Heavy quarks and new scales: Understanding subtleties of QCD

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Outline

1. **Heavy quarks**
   - Charm, beauty and truth
   - What is a heavy quark?
2. **Using heavy quarks to understand QCD**
   - Going beyond DIS and Drell Yan
   - Interpreting the initial state
   - Matrix elements
   - Interpreting the final state
3. **Conclusions**

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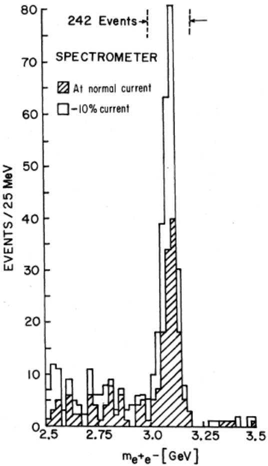
A charming discovery



The first heavy quark, charm was discovered in 1974 in $p\bar{p}$ collisions at BNL and e^+e^- at SLAC

The observations were published together:
PRL 33, 1404 (1974); PRL 33, 1406 (1974)

The J/ψ was recognized as a $c\bar{c}$ bound state
$\Rightarrow m_c \sim 1.5$ GeV

The existence of a 4th quark confirmed the Glashow-Iliopoulos-Maiani explanation for why FCNC decays ($s \to d\nu\bar{\nu}$) did not occur.
— And it loosened the shackles of $\mathrm{SU}(3)_{\mathrm{flavor}}$, Gell-Mann's "Eightfold way"

A charming crisis

While the J/ψ was clearly a quark bound state,
it had an extremely narrow width of 88 keV.

This caused a minor crisis in the fledgling QCD...

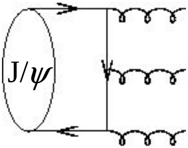
After all how could a strongly interacting state be narrow?
$\Gamma_\rho \sim 150$ MeV, $\Gamma_\omega \sim 8.5$ MeV, $\Gamma_\phi \sim 4.3$ MeV, $\Gamma_{J/\psi} \sim 88$ keV

An explanation was found by Appelquist and Politzer, PRL 34, 43 (75).

Write the width as
$\Gamma(^3S_1 \to 3 \text{ gluons}) = |R(0)|^2 |M(q\bar{q} \to ggg)|^2$

Following the model of positronium, solve the Schroedinger Eqn. for $R(r) = \frac{2}{a_0^{3/2}} e^{-r/a_0}$,

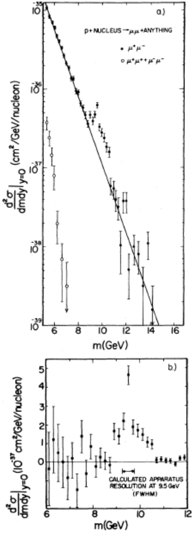
where $a_0 = \frac{1}{\alpha_s m_c/2}$.



$|M(q\bar{q} \to ggg)|^2 \sim \alpha_s^3$ — one power for each gluon

$\Rightarrow \Gamma(^3S_1 \to 3 \text{ gluons}) \sim 0.2\, \alpha_s^6\, m_c \sim 90 \text{ keV}; \alpha_s \approx 0.26$

A beautiful discovery

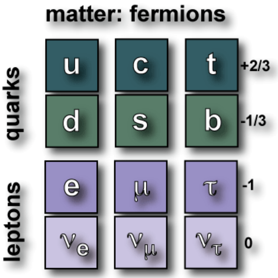


In 1975 the τ was discovered and led to the search for other 3rd-generation particles.

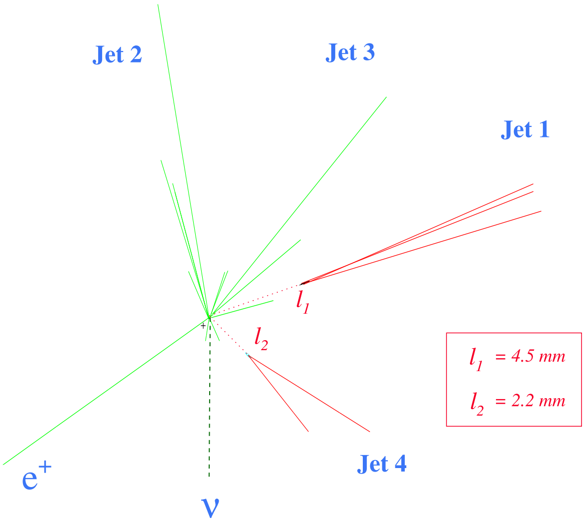
In 1977 the Upsilon (a $b\bar{b}$ bound state) was observed at the Fermilab Tevatron. PRL 39, 252 (1977)

(The Upsilon is also very narrow.)

Once the bottom quark was found it was clear that a sixth quark was needed to complete the family structure.



"This is the top quark!"



$M_{top}^{Fit} = 170 \pm 10$ GeV/c^2

24 September, 1992
run #40758, event #44414

What is a "heavy quark?"

Usual definition: A heavy quark is a quark with $m_q \gg \Lambda_{\rm QCD}$.

	Pole mass M	$\overline{\rm MS}$ mass $\overline{m}(\overline{m})$
Charm	~ 1.3–1.7 GeV	1.275 ± 0.025 GeV
Bottom	~ 4.5–5 GeV	4.18 ± 0.03 GeV
Top	$173.34 \pm 0.27 \pm 0.24 \pm 0.67$ GeV (?)	160^{+5}_{-4} GeV

PDG (+prelim 2014)

Pole Mass: $\sim \frac{1}{\not p - M}$

$\overline{\rm MS}$ Mass: Related to pole mass by

$$\frac{M}{\overline{m}(\overline{m})} = 1 + \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left(-1.0414 \ln(M^2/\overline{m}^2) + 13.4434\right) + \ldots$$

It seems kind of funny to list 2 different masses...

c and b masses are best written in $\overline{\rm MS}$ scheme.

t mass is given in pole-mass scheme.

What is a heavy quark mass?

Answer 1: A parameter of the Lagrangian $\mathcal{L} \sim m_t \bar{t} t$

A weak answer, but if the number is big enough we can expand in inverse powers of the mass to create a convergent series. (E.g., HQET)

Answer 2: An effective (Yukawa) coupling between t–t–h
$m_t = Y_t/(2\sqrt{2}G_F)^{1/2}$, $Y_t \approx 1.00$ in the SM

This is better, as the Standard Model predicts that quark masses are not fundamental, but rather an artifact of dynamical interactions.

Answer 3: The kinematic mass seen by the experiments

Right after the discovery of the top quark, Martin Smith and Scott Willenbrock asked this question about the "pole mass" of the top quark. They showed that a renormalon (the closest pole of the Borrel transform) induced an ambiguity of $\mathcal{O}(\Lambda_{QCD})$ in the definition of the pole mass.

This led to the recommended use of the $\overline{\mathrm{MS}}$ mass for top quarks.

We theorists are good at setting standards that make our life easier . . . most perturbative calculations use the $\overline{\mathrm{MS}}$ mass for simplicity.

Of course mass is NOT measured directly. Instead, it affects the distribution of events that are measured, and that distribution is used to INFER the mass by matching to a calculation. . . .

What is a heavy quark mass?

Answer 4: A new scale in the problem.

This will both complicate our calculations and lead to new insights into the meaning of QCD structures that are hidden when we ignore quark masses.

The key is context.

Depending on the other scales in the problem, a heavy quark mass may teach us something deep about the physics, or be completely irrelevant.

E.g., most mass corrections go like $\mathcal{O}(m^2/\mu^2)$

Homework: Show in the top quark width $\Gamma(t \to bW)$, dropping m_b loses terms of $\mathcal{O}(m_b^2/m_t^2) \sim 1\%$.

In the rest of this lecture we will concentrate on what we learn from corrections that go like $\mathcal{O}[\ln(m_t^2/m_b^2)]$.

Outline

Structure of an observable cross section

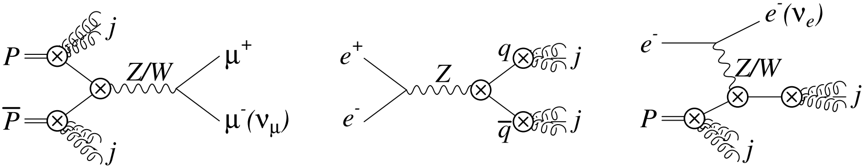
$$\sigma_{\rm obs.} = \int f_1(x_1,\mu_1) f_2(x_2,\mu_2) \otimes \overline{|M|^2} \otimes d\mathrm{P.S.} \otimes D_i(p_i)\ldots D_n(p_n)$$

Theorists factorize (break) the cross section into:

- Initial-state IR singularities swept into parton distribution "functions".
  These are not physical, but include scheme dependent finite terms:
  $\overline{\rm MS}$ — the current standard
  DIS — ill-defined in all modern PDF sets, could be fixed, but why?
- A squared matrix element, which represents the bulk of the perturbative calculation effort.
- Phase space which you may not want to completely integrate out.
  $\Rightarrow$ Exclusive cross sections (jet counting), angular correlations
- Fragmentation functions or jet definitions.
  These provide the coarse graining to hide final-state IR singularities.

Drell-Yan and DIS

The traditional testbed of perturbative QCD have been restricted to Drell-Yan production, e^+e^- to jets, or deep inelastic scattering (DIS).

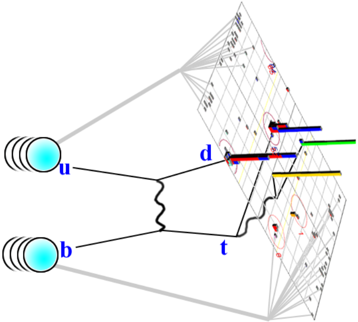


A key property that all three processes share is a complete factorization of QCD radiation between different parts of the diagrams.

- Drell-Yan $\rightarrow$ Initial-state (IS) QCD radiation only.
- $e^+e^- \rightarrow$jets $\rightarrow$ Final-state (FS) QCD radiation only.
- DIS $\rightarrow$ Proton structure and fragmentation functions probed. Simple color flow.

A heavy quark testbed for QCD: single top

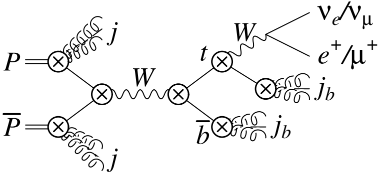
Experimentalist: Single top quark production is the observation of $b\,\ell^{\pm}\not{E}_T$ that reconstruct to a top quark mass, plus an extra jet (or two).



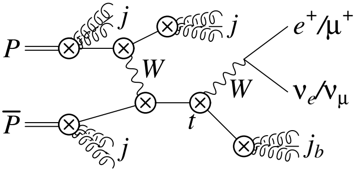
Theorist: Single top quark production is a playground in which we refine our understanding of perturbative QCD in the presence of heavy quarks.

s-/t-channel single-top-quark production (A generalized Drell-Yan and DIS)

A perfect factorization through next-to-leading order (NLO) makes single-top-quark production mathematically *identical*[†] to DY and DIS!



Generalized Drell-Yan.
IS/FS radiation are independent.



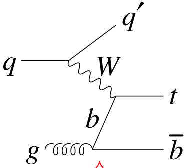
Double-DIS (DDIS) w/ 2 scales:
$\mu_l = Q^2$, $\mu_h = Q^2 + m_t^2$

Color conservation forbids the exchange of just 1 gluon between the independent fermion lines.

† Massive forms: m_t, m_b, and m_t/m_b are relevant.

Rethinking the initial state: W-gluon fusion $\rightarrow$ t-channel single-top

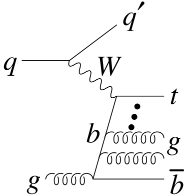
W-gluon fusion (circa 1996)



$\Uparrow$

$\sim \alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right) + \mathcal{O}(\alpha_s)$

$m_t \approx 35 m_b$! $\alpha_s \ln \sim .7$-$.8$



Each order adds

$\frac{1}{n!}\left[\alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right)\right]^n$

Looks bad for perturbative expansion…

Look at the internal b.

The propagator is

$\frac{1}{(P_g - P_{\bar b})^2 - m_b^2} = \frac{1}{-2P_g \cdot P_{\bar b}}$

$P_g = E_g(1,0,0,1)$, $P_{\bar b} = (E_b, \vec{p}_T, p_z)$

$P_g \cdot P_{\bar b} = E_g\left(p_z\sqrt{1+\frac{p_T^2+m_b^2}{p_z^2}} - p_z\right)$

$\approx E_g p_z\left(\frac{p_T^2+m_b^2}{2p_z^2}\right) \sim (p_T^2 + m_b^2)$

$\int_{p_{T\,\mathrm{cut}}} \frac{\mathrm{d}p_T^2}{p_T^2+m_b^2} \rightarrow \ln\left(\frac{1}{p_{T\,\mathrm{cut}}^2+m_b^2}\right)$

The same procedure for the W leads to the massive formula for DIS.

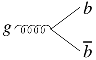
$\sigma \sim \alpha_s \ln\left(\frac{Q^2+m_t^2}{p_{T\,\mathrm{cut}}^2+m_b^2}\right)$

We now have multiple scales entering the problem: $Q, m_t, m_b, p_{T\,\mathrm{cut}}$

Resummation of large logs and b PDF

The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation sums large logs in (almost) collinear singularities in gluon splitting.

$$\frac{db(\mu^2)}{d\ln(\mu^2)} \approx \frac{\alpha_s}{2\pi} P_{bg} \otimes g + \frac{\alpha_s}{2\pi} P_{bb} \otimes b; \qquad b \ll g$$

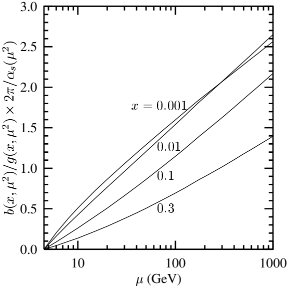


$$P_{bg}(z) = \tfrac{1}{2}[z^2 + (1-z)^2]$$

$$b(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{m_b^2}\right) \int_x^1 \frac{dz}{z} P_{bg}(z) g\left(\frac{x}{z},\mu^2\right)$$

Barnett, Haber, Soper, NPB 306, 697 (88)
Olness, Tung, NPB 308, 813 (88)
Aivazis, Collins, Olness, Tung, PRD 50, 3102 (94)

$b \propto \alpha_s \ln(\mu^2/m_b^2) \times g$



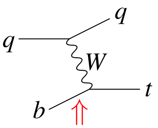
Stelzer, ZS, Willenbrock, PRD 56, 5919 (1997)

The procedure is the same for c or t.

Aside: In the $\overline{\rm MS}$ scheme, $b(\mu \le m_b) \equiv 0$.
DIS scheme is not uniquely defined for heavy quarks.
Do you choose $F_2 \equiv 0$ (traditional) or define w.r.t. $\overline{\rm MS}$?
The first attempt to calculate single-top failed because the DIS scheme was used.
Bordes, van Eijk, NPB435, 23 (95)

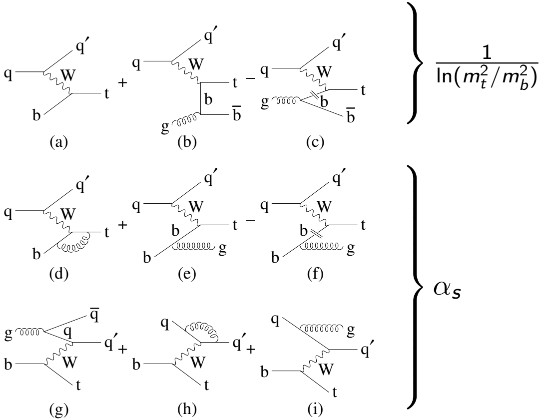
Remove 1 scale (m_b) w/improved perturbation theory

New Leading Order



$b \sim \alpha_s \ln\left(\frac{\mu^2}{m_b^2}\right) \times g$

The t-channel W exchange naturally lead to the nomenclature of t-channel production



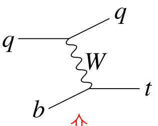
NLO: Terms that generated large logs are already resummed.

⇒ Must subtract overlap to avoid double-counting (general issue)

⇒ Reorders PT into 2 types of corrections: α_s and $\frac{1}{\ln(m_t^2/m_b^2)}$ w.r.t. LO

New nomenclature and classification

New Leading Order



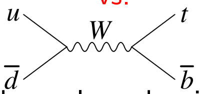
$(P_W^2 < 0)$

⇑

$b \sim \alpha_s \ln\left(\frac{\mu^2}{m_b^2}\right) \times g$

t-channel production
Named for the "*t*-channel" exchange of a *W* boson.

vs.

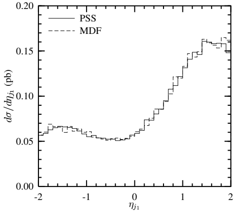


$(P_W^2 > 0)$

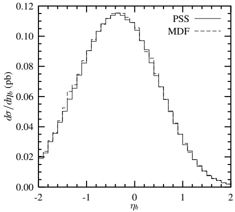
s-channel production
Named for the "*s*-channel" exchange of a *W* boson.

Classifying processes by analytical structure leads directly to kinematic insight:

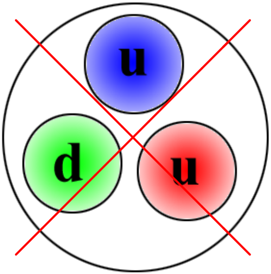
Jets from *t*-channel processes are more forward than those from *s*-channel.

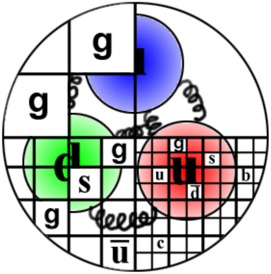


jet from *t*-channel



b jet from *s*-channel



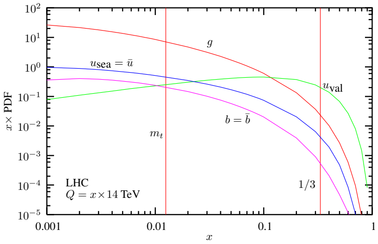


Using DGLAP was NOT just a math trick!

This "valence" picture of the proton is not complete.

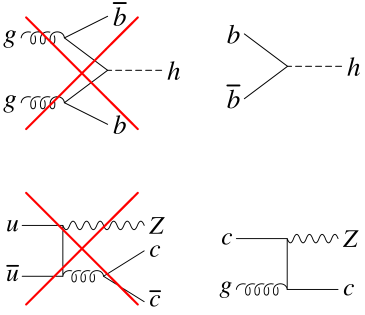
Larger energies resolve smaller structures.

The probability of finding a particle inside the proton is given by PDFs (Parton Distribution Functions)



b (and c) quarks are full-fledged members of the proton structure.

Rethinking several physical processes



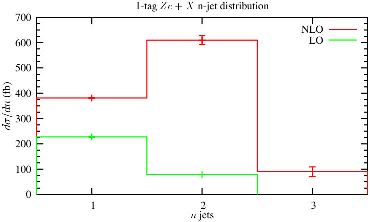
Starting with a c/b gives us:

$b\bar{b} \to h$	Largest SUSY Higgs cross section
Zb/Zc	Affects LHC luminosity monitor
Zbj/Zcj	Higgs background
Wbj	Largest single-top background

etc.

Why is this important?

Zc at Tevatron



Parton luminosity can be more important than counting powers of α_s!

This is exaggerated > 10 TeV:

$Z \approx Z + 1\text{ jet} \approx Z + 2\text{ jets!}$

(True of $W + X$ as well!)

How well-defined (theoretically) is jet counting at LHC?

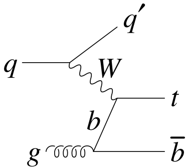
Rethinking the matrix element: A practical problem for experiments

The same large logs that lead to a reordered perturbation for t-channel single-top, implied a potentially large uncertainty in measurable cross sections when cuts were applied.

Recall: t-channel and s-channel are distinguished by the number of b-jets.

A problem: About 20% of the time, the extra $\bar{b}$-jet from the t-channel process is hard and central.

Real problem: Is the b contamination 20%, 30%, 10%?



Another problem: To distinguish from $t\bar{t}$, the cross section in the $W + 2$ jet bin has to be known.

Counting jets is IDENTICAL to performing a jet veto.

Inclusive cross sections are not enough, we need to calculate exclusive cross sections

Fully Differential NLO Techniques

- In 2001, there were few matrix-element techniques or calculations that could deal IR singularities in processes with massive particles.
- Experiments were mostly stuck using LO matrix elements to predict semi-inclusive or exclusive final states.
- We needed methods to provide the 4-vectors, spins, and corresponding weights of exclusives final-state configurations.

These needs led to work on 3 techniques:

- Phase space slicing method with 2 cutoffs.
  L.J. Bergmann, Ph.D. Thesis, FSU (89)
  cf. H. Baer, J. Ohnemus, J.F. Owens, PRD 40, 2844 (89)
  B.W. Harris, J.F. Owens, PRD 65, 094032 (02)
- Phase space slicing method with 1 cutoff.
  W.T. Giele, E.W.N. Glover, PRD 46, 1980 (92)
  cf. W.T. Giele, E.W.N. Glover, D.A. Kosower, NPB 403, 633 (93)
  E. Laenen, S. Keller, PRD 59, 114004 (99)
- Massive dipole formalism (a subtraction method) coupled with a helicity-spinor calculation. Invented to solve single-top production.
  cf. L. Phaf, S. Weinzierl, JHEP 0104, 006 (01)
  S. Catani, S. Dittmaier, M. Seymour, Z. Trocsanyi, NPB 627,189 (02)

Phase Space Slicing Method (2 cutoffs)

B.W. Harris, J.F. Owens, PRD 65, 094032 (02)

The essential challenge of NLO differential calculations is dealing with initial- and final-state soft or collinear IR divergences.

$$\sigma_{\mathrm{obs.}} \sim \int \frac{1}{s_{ij}} \sim \int \frac{dE_i dE_j d\cos\theta_{ij}}{E_i E_j (1-\cos\theta_{ij})}$$

If $E_{i,j} \to 0$ "soft" singularity
If $\theta_{ij} \to 0$ "collinear" singularity

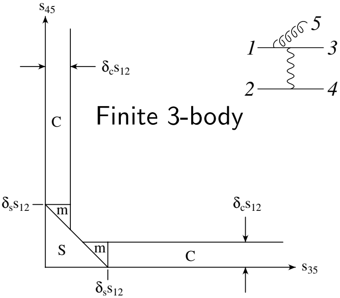
IDEA: Introduce arbitrary cutoffs (δ_s, δ_c) to remove the singular regions...

We traded dependence on physical observables (energy, angles) for logarithmic dependence on arbitrary parameters ($\ln\delta_s$, $\ln\delta_c$)

Divide phase space into 3 regions:

1. soft: $E_g \le \delta_s \sqrt{\hat{s}}/2$ gluons only
2. collinear: $\hat{s}_{35}, \hat{s}_{45}, \ldots < \delta_c \hat{s}$;
3. hard non-collinear: (finite, particles well separated, $E > 0$)

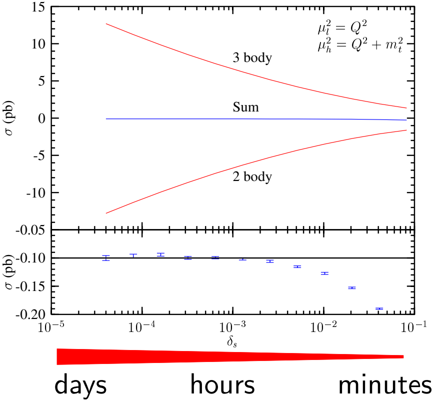
Phase space plane (s_{35}, s_{45})



Cut-off dependence of NLO correction

Each term is logarithmically divergent for small δ_s, δ_c

Logarithmic dependence on the cutoffs cancels in any IR-safe observable at the histogramming stage.



At the end we take δ_s and δ_c to zero via numerical computation. This can take a long time. . .

Massive Dipole Formalism (subtraction)

$$\sigma_{NLO} = \int_{n+1} d\sigma^{\text{Real}} + \int_n d\sigma^{\text{Virtual}}$$
$$= \int_{n+1} \left(d\sigma^R - d\sigma^A\right) + \int_n \left(d\sigma^V + \int_1 d\sigma^A\right)$$

- $d\sigma^A$ is a sum of color-ordered dipole terms.
  - $d\sigma^A$ must have the same point-wise singular behavior in D dimensions as $d\sigma^R$.
    $\Rightarrow$ $d\sigma^A$ is a local counterterm for $d\sigma^R$.
  - $\int_1 d\sigma^A$ is analytic in D dimensions, and reproduces the soft and collinear divergences of $d\sigma^R$.
- Some advantages over Phase Space Slicing are:
  - You can easily project out spin eigenstates.
    $\Rightarrow$ Explicitly test different spin bases at NLO after cuts.
  - Event generators use color-ordered matrix elements.
- Both methods have some contribution to n-body final states from $n+1$ phase-space. Hence, you must do 2 separate integrations.

Subtraction vs. phase space slicing

In practical terms, the difference in methods is in how to integrate in the presence of infrared singularities.

$$I = \lim_{\epsilon \to 0^+} \left\{ \int_0^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\}$$

Subtraction: Add and subtract $F(0)$ under the integral

$$\begin{aligned} I &= \lim_{\epsilon \to 0^+} \left\{ \int_0^1 \frac{dx}{x} x^\epsilon \left[F(x) - F(0) + F(0)\right] - \frac{1}{\epsilon} F(0) \right\} \\ &= \int_0^1 \frac{dx}{x} \left[F(x) - F(0)\right], \quad \text{finite up to machine precision} \end{aligned}$$

PSS: Integration region divided into two parts $0 < x < \delta$ and $\delta < x < 1$, with $\delta \ll 1$. A Maclaurin expansion of $F(x)$ yields

$$\begin{aligned} I &= \lim_{\epsilon \to 0^+} \left\{ \int_0^\delta \frac{dx}{x} x^\epsilon F(x) + \int_\delta^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\} \\ &= \int_\delta^1 \frac{dx}{x} F(x) + F(0) \ln \delta + \mathcal{O}(\delta), \quad \text{take } \lim_{\delta \to 0} \text{ numerically} \end{aligned}$$

Remaining $\ln \delta$ singularities removed by summing all integrals I_i.

Rethinking jet definitions and phase space: Experiments need exclusive $t + 1$ jet at NLO

ZTOP, Z.S., PRD 70, 114012 (2004) [hep-ph/0408049]

	# b-jets	tj (Wbj)	tjj ($Wbjj$)
s-channel	$= 2$	0.620 pb $^{+13}_{-11}$%	0.168 pb $^{+24}_{-19}$%
	$= 1$	0.022 pb $^{+24}_{-19}$%	(NNLO)
t-channel	$= 1$	0.950 pb $^{+16}_{-15}$%	0.152 pb $^{+17}_{-14}$%
	$= 2$	0.146 pb $^{+21}_{-16}$%	0.278 pb $^{+21}_{-16}$%

Cuts: $p_{Tj} > 15$ GeV, $|\eta_j| < 2.5$, no cuts on t

Jet definition: $\Delta R_{k_T} < 1.0$ ($\approx \Delta R_{\rm cone} < 0.74$)

Breakdown of *shape-independent* uncertainties

Process		$\times\delta m_t$(GeV)	$\mu/2$–2μ	PDF	b mass	$\alpha_s(\delta_{\rm NLO})$
s-channel	$p\bar{p}$	$^{-2.33}_{+2.71}$%	$^{+5.7}_{-5.0}$%	$^{+4.7}_{-3.9}$%	$< 0.5\%$	$\pm1.4\%$
	pp	$^{-1.97}_{+2.26}$%	$\pm2\%$	$^{+3.3}_{-3.9}$%	$< 0.4\%$	$\pm1.2\%$
t-channel	$p\bar{p}$	$^{-1.6}_{+1.75}$%	$\pm4\%$	$^{+11.3}_{-8.1}$%	$< 1\%$	$\pm0.01\%$
	pp	$^{-0.73}_{+0.78}$%	$\pm3\%$	$^{+1.3}_{-2.2}$%	$< 1\%$	$\pm0.1\%$

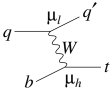
Every number here, even the concept of t-channel single-top, required a new or revised understanding of QCD.

- b PDFs $\rightarrow$ t-channel
- PDF uncertainties
- multiple scales: m_t/m_b
- 2 expansions: α_s, $1/\ln$
- Fully differential NLO jet calculations

. . .

Scale (μ) dependence of the t-channel jets and top

- The shapes of the p_T and η distributions do not change if you vary the scales. Only the normalization changes.
- If you vary the 4 independent scales† at the same time you underestimate the uncertainty.



$\mu/2 - 2\mu$	LO_t (m_t)	NLO_t (m_t)	LO_t (DDIS)	NLO_t (DDIS)
fixed	0.95 pb	1.03 pb	1.07 pb	1.06 pb
μ_l & μ_h	$\pm 1\%$	$\pm 2.5\%$	${}^{+0.1}_{-2}\%$	$\pm 3.5\%$
μ_h	${}^{-7.5}_{+5.5}\%$	${}^{-3.5}_{+4}\%$	${}^{-7.2}_{+5.2}\%$	${}^{-3}_{+4}\%$
μ_l	${}^{+6.7}_{-5.8}\%$	$\pm 1\%$	${}^{+8}_{-6.8}\%$	$\pm 0.6\%$

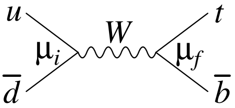
- Summing the independent variations in quadrature predicts $\sim \pm 11\%$ uncertainty at LO (consistent with the results).
- At NLO we get $\sim \pm 4\%$ uncertainty due to scale variation.

† (2 factorization, 2 renormalization)

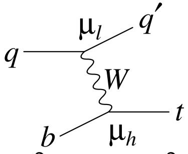
Scale variation to estimate higher-order uncertainty

Standard lore says that the choice of scale in a perturbative calculation is arbitrary... Standard lore is not quite correct.

If single-top-quark production were exactly Drell-Yan or DIS, then there are unique scale choices.



$\mu_f \approx \mu_i \approx M_{tb}^2$



$\mu_l = Q^2$, $\mu_h = Q^2 + m_t^2$.

The PDFs were extracted assuming these scales. Therefore, it is mathematically inconsistent to choose any other scale for DY or DIS.

This means there is a subtle (small?) systematic error in all calculations that had not been previously recognized.

With the DDIS choice of scales, the NLO correction to the inclusive cross section is zero within errors.

This will be true for some particle distributions as well!

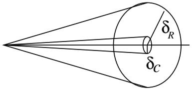
How do we interpret exclusive NLO calculations?

Z.S., PRD 70, 114012 (2004)

"Paradigm of jet calculations"

- We are calculating extensive objects,
  i.e., jets not "improved quarks."
- Unlike inclusive NLO calculations, exclusive NLO calculations are only well-defined in the presence of a jet definition or hadronization function. ($D_i(p_i)$)

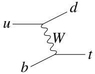
  $\Rightarrow$ The mathematics of quantum field theory tells us we cannot resolve the quarks inside of these jets!



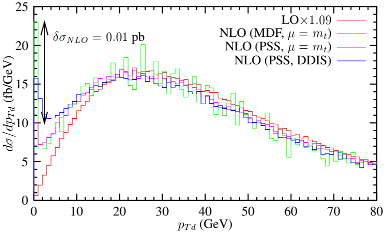
- "Bad things" happen if you treat jets as NLO partons. . .

Transverse momenta distributions at NLO

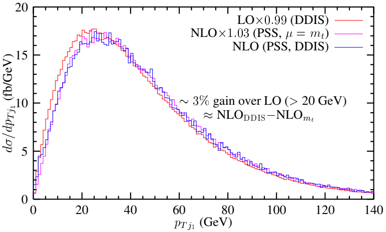
At LO, a d-quark recoils against the top quark in t-channel.



NLO "d-jet" (no cuts)



We measure the highest E_T jet



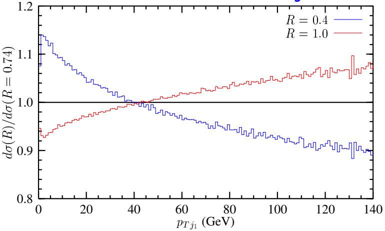
- Perturbation theory is not terribly stable at low p_{Td} (or even high p_{Td}).
- This is not what we want.
  Be careful what you ask for!

The highest E_T jet recoils against the top. The measurable change in shape is comparable to the scale uncertainty.

Jet distributions depend on jet definition

Just like the experimentalists, theorists must study the effect jet algorithms with different cone sizes R will have on measurable properties.

Ratio of $d\sigma(R)/dp_{Tj}$ to $d\sigma(R = 0.74)/dp_{Tj}$



For "reasonable" values of R the variation is $< 10\%$, but this must be checked for all observables. (Note: theoretical uncertainty $< 5\%$)

Upshot: NLO exclusive calculations give jets not partons.

Without some thought, mismatches between theory and experiment can be larger than the theory error alone would indicate.

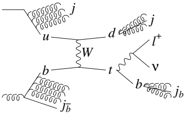

THEORY
Experiment

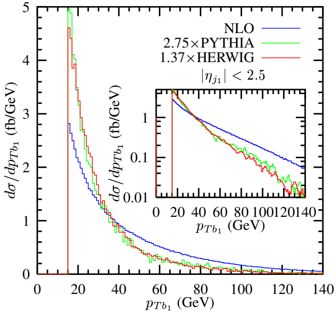
Event generators vs. NLO t-channel $t\bar{b}$ ($Wb\bar{b}$)

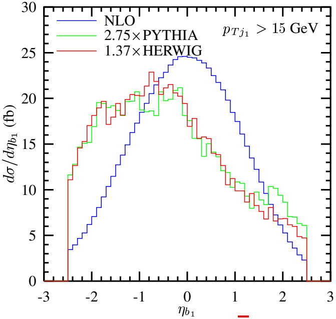
Z.S., PRD 70, 114012 (2004)

Initial-state radiation (ISR) is generated by backward evolution of angular-ordered showers.

$\Rightarrow$ The jet containing the extra $\bar{b}$ comes from soft ISR.







- PYTHIA/HERWIG completely underestimate the $Wb\bar{b}$ final state.
- Lesson: n-jets+showers $\neq n+1$ jets. $\Rightarrow$ Need NLO matching. (Schemes have since proliferated: MLM, CKKW, SCET, . . .)

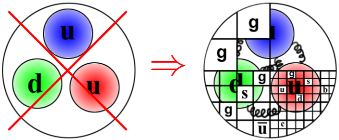
Conclusions

"Heavy quarks" (c, b, and t) are interesting because their mass adds a new scale to any problem.

- Tomorrow you will head more about the top quark from Prof. Yang
- Today we used the top quark as a tool to better understand some aspects of perturbative QCD.

Single-top-quark production is the new DIS and Drell-Yan

$$\sigma_{\text{obs.}} = \int f_1(x_1, \mu_1) f_2(x_2, \mu_2) \otimes \overline{|M|^2} \otimes d\text{P.S.} \otimes D_i(p_i) \ldots D_n(p_n)$$

- 

  — b/c PDFs are inside the proton
  — For construction and uncertainties cf. Prof. Nadolsky
  ⇒ New processes & new questions

- Exclusive jet observables require careful mathematical techniques
  - Requires an understanding of jet definitions, cf. Prof. Li
  - Experiments need to use NLO matched Monte Carlo programs, cf. Profs. Schoenherr and Prestel

Conclusions

- Heavy quarks generally introduce terms to a cross section of the form:

$$\sigma \sim \alpha_s \ln\left(\frac{\mu^2}{p_{T\,\rm cut}^2 + m_Q^2}\right)$$

  These terms can appear in the initial or final state, and need to be resummed.

  When you see logs of this type, it is often a hint there is something deeper to be learned.

- There was not enough time to describe heavy quark jet substructure, but heavy quarks will play an increasingly important role at the high energy LHC as they leave distinctive detector signatures you will hear in a few days from Prof. Li.
- Prof. Chen will describe experimental results for heavy quarks

Homework: Show in the top quark width $\Gamma(t \to bW)$, dropping m_b loses terms of $\mathcal{O}(m_b^2/m_t^2) \sim 1\%$.

THANK YOU

Additional slides

The top quark as a parton

In general, we do not consider the top quark when discussing proton structure.

The reason is simple: We do not tend to measure at scales far enough above m_t to ignore its mass.

Dawson, Ismail, and Low (1405.6211) recently revisited this issue and demonstrated it was indeed not sensible for <u>inclusive</u> cross sections at <u>100</u> TeV.

However, the p_T distributions for some processes, such as $H^+ + X$ production need a top PDF to get the correct result.