gluon phase space density

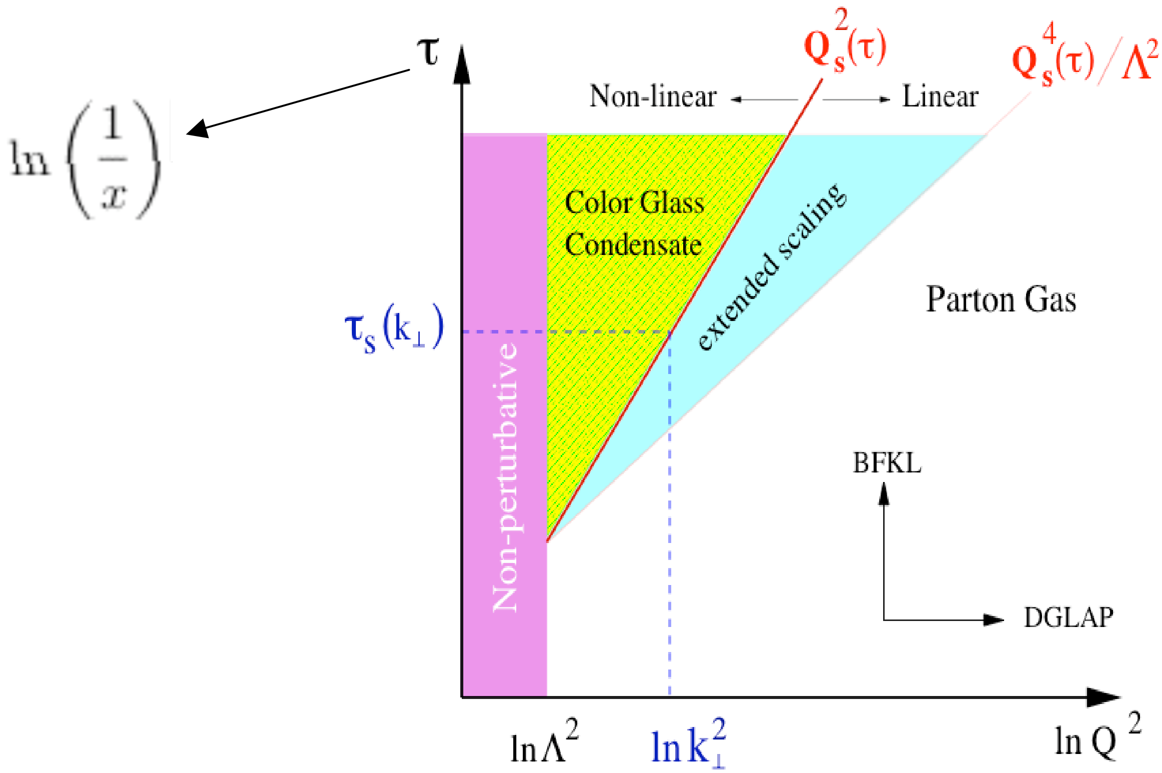
$$\ln k^2 >> \alpha_s Y (\mathrm{MV, DGLAP}) : \phi \approx \frac{\mu_A^2}{k^2}$$

$$\ln k^2 \sim \alpha_s Y \text{ but } k^2 >> Q_s^2(Y)\, (\mathrm{BFKL}) : \phi \approx \left(\frac{\mu_A^2}{k^2}\right)^{1/2} e^{\omega \alpha_S Y}$$

$$k^2 << Q_s(Y) : \phi \approx \frac{1}{\alpha_S} \ln\left(\frac{Q_s^2(Y)}{k^2}\right)$$

How does one compute Q_s(Y) ?

Novel regime of QCD evolution at high energies



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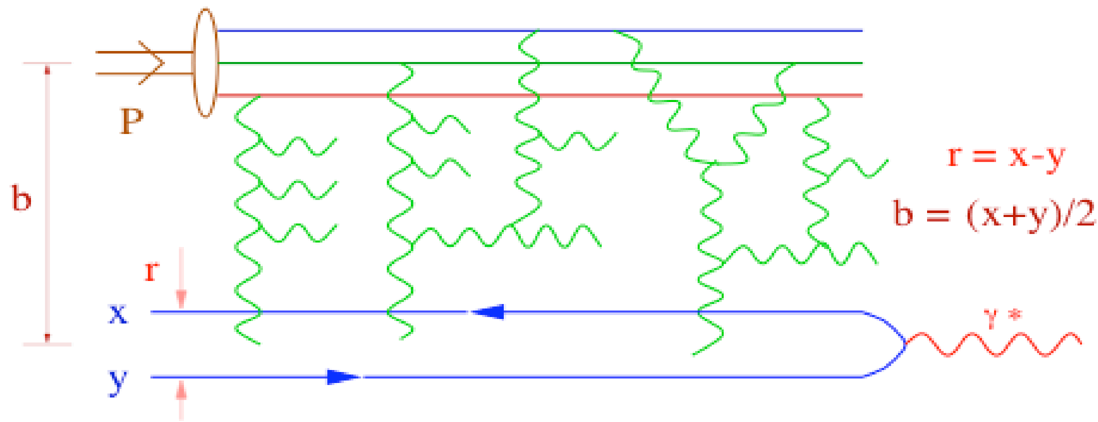
The Balitsky-Kovchegov equation

I. Balitsky;
Y. Kovchegov

DIS:



$$\sigma^{\gamma^* p}(x, Q^2) = \int_0^1 dz \int d^2 r |\psi(z, r; Q^2)|^2 \sigma_{\text{dipole}}(x, r)$$

where $\sigma_{\text{dipole}}(x, r) = 2 \int d^2 b \,(1 - S(x, r, b))$

McLerran,RV

with $S(x, r, b) = \frac{1}{N_c} < \text{Tr} V^\dagger(x) V(y) >_Y \equiv 1 - \mathcal{N}_Y(r, b)$

s-matrix   amplitude

Path ordered exponential $V^\dagger(x) = \mathcal{P} \exp\left(ig \int dx^- \alpha_a(x^-, x) T^a \right)$

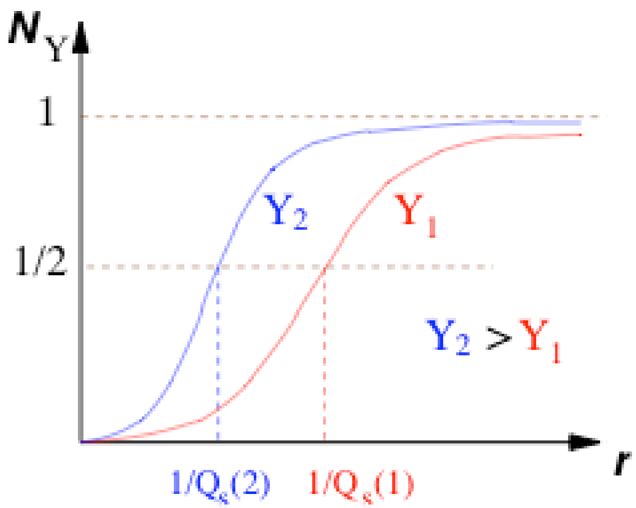
- Weak field limit: $V^\dagger(x) \approx 1 + ig\alpha(x)$; $g\alpha << 1$

=> $\mathcal{N}_Y(r) \sim \alpha_s r^2 \frac{xG(x,1/r^2)}{\pi R^2}$ *violates unitarity bound if N > 1*

- For $r > 1/Q_s(Y)$ dipole probes strong fields $(g\alpha \sim 1)$

Iancu-McLerran RPA => $< V^\dagger(x)V(y) >_Y << 1$ for $|x-y| >> 1/Q_s(Y)$

=> N ~ 1 - dipole unitarizes



Choose $\mathcal{N} = \frac{1}{2}$ as saturation condition to determine Q_s

BK: Evolution eqn. for the dipole cross-section

- The 2-point correlator $< V^\dagger(x) V(y) >$ in JIMWLK has

a closed form expression for $N_c \to \infty$ and $A >> 1$

$$\frac{\partial \mathcal{N}_Y(x,y)}{\partial Y} = \bar{\alpha}_s \int_z \frac{(x-y)^2}{(x-z)^2(y-z)^2} \{\underbrace{\mathcal{N}_Y(x,z) + \mathcal{N}_Y(z,y) - \mathcal{N}_Y(x,y)}_{\text{BFKL}} - \underbrace{\mathcal{N}_Y(x,z)\mathcal{N}_Y(z,y)}_{\text{Non-linear}}\}$$

- For small dipole , $(r << 1/Q_s(Y)) => \text{BFKL eqn.}$

$$\mathcal{N}_Y(r) \approx (r^2 Q_0^2)^{1/2} e^{\omega \bar{\alpha}_s Y} \exp\left(-\frac{\ln^2(1/r^2 Q_0^2)}{2\beta \bar{\alpha}_s Y}\right)$$

- From saturation condition,

$$\mathcal{N} = 1/2 \text{ when } r \sim 1/Q_s(Y) => \boxed{Q_s^2(Y) \approx Q_0^2\, e^{\lambda Y} \text{ with } \lambda \sim 4.8\,\alpha_s}$$

- For large dipole, $(r >> 1/Q_s(Y))$

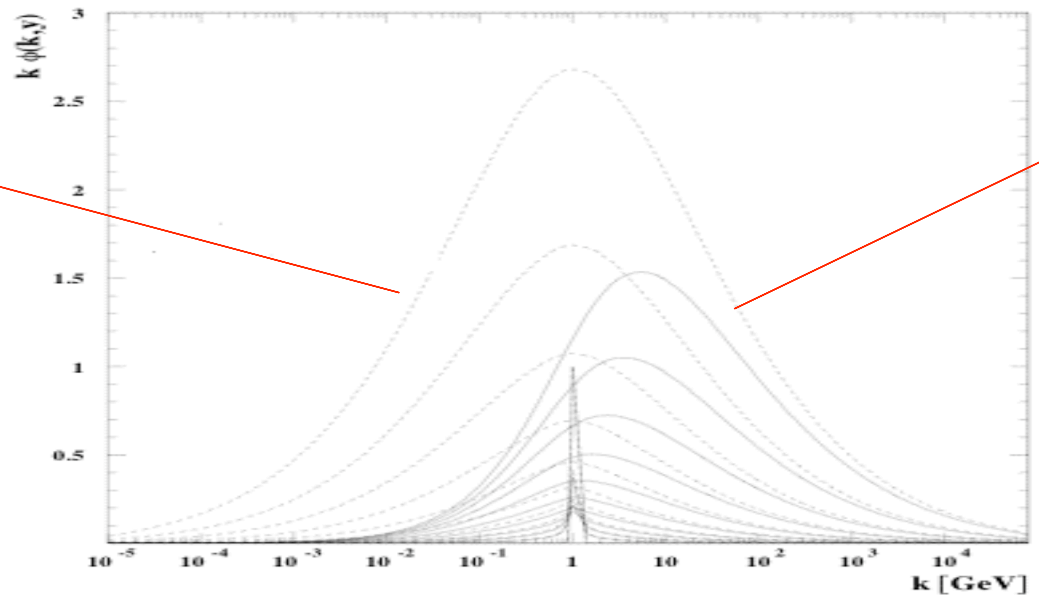
$$\mathcal{N}_Y(r) \approx 1 - \kappa \exp\left(-\frac{1}{4c} \ln^2(r^2 Q_s^2(Y))\right)$$

$c \approx 4.8$

Levin,Tuchin;
Iancu,McLerran;Mueller

Numerical solutions of the BK-Eqn.

BFKL



BK

Figure 1: The functions $k\phi(k, Y)$ constructed from solutions to the BFKL and the Balitsky-Kovchegov equations for different values of the evolution parameter $Y = \ln(1/x)$ ranging from 1 to 10. The coupling constant $\alpha_s = 0.2$.

From K. Golec-Biernat, L. Motyka, A. M. Stasto, Phys Rev D65 (2002) 074037; hep-ph/0110325

No infrared diffusion a la BFKL in BK

Exact analogy to travelling waves => Munier,Peschanski

Synopsis of CGC numerics

Numerical simulations of BK-eqn display Geometrical Scaling
(Armesto,Braun; Golec-Biernat,Stasto,Motyka)

Infrared diffusion pathology of BFKL is cured.

State of the art: numerical simulations of JIMWLK n-point correlators by Rummukainen & Weigert

Running coupling effects important & still to be understood…

Geometrical Scaling

Iancu, Itakura, McLerran; Mueller, Triantafyllopolous

Can write the solution of BFKL as:

$$\mathcal{N}_Y(r_\perp) \approx \exp\left(\omega\bar{\alpha}_s Y - \frac{\rho}{2} - \frac{\rho^2}{2\beta\bar{\alpha}_s Y}\right) \text{ with } \rho = \ln\frac{1}{r^2 Q_0^2}$$

ρ_S soln. where argument vanishes

$$=> Q_s^2 = Q_0^2\, e^{c\bar{\alpha}_s Y}, \text{with } c = 4.84$$

For $r_\perp < 1/Q_s$ (but close!), can write

$$\rho = \rho_S(Y) + \ln\frac{1}{r_\perp^2 Q_s^2} \equiv \rho_S + \delta\rho$$

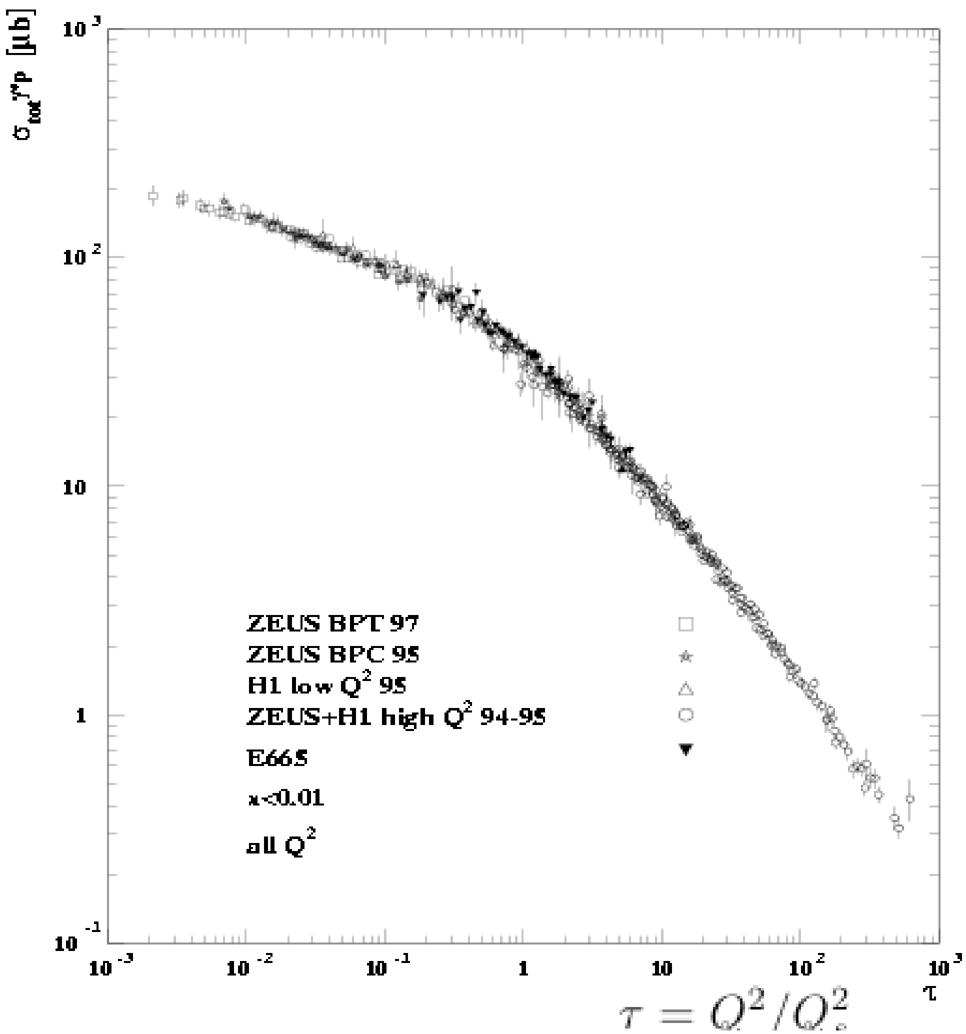
Plugging into N_Y, can show simply

$$\mathcal{N}_Y \approx \left(r_\perp^2 Q_s^2(Y)\right)^\gamma \text{ for } Q_s^2 << Q^2 << \frac{Q_s^4}{Q_0^2}$$

$\gamma \sim 0.64$ is large than BFKL anomalous dimension ~0.5

Geometrical scaling at HERA

(Golec-Biernat,Kwiecinski,Stasto)



Scaling seen for all x < 0.01 and $0.045 < Q^2 < 450\,\mathrm{GeV}^2$

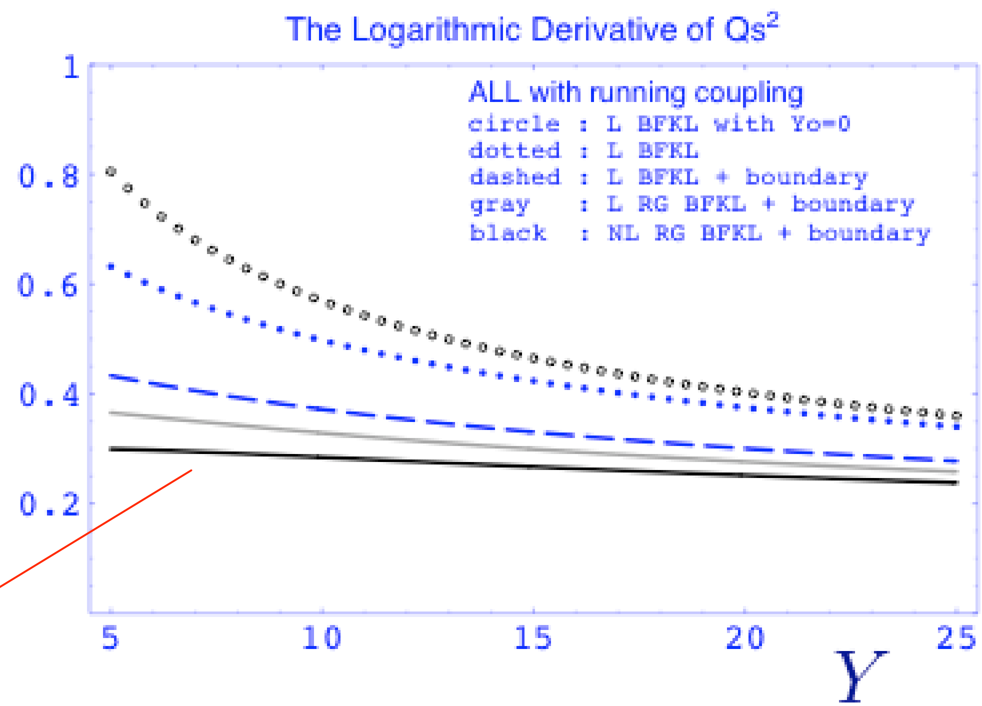
How does Q_s behave as function of Y?

Fixed coupling LO BFKL: $Q_s^2 = Q_0^2 e^{c\,\bar{\alpha}_s Y}$

LO BFKL+ running coupling: $Q_s^2 = \Lambda_{\rm QCD}^2 e^{\sqrt{2b_0 c(Y+Y_0)}}$

Re-summed NLO BFKL + CGC:

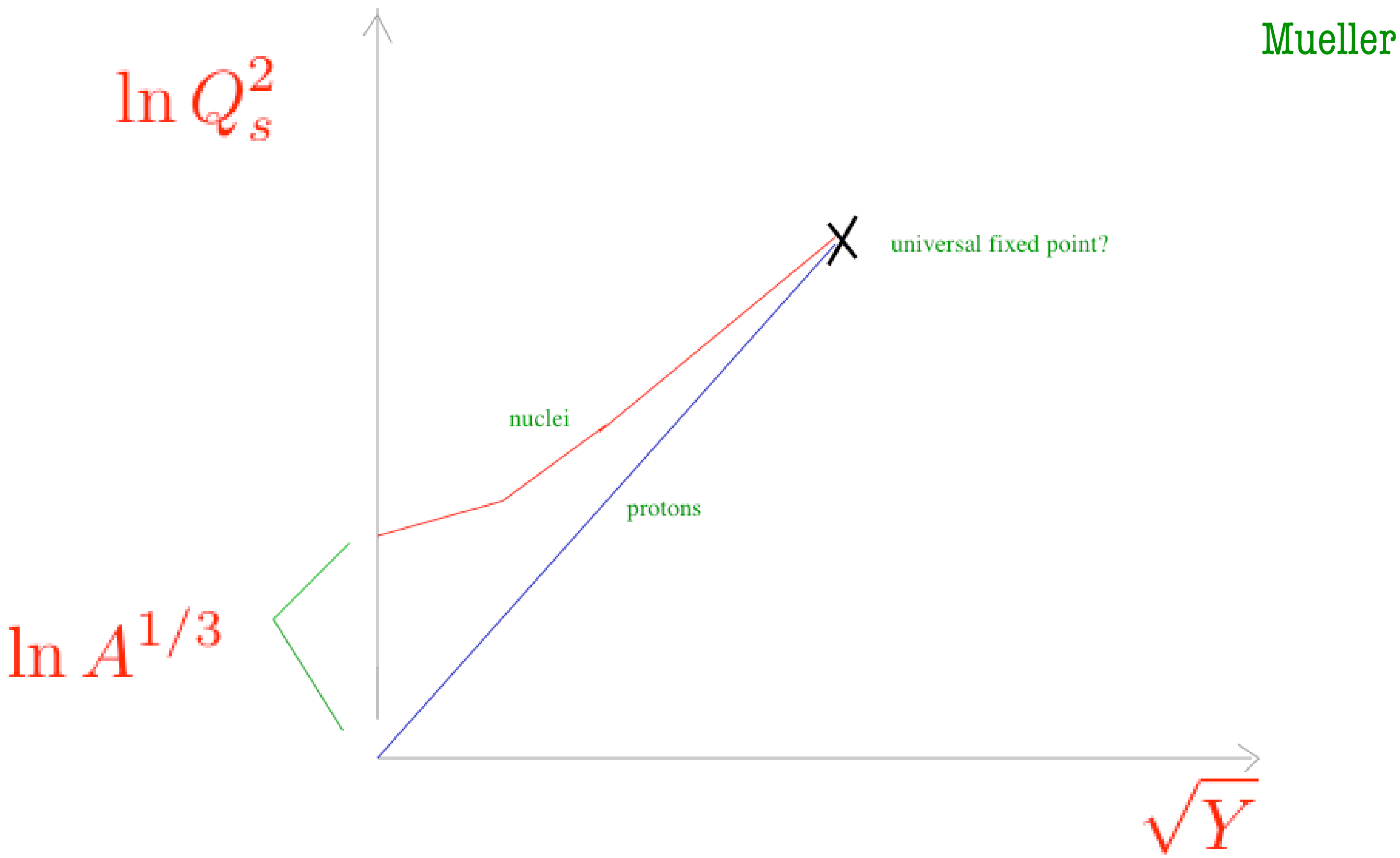
$$\lambda \equiv \frac{d \ln Q_s^2}{dY}$$



Triantafyllopolous

Very close to HERA result!

A-dependence of saturation scale



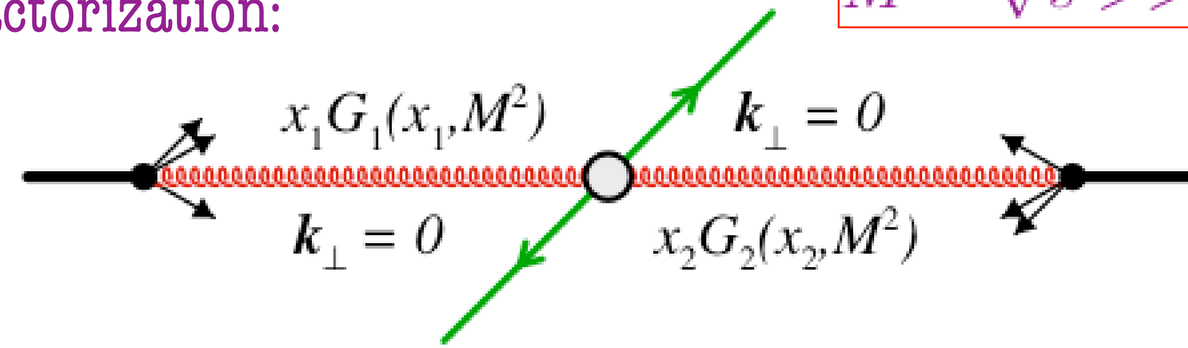
Such interesting systematics may be tested at LHC & eRHIC

Hadron & Nuclear Scattering at high energies

I: Universality: collinear versus k_t factorization

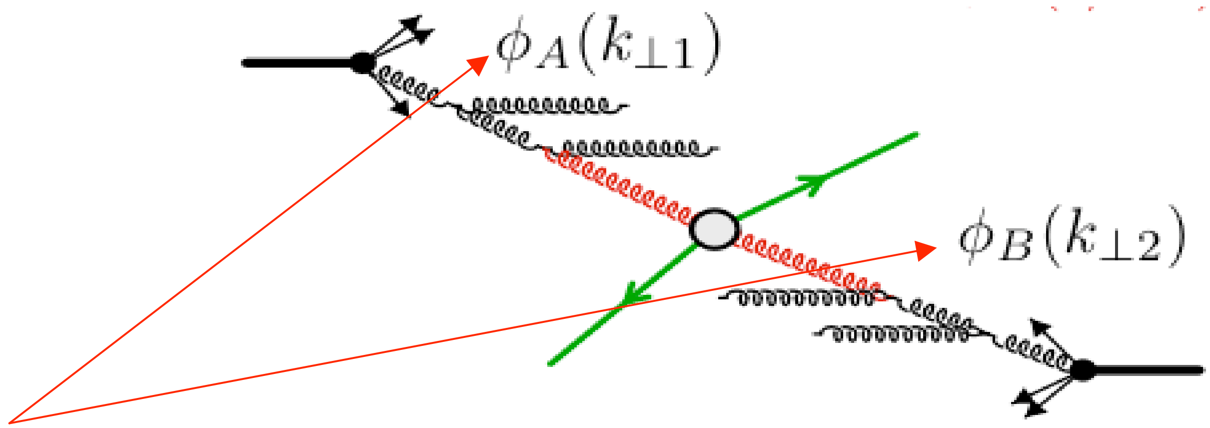
Collinear factorization:

$M \sim \sqrt{s} >> \Lambda_{QCD}$



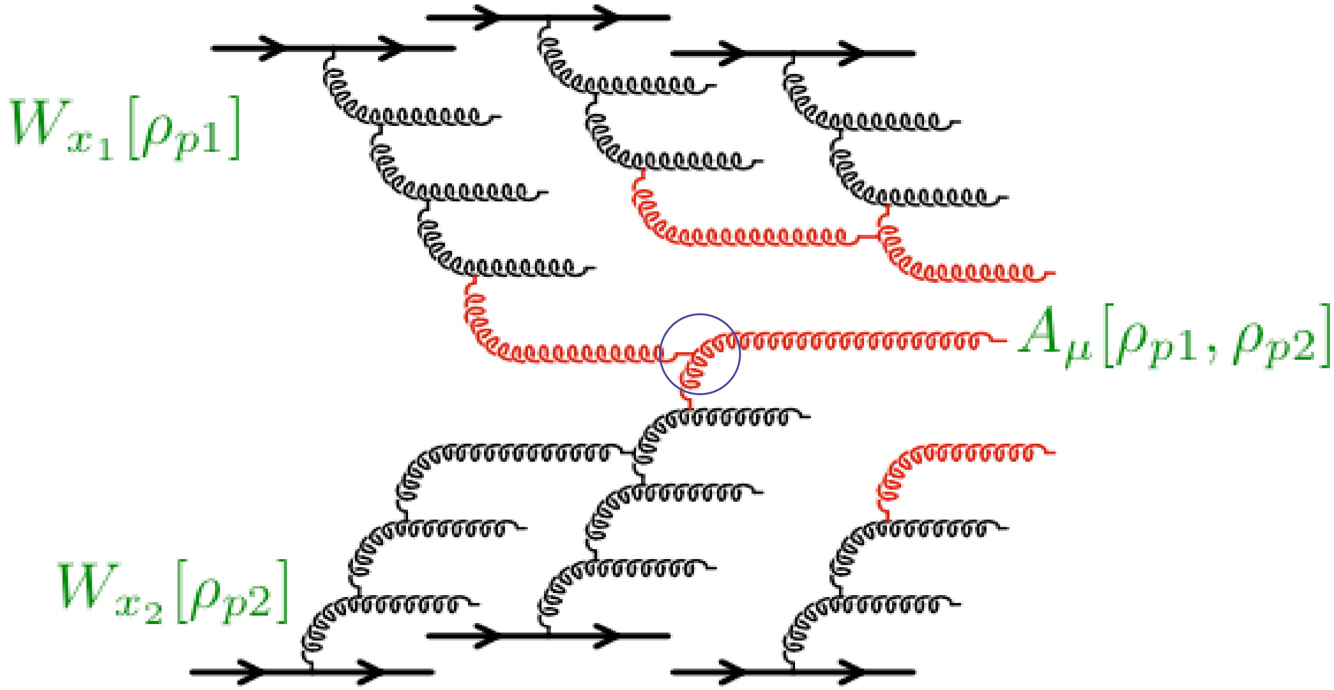
$\Lambda_{QCD} << M << \sqrt{s}$

k_t factorization:



Are these objects universal? Very important for extraction of "gluon" distributions.

Hadronic collisions in the CGC framework



Solve Yang-Mills equations for two light cone sources: ρ_{p1} & ρ_{p2}

For observables $O(A_\mu(\rho_{p1}, \rho_{p2}))$ average over $W_{x1}[\rho_{p1}]$ & $W[\rho_{p2}]$

Gluon production in high energy pA collisions:

$$N_g = \frac{4g^2 N_c}{\pi^2 (N_c^2 - 1) q_\perp^2} \int \frac{d^3 q}{(2\pi)^3 2E_q} \frac{d^2 k_\perp}{(2\pi)^2} \int d^2 x_\perp \frac{d\phi_p(k_\perp, x_\perp)}{d^2 X_\perp} \frac{d\phi_A(q_\perp - k_\perp, x_\perp - b)}{d^2 X_\perp}$$

$\phi_A(k_\perp, x_\perp) \propto < U^\dagger_{ab} U_{bc} >_{\rho_A}$ is non-linear in the gluon density to all orders-recover unintegrated gluon dist at large k_t

Quark production:

$$\frac{d\sigma^{pA \to q\bar{q}X}}{dy_p dy_A d^2 p_\perp q_\perp} \propto \phi_p \times [A\phi_{g,g} + (B\phi_{g;q\bar{q}} + c.c) + C\phi_{q\bar{q};q\bar{q}}]$$

$$< U_A(x_\perp) U_A^\dagger(y_\perp) >$$

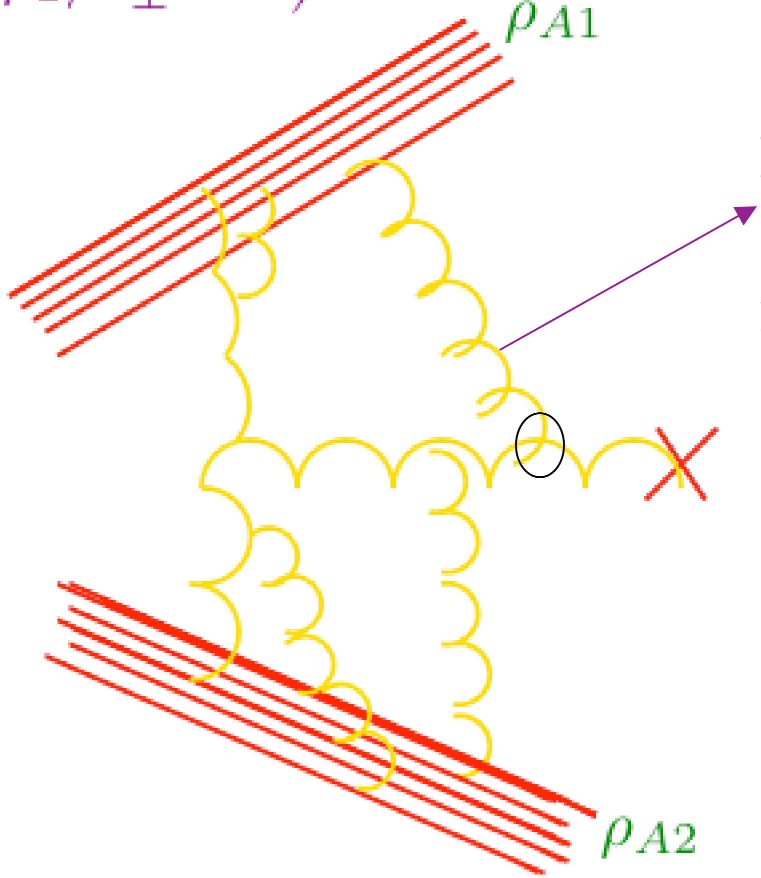
$$< U_F(x_\perp) \tau^a U_F^\dagger(y_\perp) \tau^{b'} (U_A^{a'b'})^\dagger(y'_\perp) >$$

$$< U_F(x_\perp) \tau^a U_F^\dagger(y_\perp) U_F(y'_\perp) \tau^b U_F(x'_\perp) >$$

Not k_t factorizable!

Gluon & Quark production in the dense/AA region

$(\rho_1/k_\perp^2 \sim \rho_2/k_\perp^2 \sim 1)$

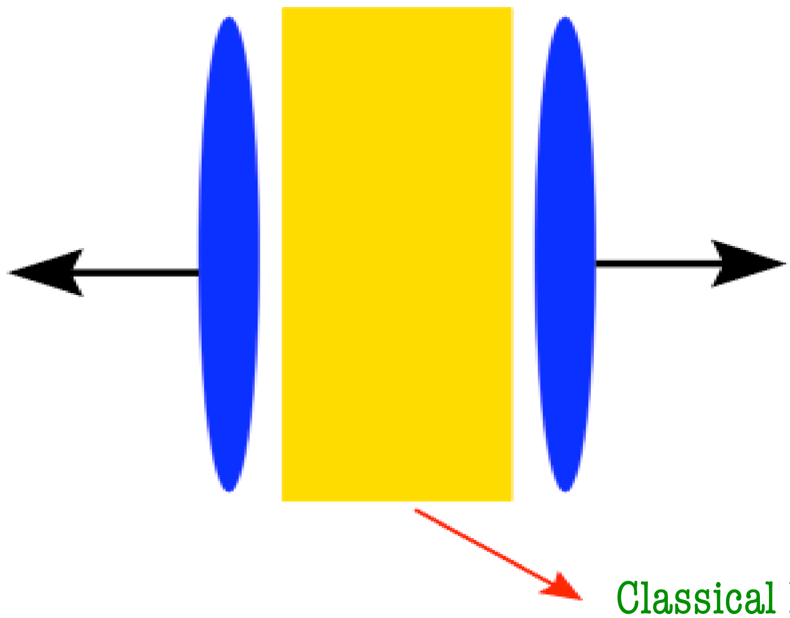


Breaks k_t factorization?
Contribution is same order in coupling

- Wave-fn evolution effects (beyond MV) difficult to include-work of Rummukainen & Weigert is promising.

- Classical evolution shows re-scattering-hence energy loss at eta=0 must be especially strong in AA!

Colliding Sheets of Colored Glass at High Energies



Classical Fields with occupation # f= $\frac{1}{\alpha_S}$

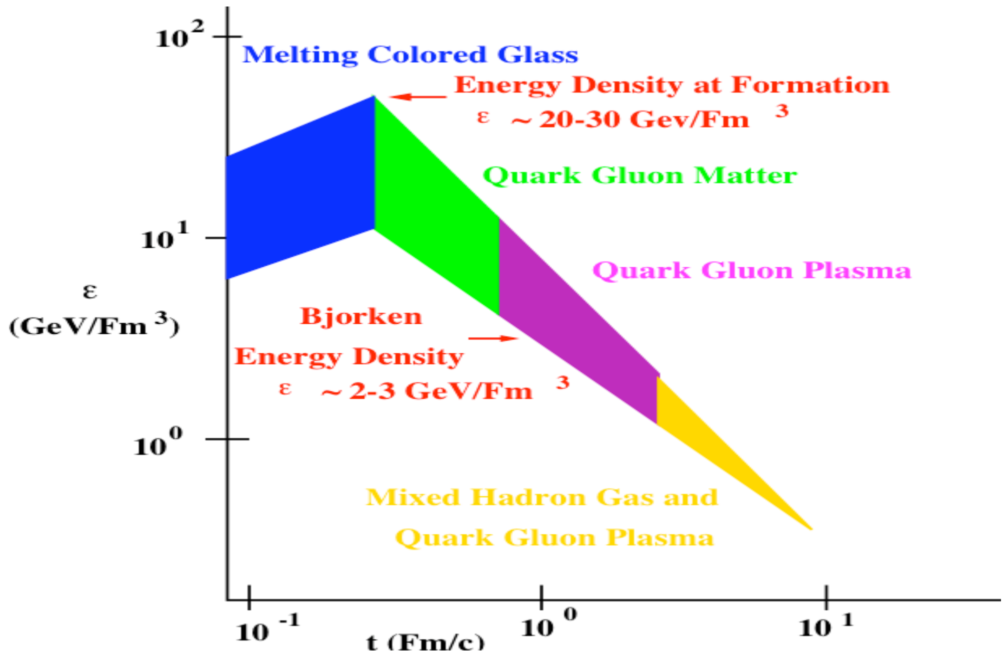
Initial energy and multiplicity of produced gluons depends on Q_s

$$\frac{1}{\pi R^2}\frac{dE_\perp}{d\eta} = \frac{0.25}{g^2} Q_s^3 \qquad \frac{1}{\pi R^2}\frac{dN}{d\eta} = \frac{0.3}{g^2} Q_s^2$$

Classical approach breaks down at late times when f <<1

$$\tau >> \frac{1}{Q_s} \text{ but } \tau << R$$

Space-time evolution in heavy ion collisions



Initial conditions determined by saturation scale Q_s >> T_i

$$\varepsilon \propto \frac{Q_s^4}{\alpha_S} \text{ at } \tau \sim \frac{1}{Q_s}$$

Q_s=1.4-2 GeV from HERA data
extrapolations and numerical simulations

Do initial state (wave fn.) effects or final state (parton re-scattering) dominate in the space-time evolution?