Introduction to Jets

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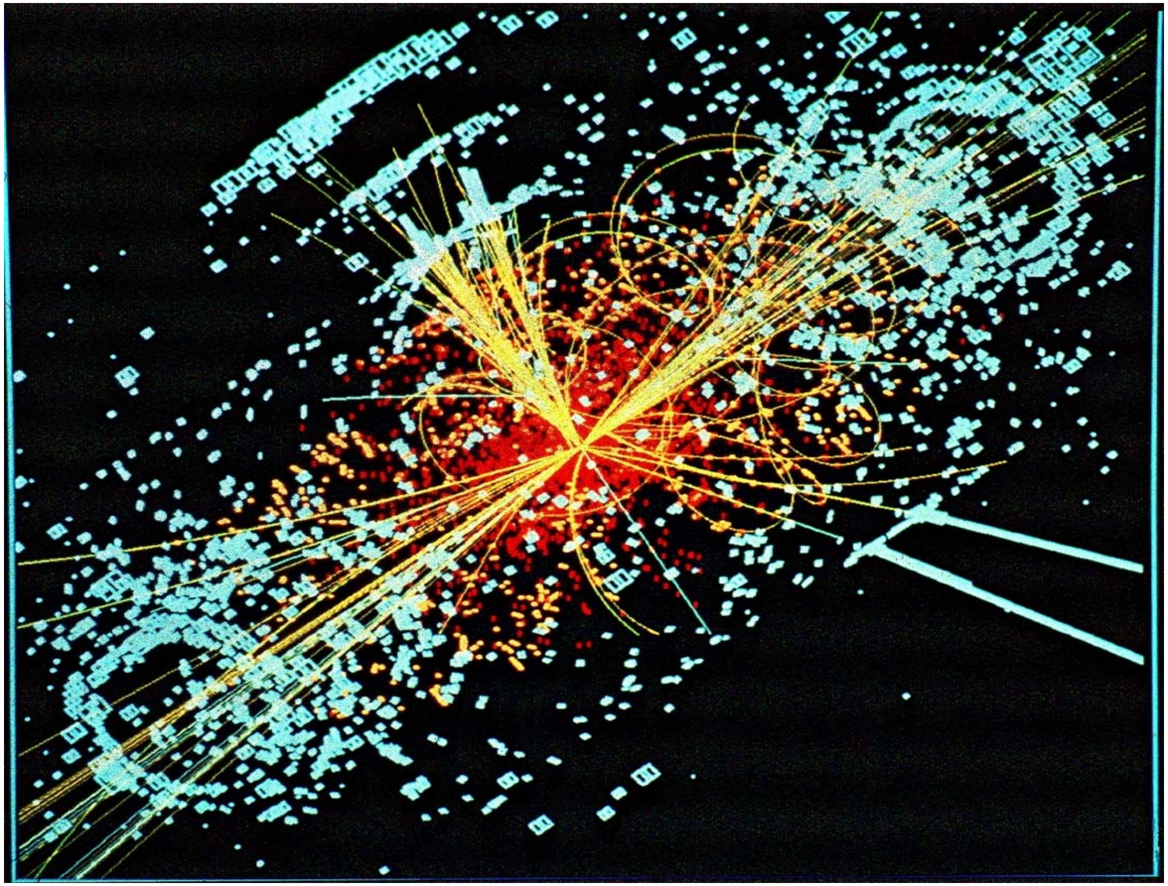
Outlines

- Introduction
- e+e- annihilation and jets
- Jets in experiment
- Jets in theory
- Summary

Introduction

- Jets are abundantly produced at colliders
- Jets carry information of underlying events, hard dynamics (strong and weak), and parent particles, including particles beyond the Standard Model
- Study of jets is crucial; comparison between theory and experiment is nontrivial
- Usually use event generators
- How much can be done in PQCD?

A jet event



- What can we learn from this mess?

Identification of heavy particles

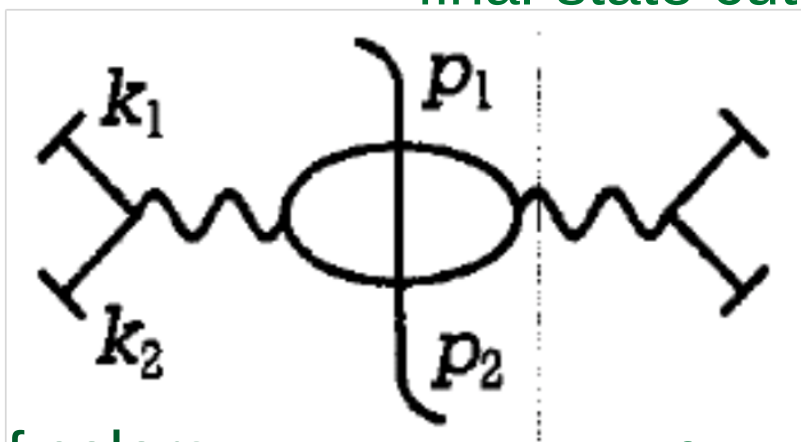
- Heavy particles decay quickly
- Easier to collect decay products of boosted heavy particles
- W, Z -> q q’, H -> b b, t-> W b -> q q’ b
- Need to differentiate signal p p -> W -> q q’ from background p p -> q q’
- Jet substructures help identification of boosted heavy particles

e^+e^- annihilation and jets

Soper's lecture

e+e- annihilation

- cross section = |amplitude|^2
- Born level



number of colors

$$\sigma_{\text{tot}} = N(4\pi\alpha^2/3q^2)\left(\sum_f Q_f^{\,2}\right)$$

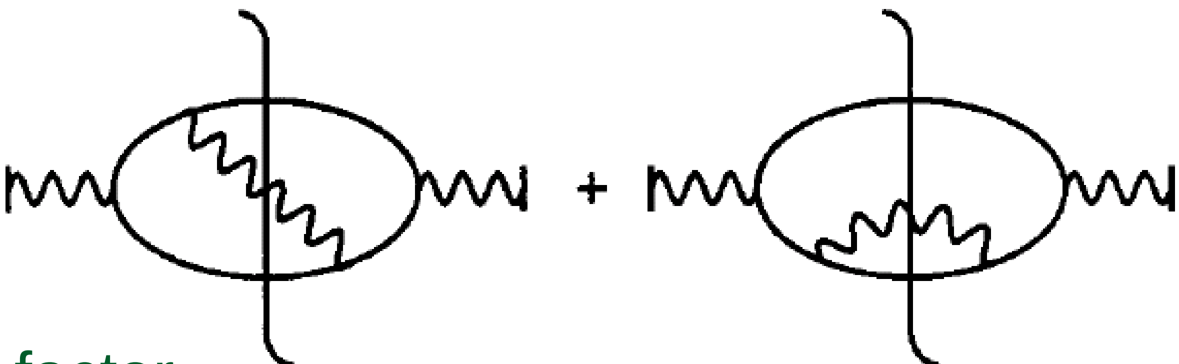
fermion charge

momentum transfer squared

Real corrections

- Radiative corrections reveal two types of infrared divergences from on-shell gluons
- Collinear divergence: l parallel P1, P2
- Soft divergence: l approaches zero

overlap of collinear and soft divergences

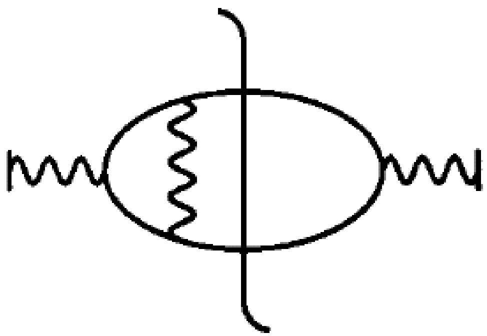


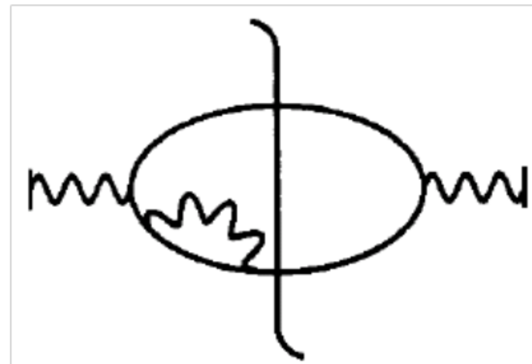
color factor

$$2NC_2(F)Q_f{}^2(\alpha\alpha_s/\pi)q^2(4\pi\mu^2/q^2)^{2\epsilon}[(1-\epsilon)/\Gamma(2-2\epsilon)]$$
$$\times\,[\epsilon^{-2}+\tfrac{3}{2}\epsilon^{-1}-\tfrac{1}{2}\pi^2+\tfrac{19}{4}+O(\epsilon)].$$

Virtual corrections

- Double infrared pole also appears in virtual corrections with a minus sign





$$-2NC_2(\mathrm{F})Q_f^{\,2}(\alpha\alpha_s/\pi)q^2(4\pi\mu^2/q^2)^{2\epsilon}[(1-\epsilon)/\Gamma(2-2\epsilon)]$$

$$\times\,[\epsilon^{-2}+\tfrac{3}{2}\epsilon^{-1}-\tfrac{1}{2}\pi^2+4+\mathrm{O}(\epsilon)]$$

overlap of collinear and
soft divergences

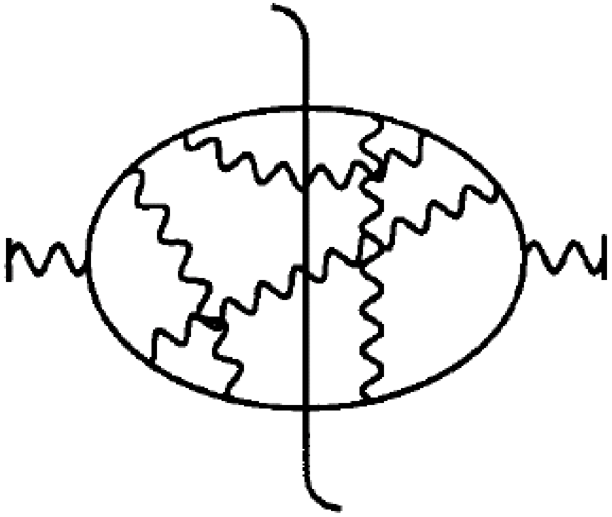
Infrared safety

- Infrared divergences cancel between real and virtual corrections
- e+e- -> X is imaginary part of off-shell photon self-energy corrections
- Total cross section (physical quantity) of e+e- -> X is infrared safe

$$\mathrm{Im}\frac{-i}{p^2+i\varepsilon} \propto \delta(p^2)$$

propagator

on-shell
final state



KLN theorem

- Kinoshita-Lee-Neuberger theorem:
  IR cancellation occurs as integrating over all phase space of final states
- Naïve perturbation applies

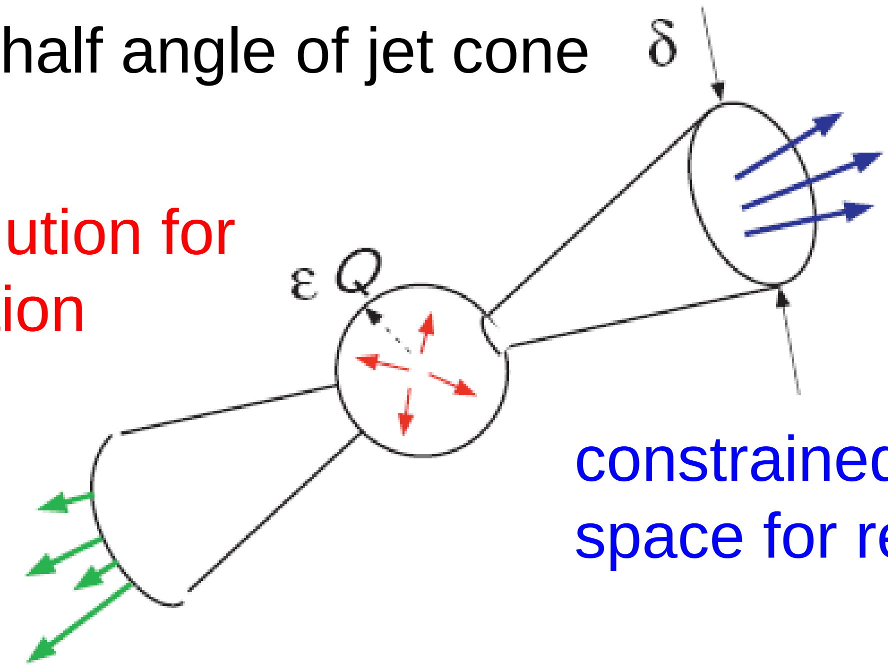
$$\sigma_{\rm tot}(q^2) = N(4\pi\alpha^2/3q^2)\sum_f Q_f{}^2[1 + (\alpha_s/\pi)\tfrac{3}{4}C_2(\mathrm{F})]$$

- Used to determine the coupling constant

Dijet in e+e- annihiation

- Dijet production is part of total cross section
- Born cross section is the same as total cross section

$$\sigma_{2j}{}^{(0)}(Q, \epsilon, \delta) = N\left(\sum_f Q_f{}^2\right)\frac{4\pi\alpha^2}{3Q^2}$$



NLO corrections

- Isotropic soft gluons within energy resolution

$$\left[2\ln^2(2\epsilon E/\mu) - \pi^2/6\right]$$

- Collinear gluons in cone with energy higher than resolution

$$\left[-3\ln(E\delta/\mu) - 2\ln^2 2\epsilon - 4\ln(E\delta/\mu)\ln(2\epsilon) + \tfrac{17}{4} - \pi^2/3\right]$$

- Virtual corrections

$$\left[-2\ln^2(E/\mu) + 3\ln(E/\mu) - \tfrac{7}{4} + \pi^2/6\right]$$

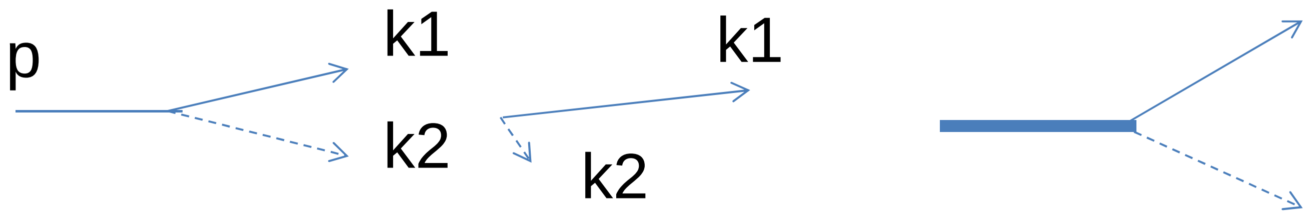
- Dijet cross section is infrared safe, but logarithmically enhanced, dominates total one

$$\left(3\ln\delta + 4\ln\delta\ln 2\epsilon + \pi^2/3 - \tfrac{5}{2}\right)$$

overlap of collinear and soft logs

Uncertainty principle

- Radiation from a moving electron, m ~ 0

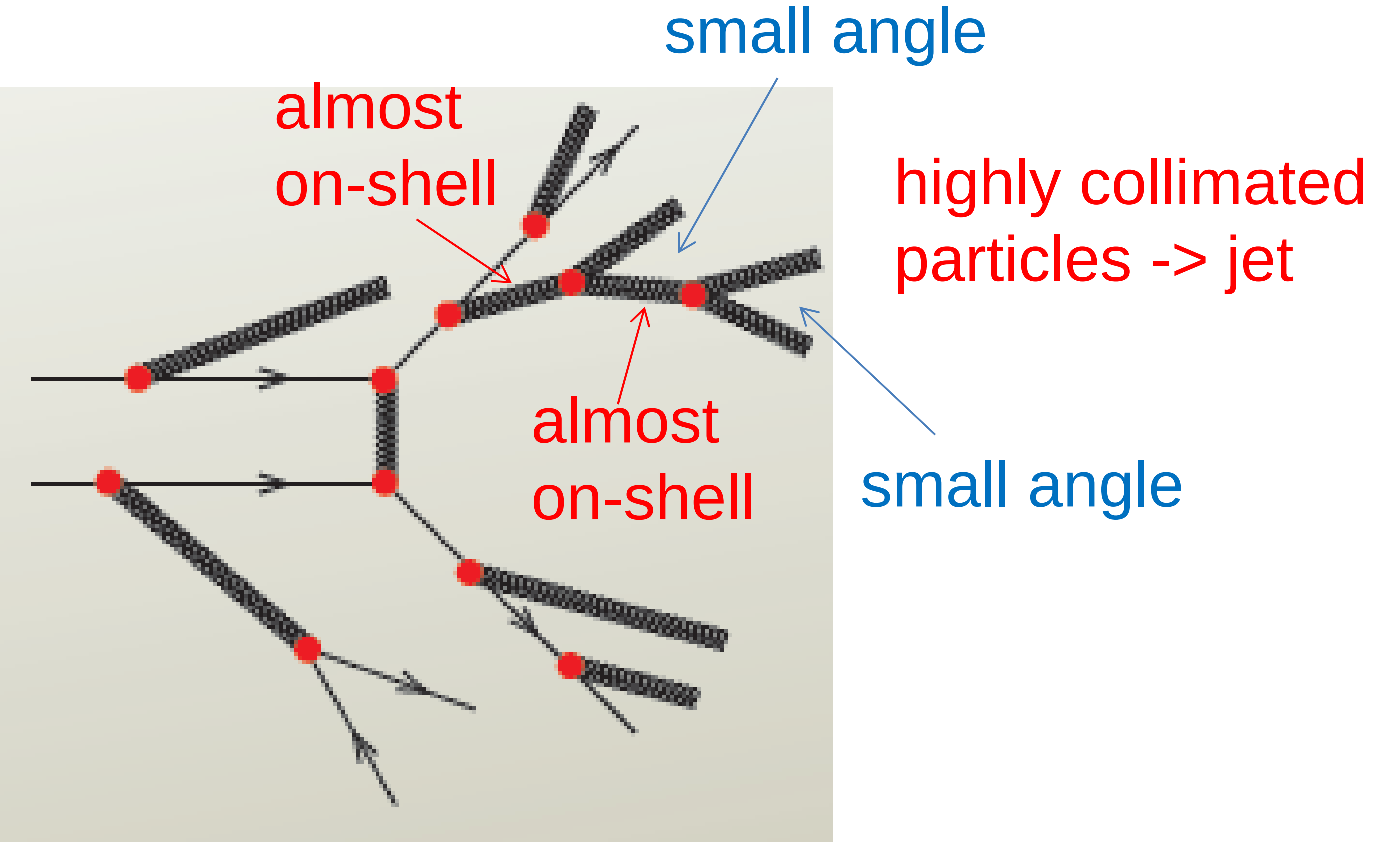


$$(k_1 + k_2)^2 \sim 0 \qquad\qquad (k_1 + k_2)^2 >> 0$$

$$\Delta E \sim 0 \qquad\qquad \Delta E >> 0$$

- Long-lived short-lived
- High probability low probability
- Either k1, k2 collimate, or one of them is soft
- This is the infrared enhancement

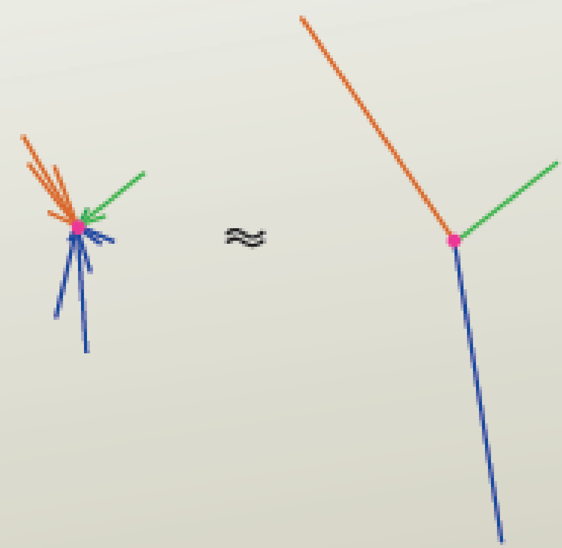
Infrared enhancement -> jets



- Probability is big to get a spray of collimated particles plus some soft particles at wide angle

Infrared safety of jets

- If any p_i becomes very small, we should get the same jets by leaving particle i out.
- If any two momenta p_i and p_j become collinear, we should get the same jets by replacing the particles by one with momentum $p_i + p_j$.
- The physical meaning is that for an IR-safe quantity, the physical event with hadron jets should give approximately the same measurement as a parton event.



Jet phenomenology

VOLUME 39, NUMBER 23 PHYSICAL REVIEW LETTERS 5 DECEMBER 1977

Jets from Quantum Chromodynamics

George Sterman
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

and

Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 26 July 1977)

double-log enhancement

The properties of hadronic jets in e^+e^- annihilation are examined in quantum chromodynamics, without using the assumptions of the parton model. We find that two-jet events dominate the cross section at high energy, and have the experimentally observed angular distribution. Estimates are given for the jet angular radius and its energy dependence. We argue that the detailed results of perturbation theory for production of arbitrary numbers of quarks and gluons can be reinterpreted in quantum chromodynamics as predictions for the production of jets.

jet as an observable (jet physics)
not quarks and gluons

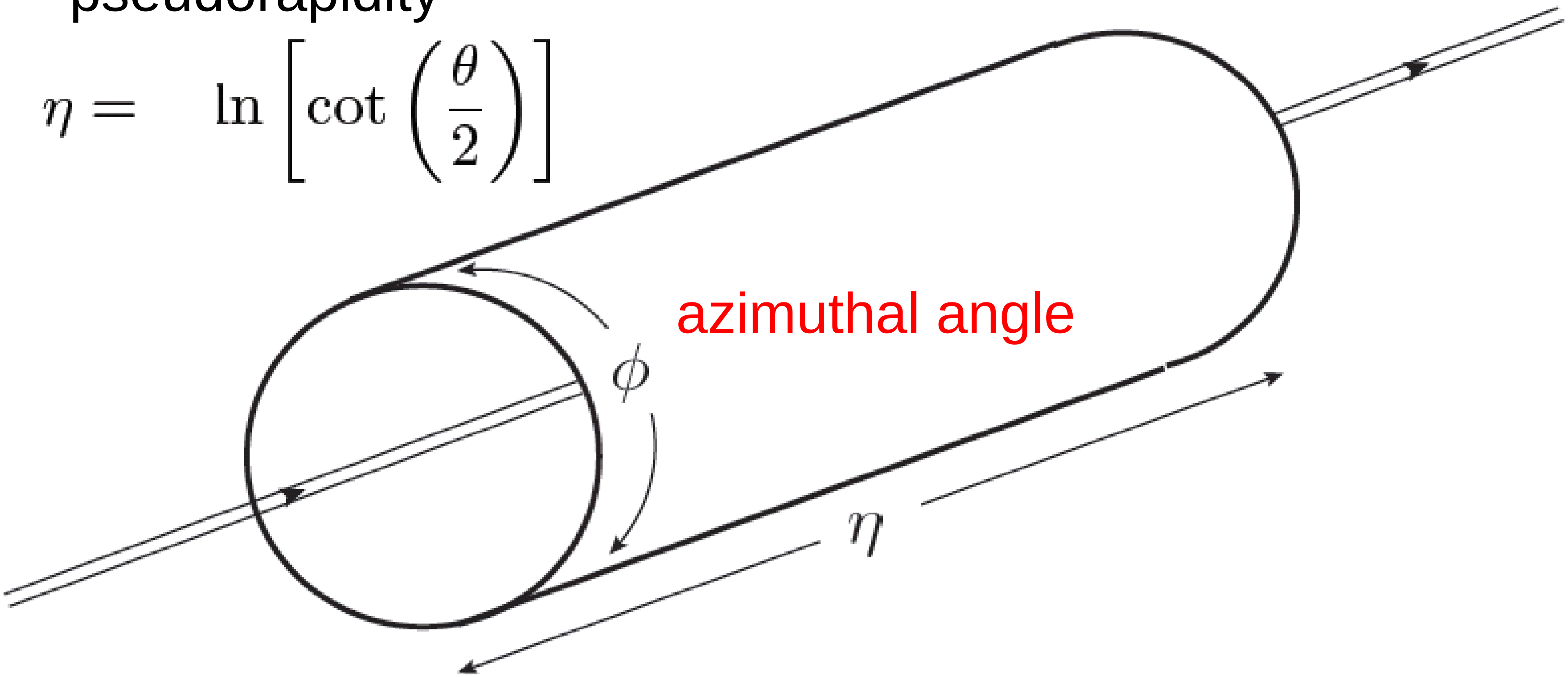
jet substructures

Jets in experiment

Coordinates for jets

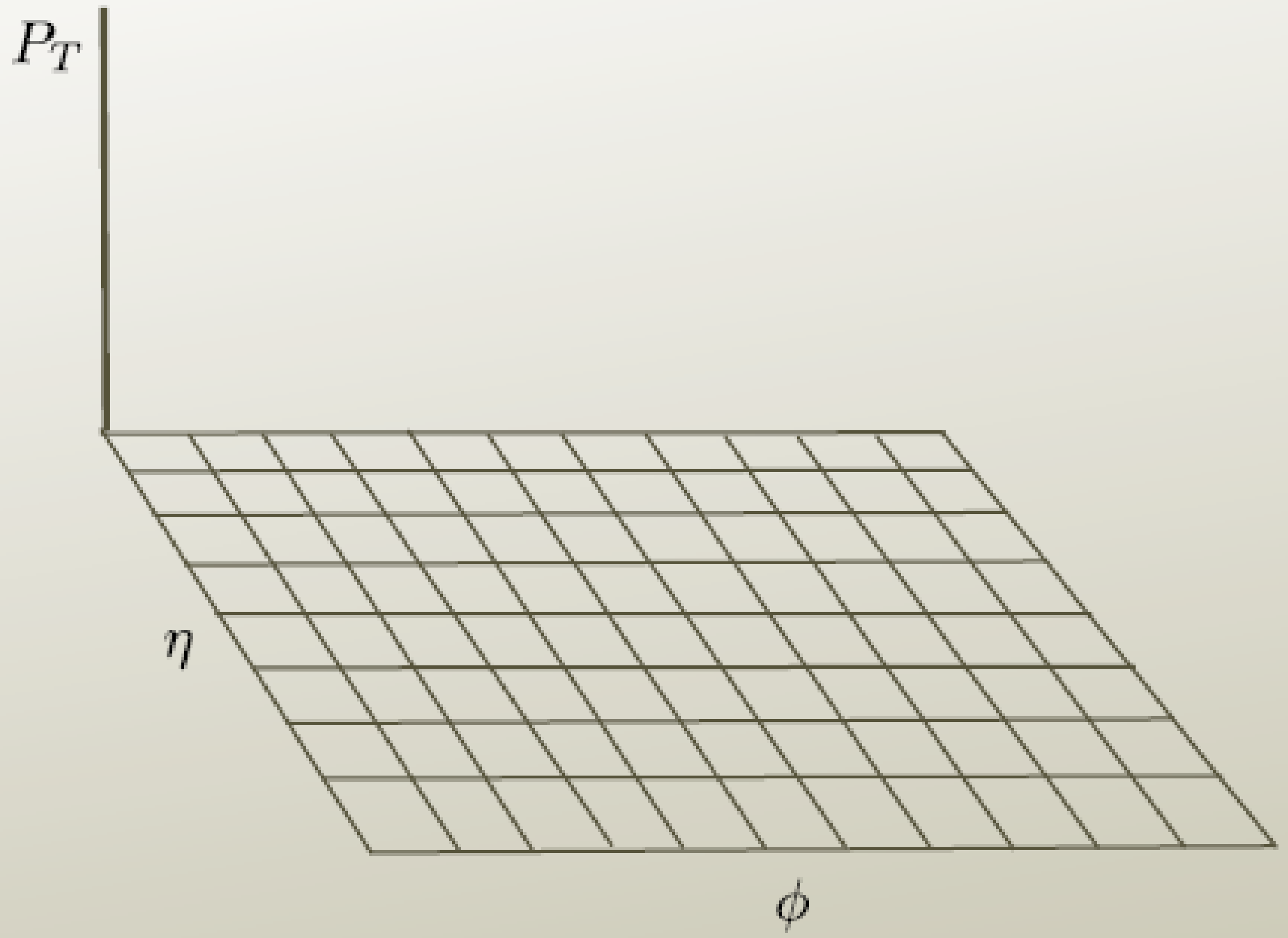
angles in detector
pseudorapidity

$$\eta = \quad \ln\left[\cot\left(\frac{\theta}{2}\right)\right]$$

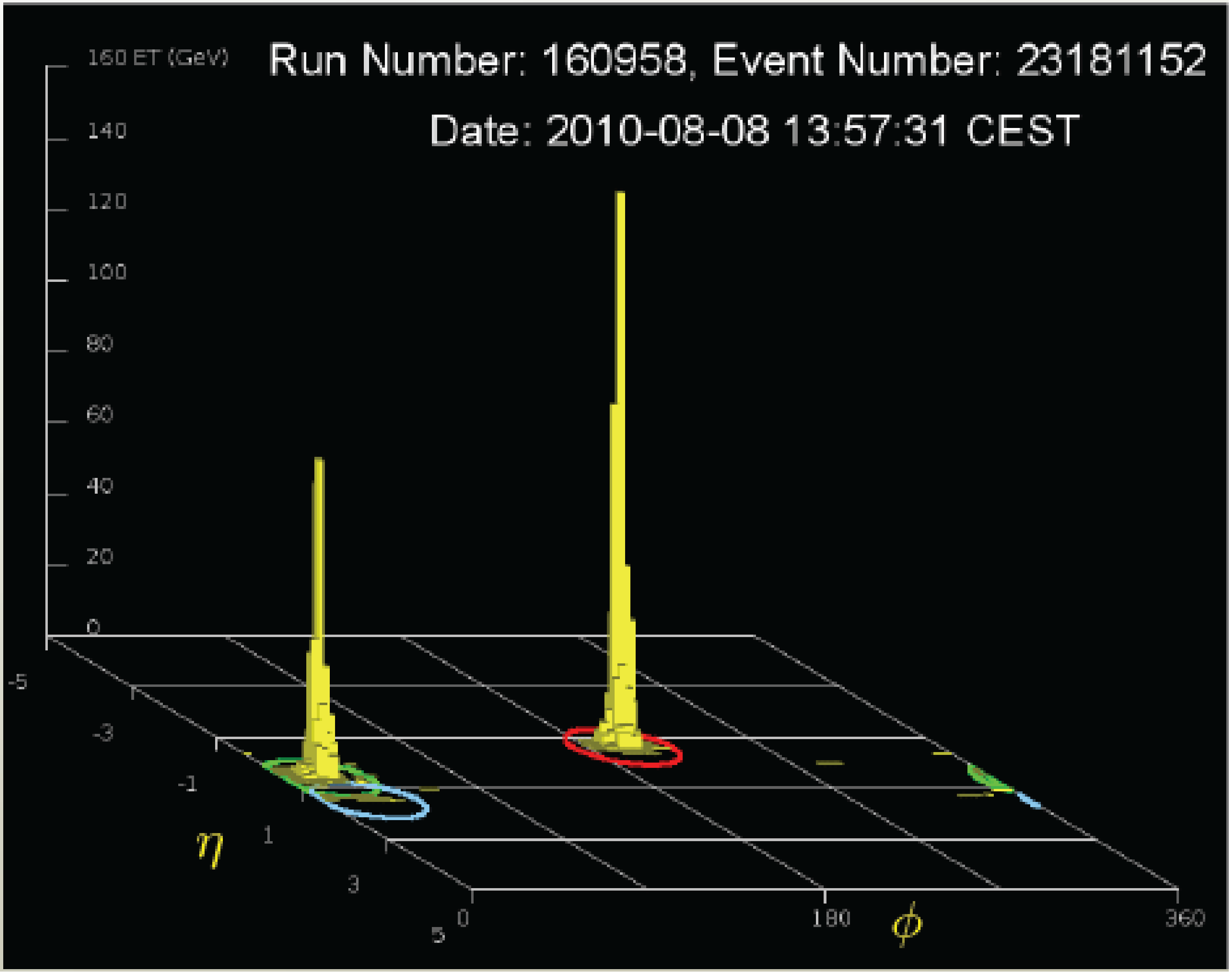


$$\theta = 0 \Rightarrow \eta = \infty, \quad \theta = 90^o \Rightarrow \eta = 0, \quad \theta = 180^o \Rightarrow \eta = -\infty$$

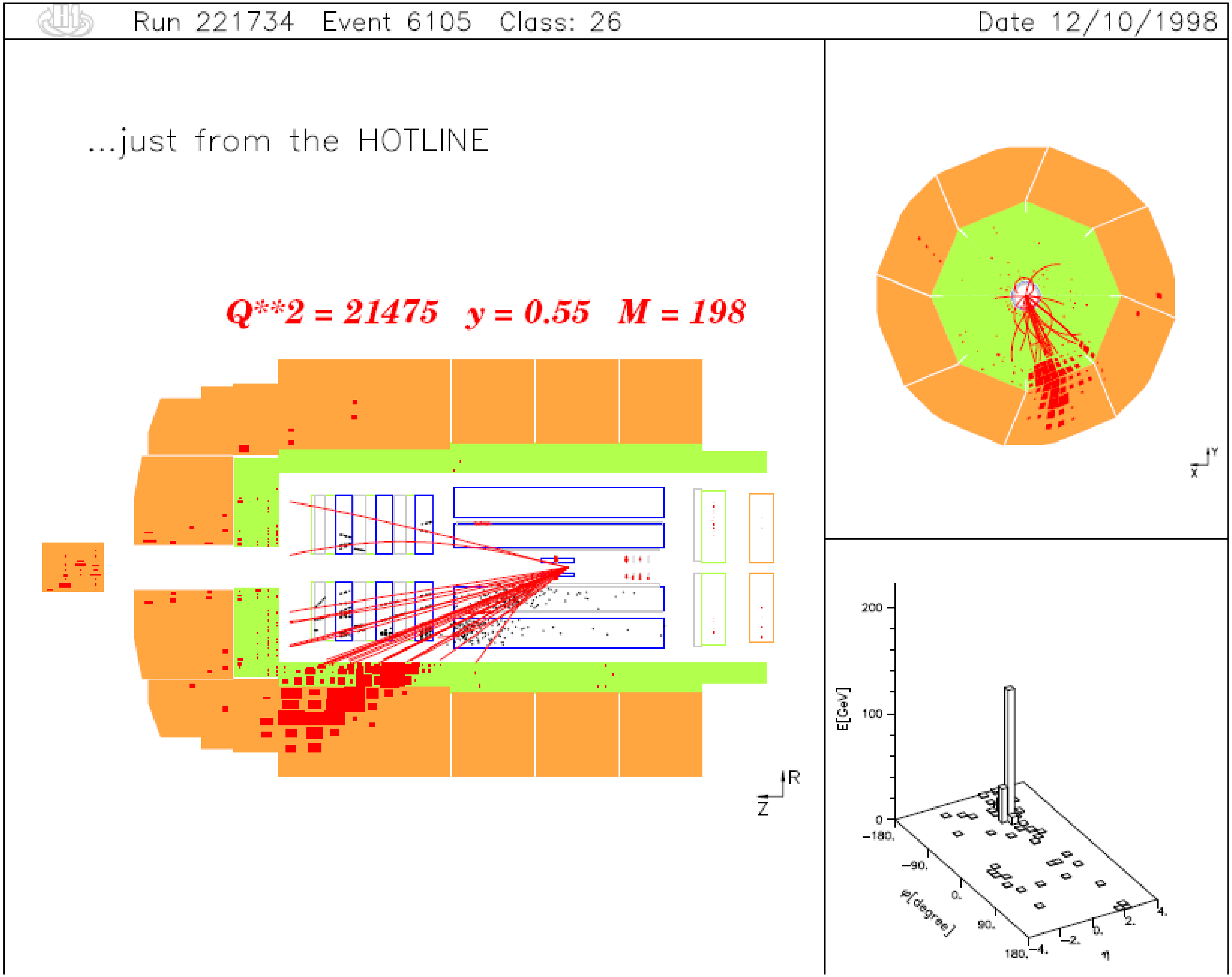
- Lego plot event display.
- Plot $P_T = |\vec{P}_T|$ in each calorimeter cell versus η and ϕ.



- An Atlas event.

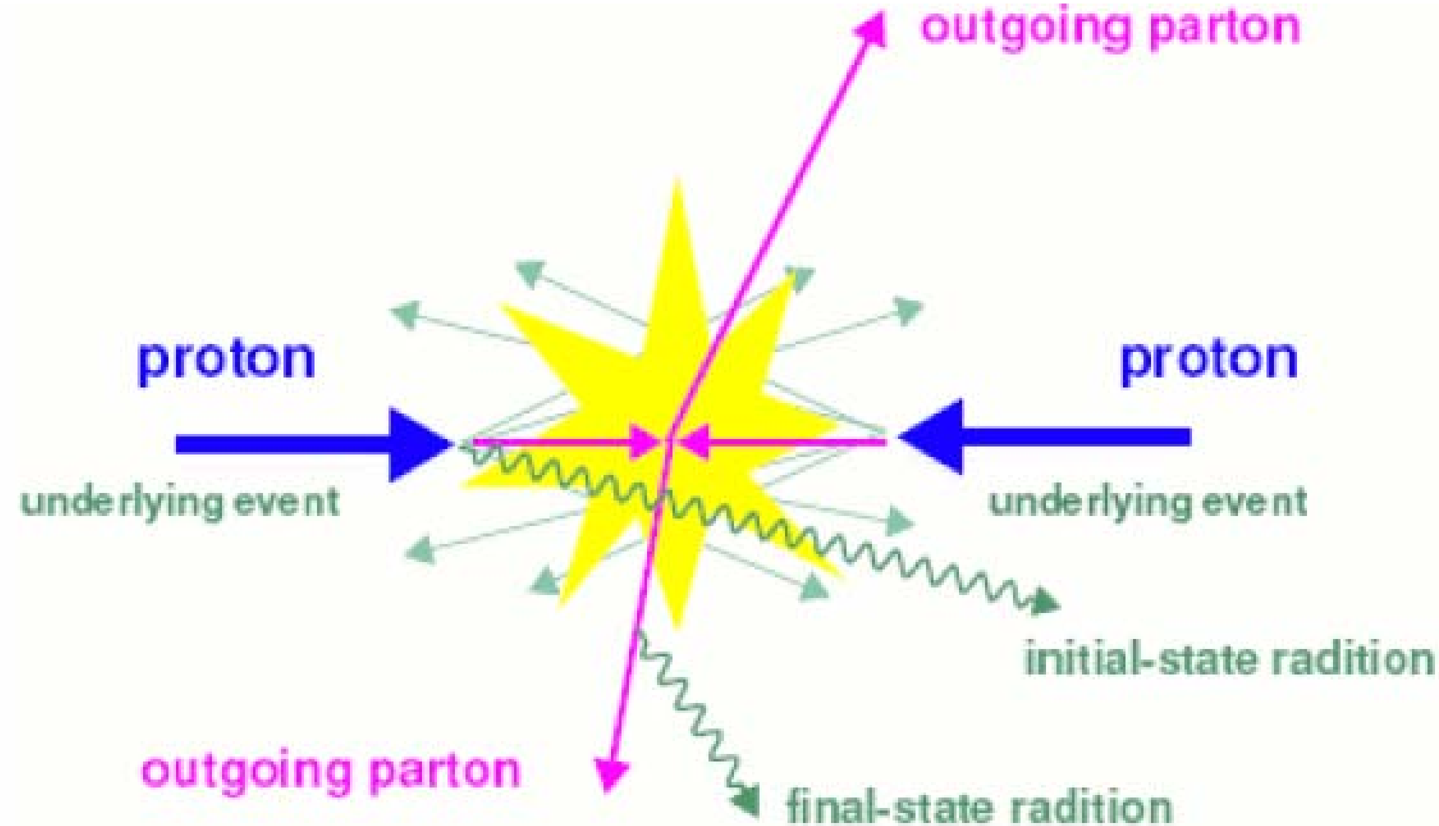


Jet from H1 Collaboration

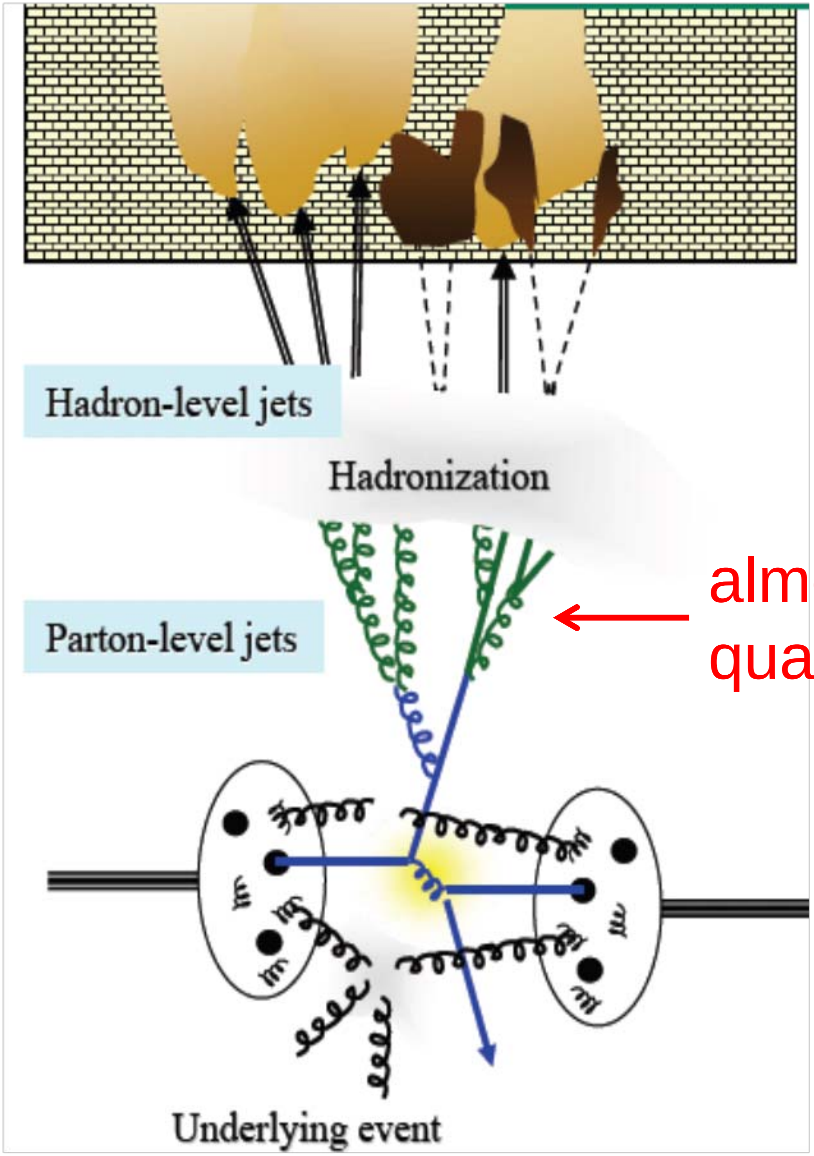


Underlying events

- Everything but hard scattering
- Initial-state radiation, final-state radiation, multi-parton interaction all contribute to jets

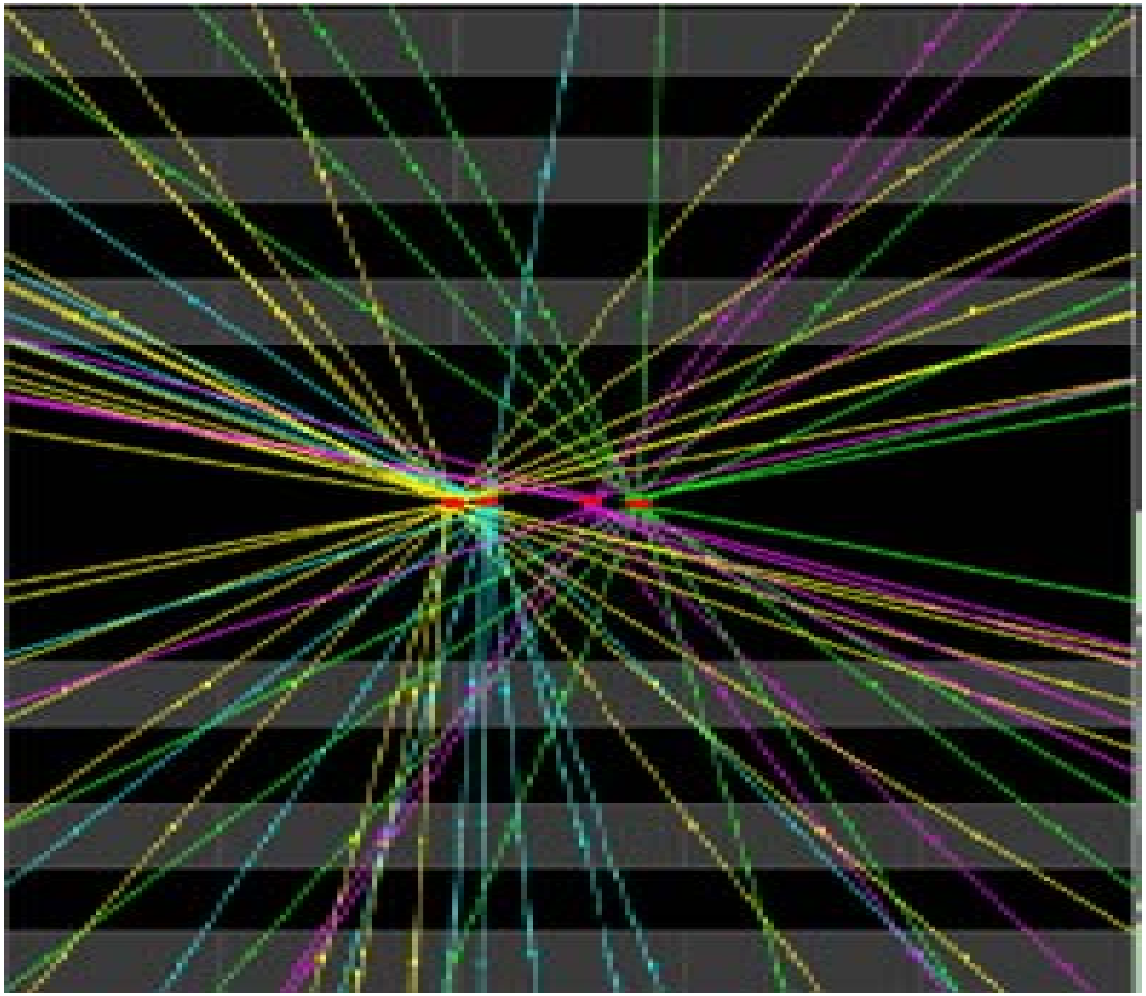


Cascades in collider



almost parallel
quarks, gluons

Pile-up events should be excluded



4 pile-up vertices

Jet algorithms

- Comparison of theory with experiment is nontrivial
- Need jet algorithms
- Algorithms should be well-defined so that they map experimental measurements with theoretical calculations as close as possible
- Infrared safety is important guideline, because jets are infrared safe

Types of algorithms

- Two main classes of jet algorithms
- Cone algorithms: stamp out jets as with a cookie cutter

  Geometrical method

- Sequential algorithms: combine parton four-momenta one by one

  Depend on particle kinematics

Seeded cone algorithm

- Find stable cones via iterative-cone procedure
- Start from seed particle i and consider set of particles j with separations smaller than jet cone

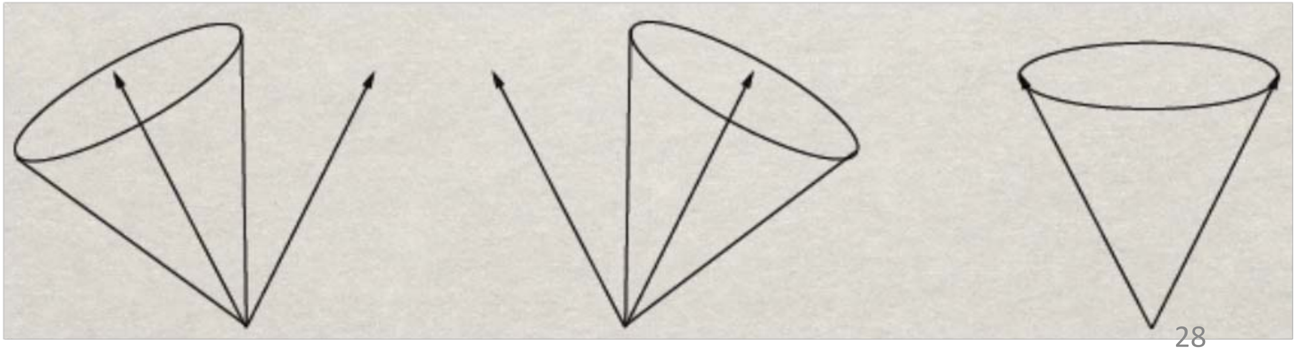
$$\Delta R_{ij} \equiv (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2 < R$$

- If the cone is stable, procedure stops. Otherwise the cone center J is taken as a new seed, and repeat the above procedure
- A stable cone is a set of particles i satisfying

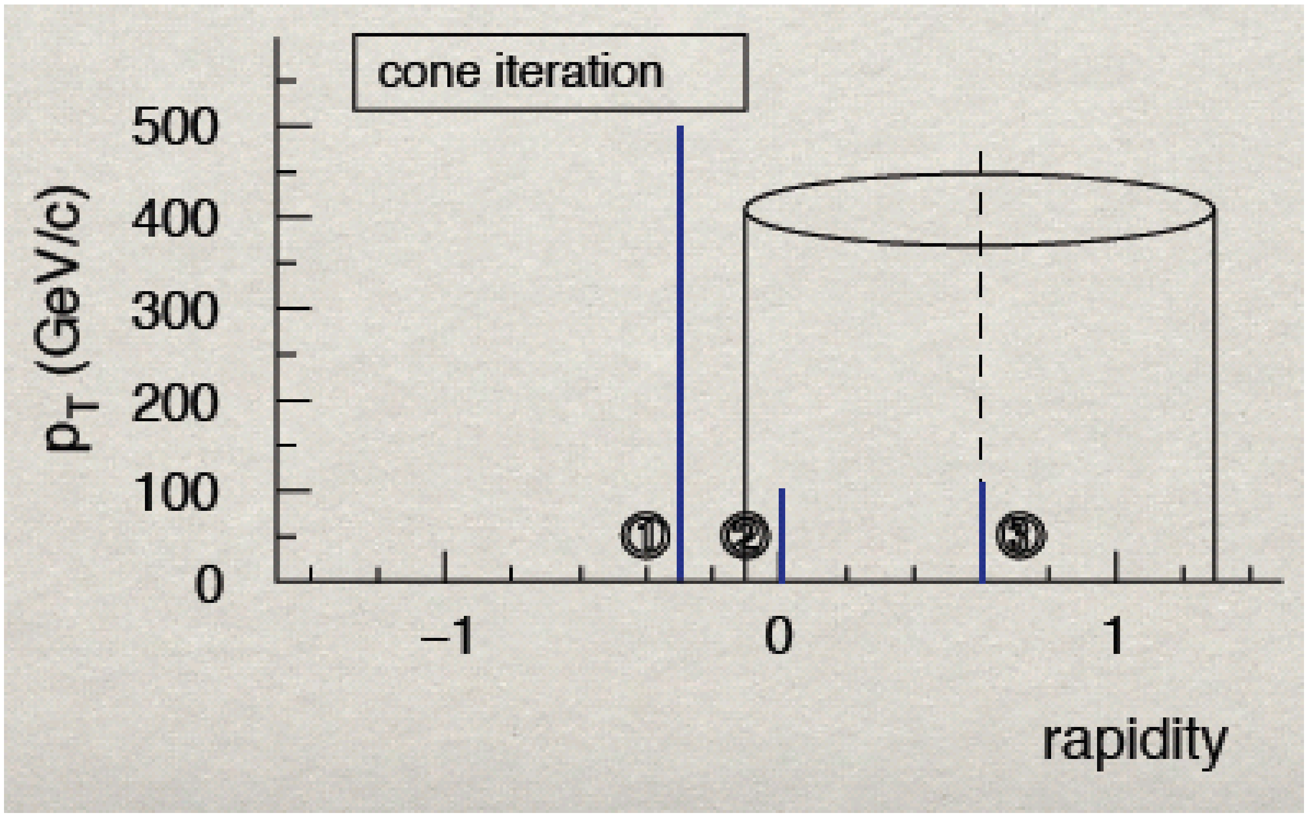
$$\Delta R_{iJ} < R$$

- Examples:

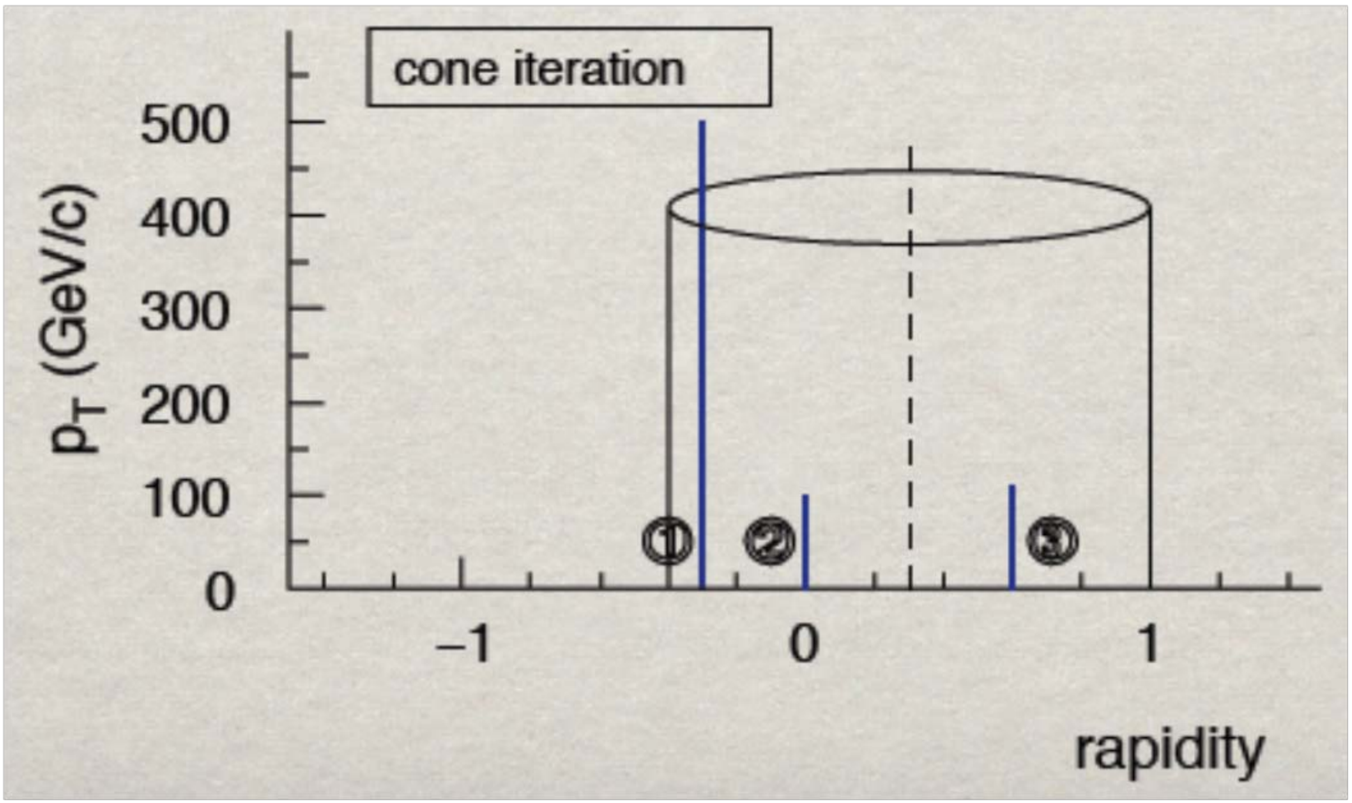
$$R < R_{12} < 2R$$



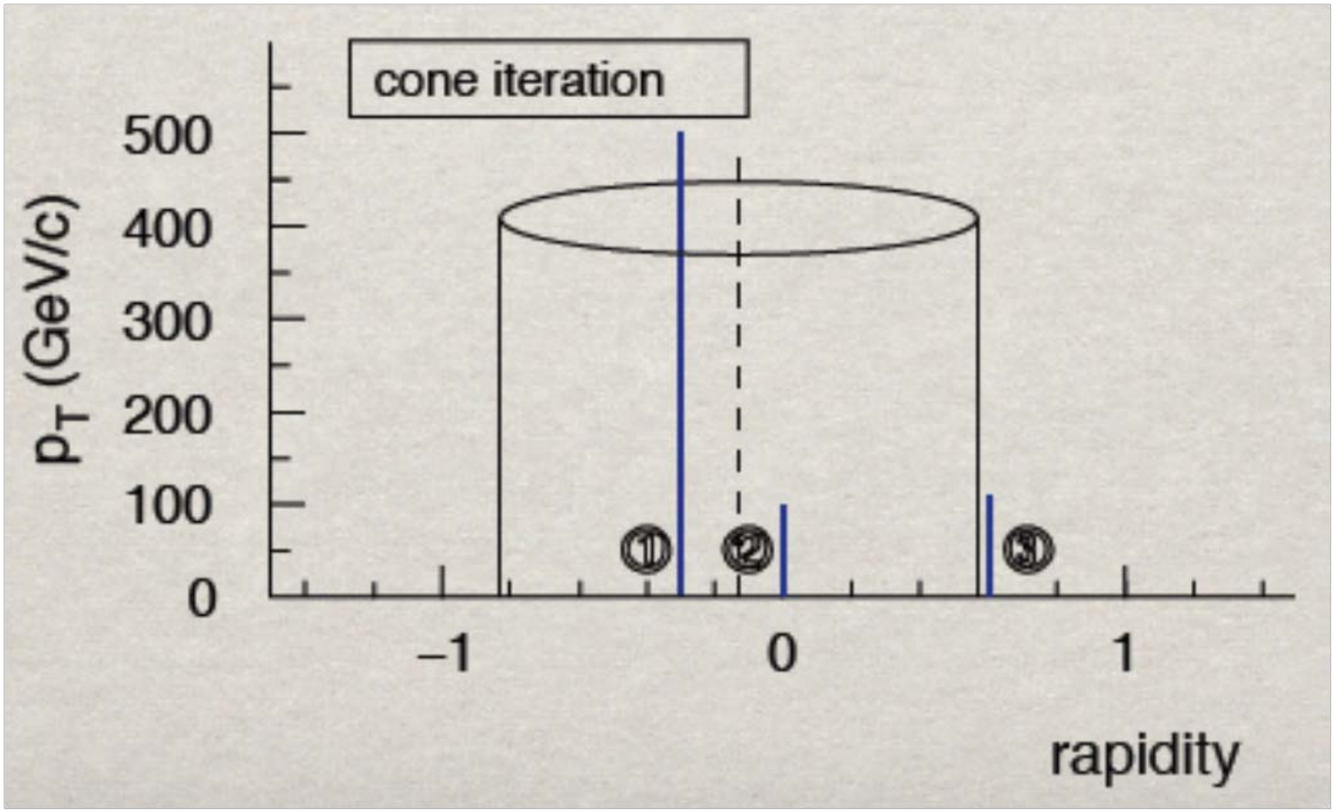
Iterative step 1



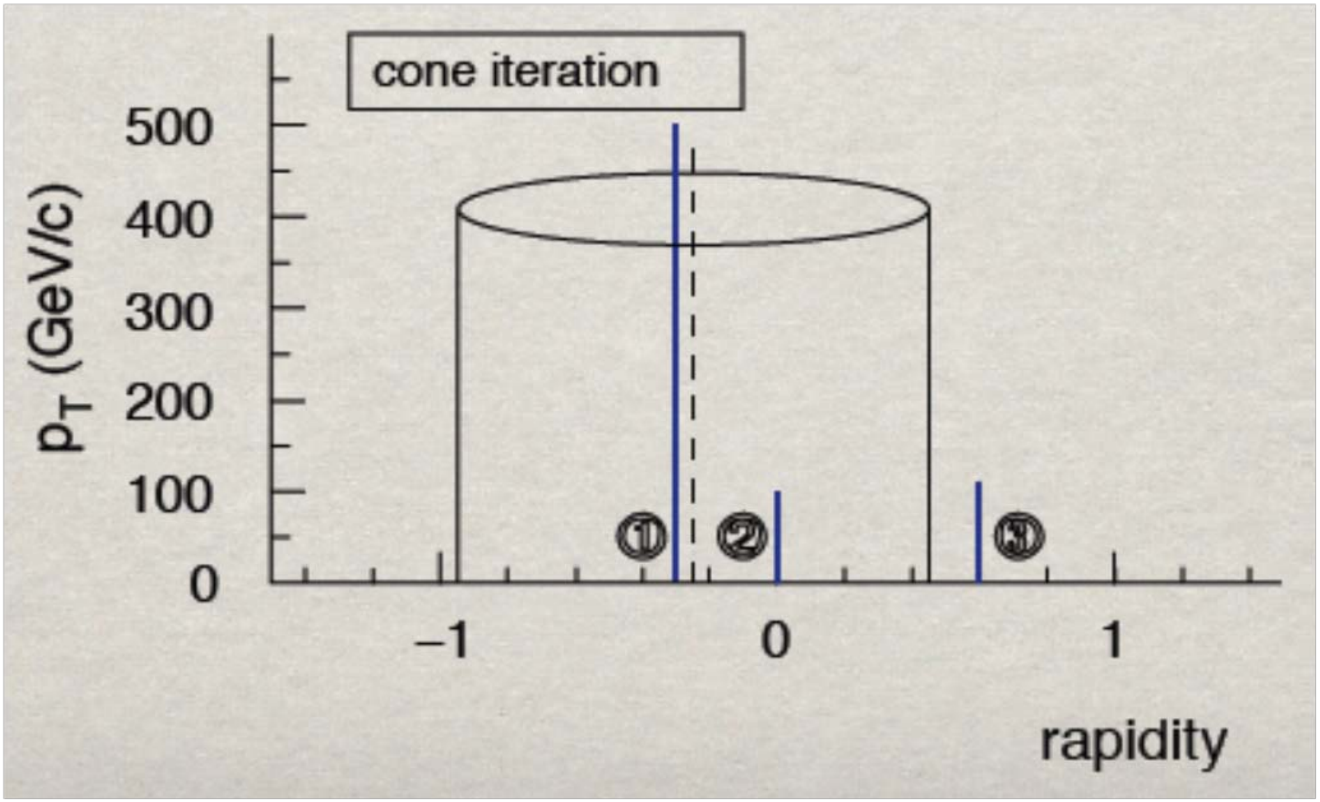
Iterative step 2



Iterative step 3

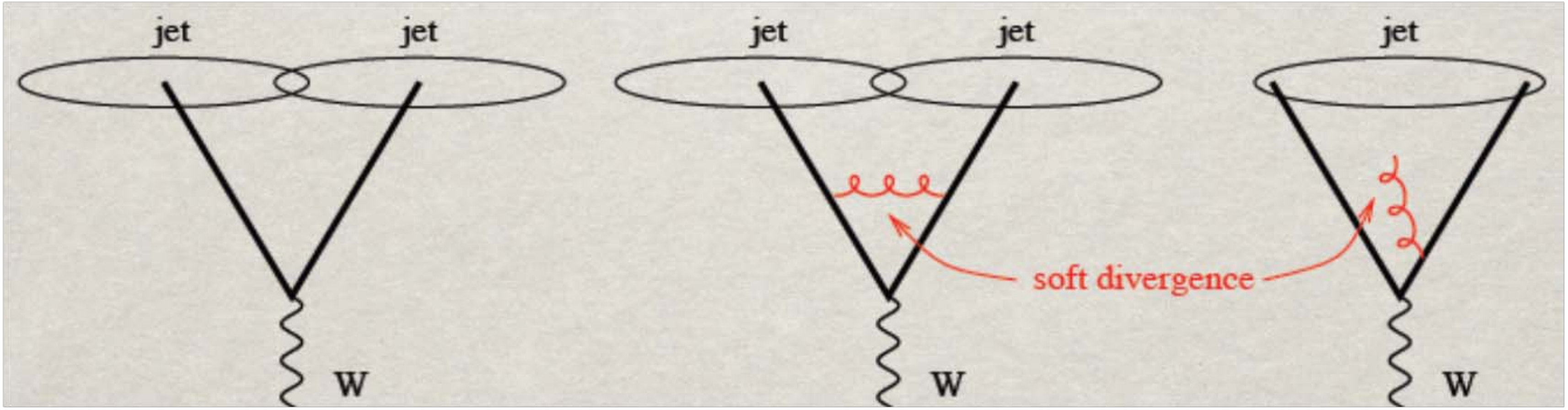


Iterative step 4



Problem of seeded cone

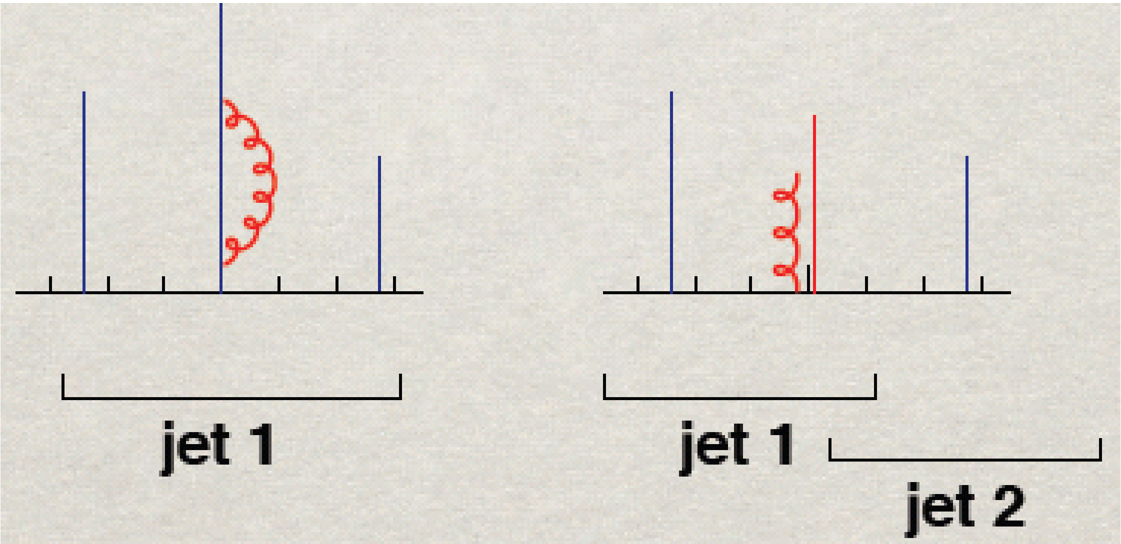
- Geometrical algorithm does not differentiate infrared gluons from ordinary gluons
- Final results (split-merge) depend on soft radiation and collinear splitting



- Virtual (real) soft gluon contributes to two (single) jet cross section, no cancellation

Not infrared safe

- How about starting from the hardest particle?
- Collinear splitting change final results



- Virtual (real) gluon contributes to single (two) jet cross section, no cancellation
- Seeded cone algorithm is not infrared safe

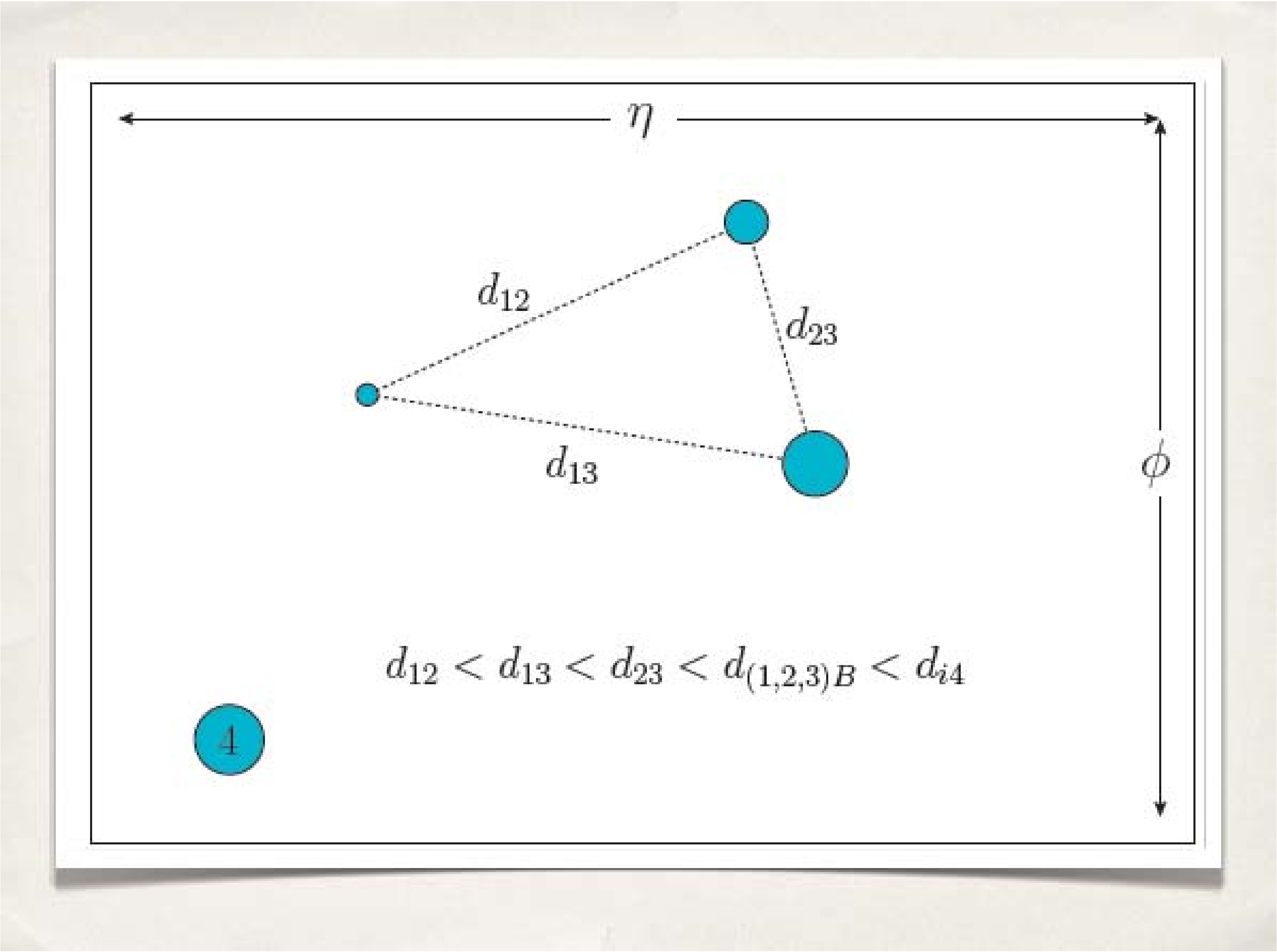
Sequential algorithms

- Take kT algorithm as an example.
- For any pair of particles i and j, find the minimum of

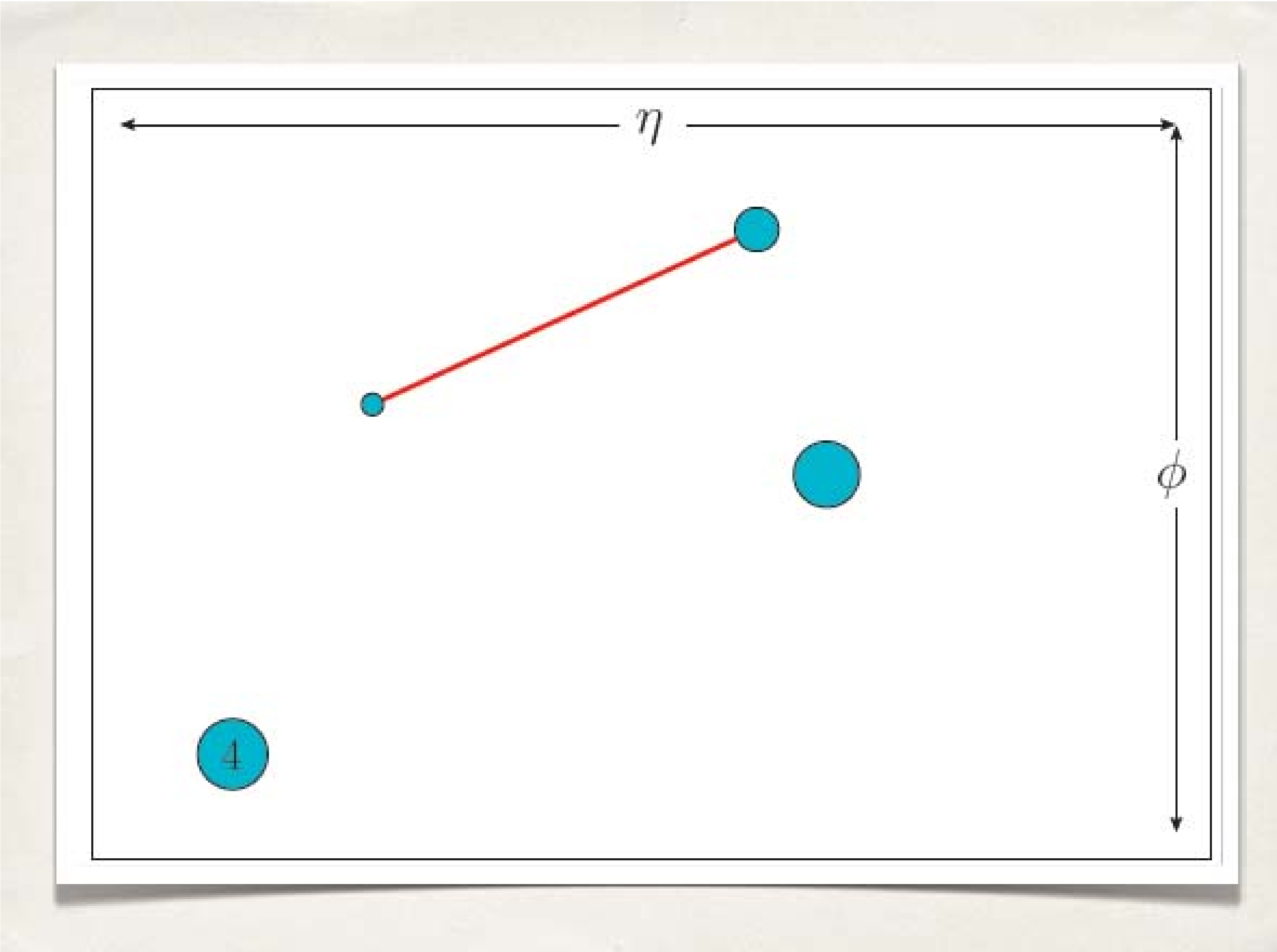
$$d_{ij} = \frac{\min\{k_{ti}^2, k_{tj}^2\}}{R^2}\Delta R_{ij}^2 \simeq k_{t,ij}^2, \quad d_{iB} = k_{ti}^2, \quad d_{jB} = k_{tj}^2$$

- If it is diB or djB, i or j is a jet, removed from the list of particles. Otherwise, i and j merged
- Repeat procedure until no particles are left
- Differentiate infrared and ordinary gluons

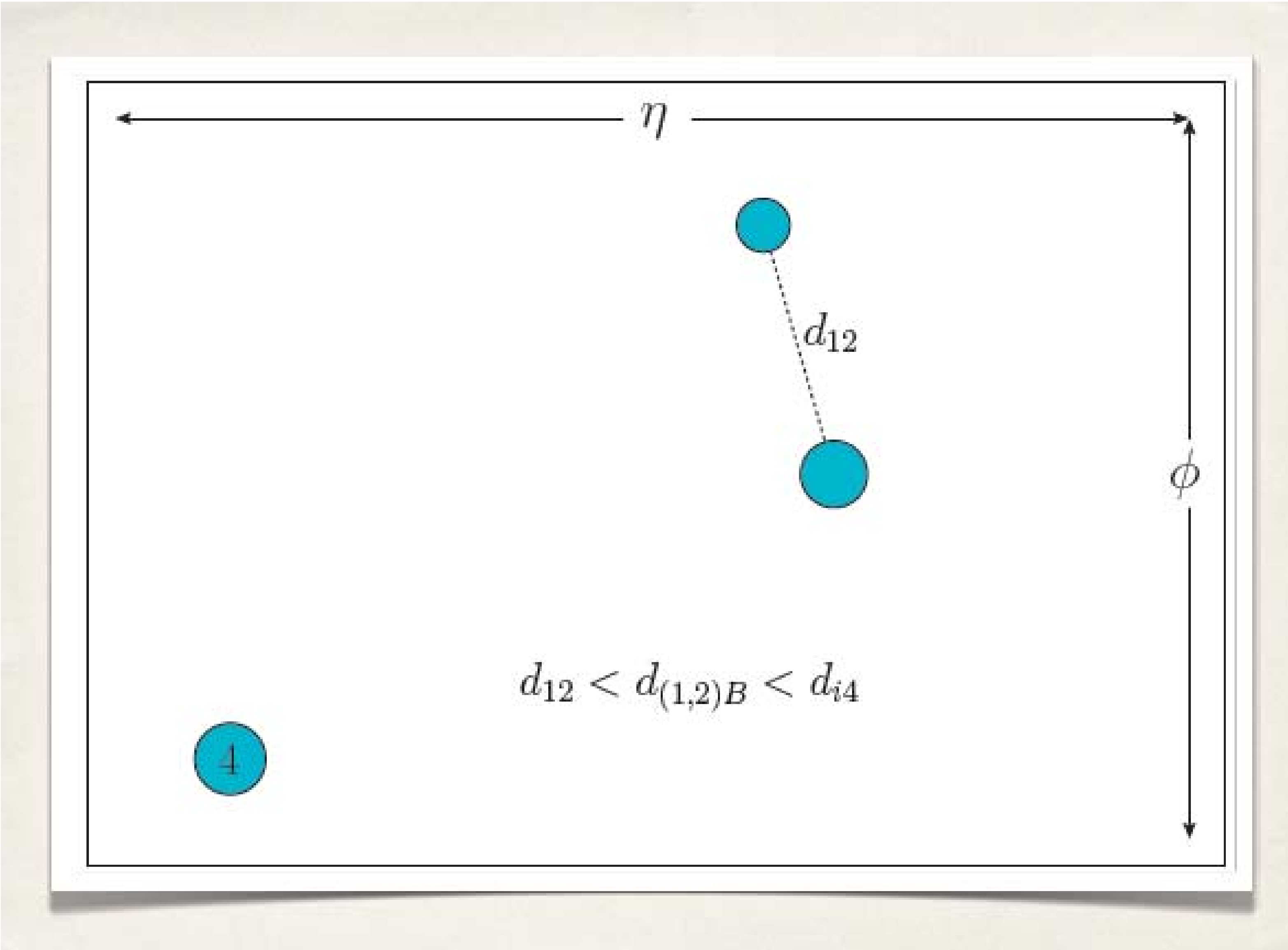
Step 1



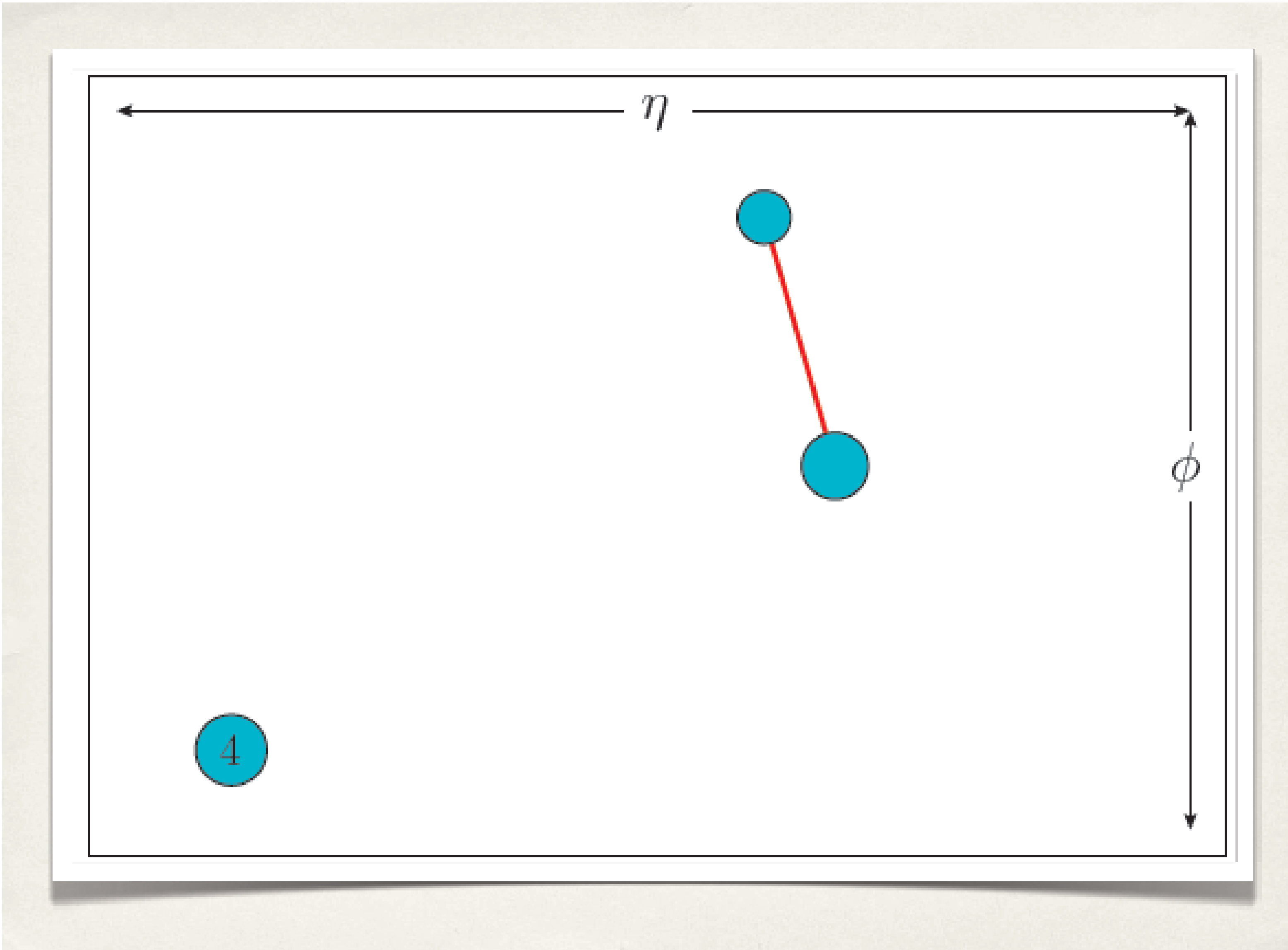
Step 2



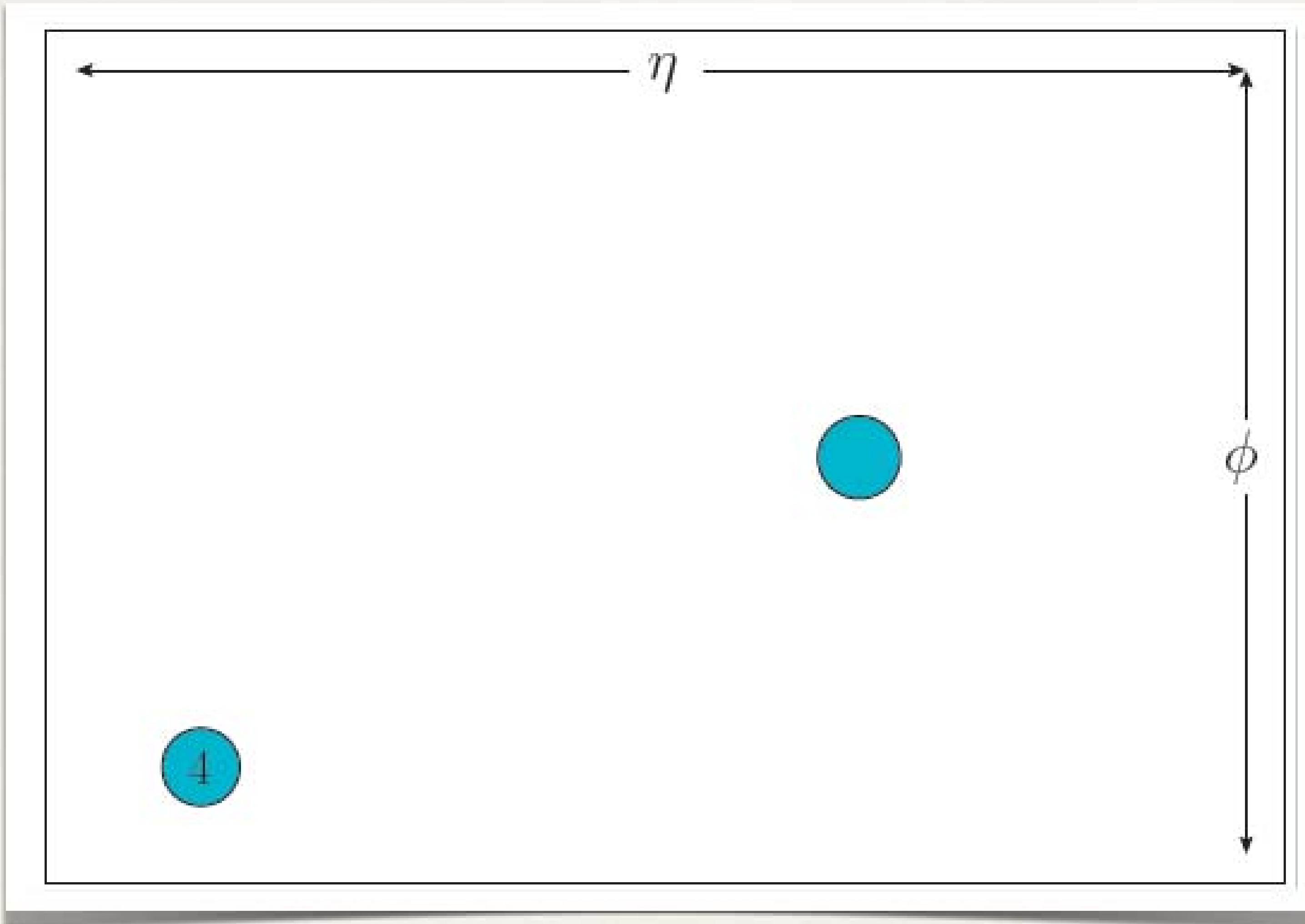
Step 3



Step 4



Step 5



Done!

Features of kT algorithm

- Take $d_{1B} < d_{2B} < d_{3B}$ as an example
- To pick up d_{23}, jet 2 and jet 3 must be very collimated. Chance is lower.
- It is more likely to pick up d_{12}.
- kT algorithm starts clustering from soft partons
- It is possible to cluster soft partons from neighboring jets -> irregular jet shape

Shower histories

- The graph of parton joinings (read right to left) can be thought of as a graph of parton splittings (read left to right) in a parton shower.
- If we use the k_T jet algorithm, then the parton splittings go from harder (high k_T) to softer (low k_T).

Recombination Algorithms

- k_T algorithm   start with softer particles

$$d_{ij} = \min(p_{Ti}^2, p_{Tj}^2)\left(\frac{\Delta R}{R_0}\right)^2, \; d_{iB} = p_{Ti}^2$$

- C/A algorithm

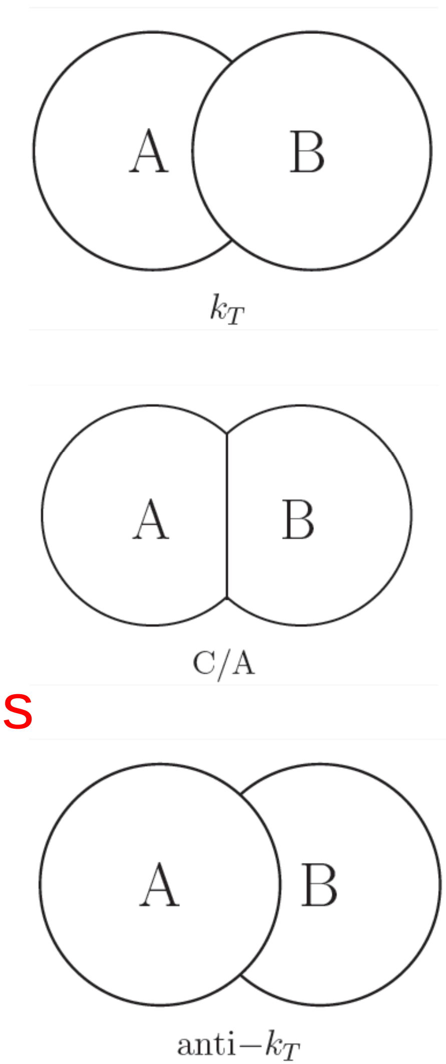
$$d_{ij} = \left(\frac{\Delta R}{R_0}\right)^2, \; d_{iB} = 1$$

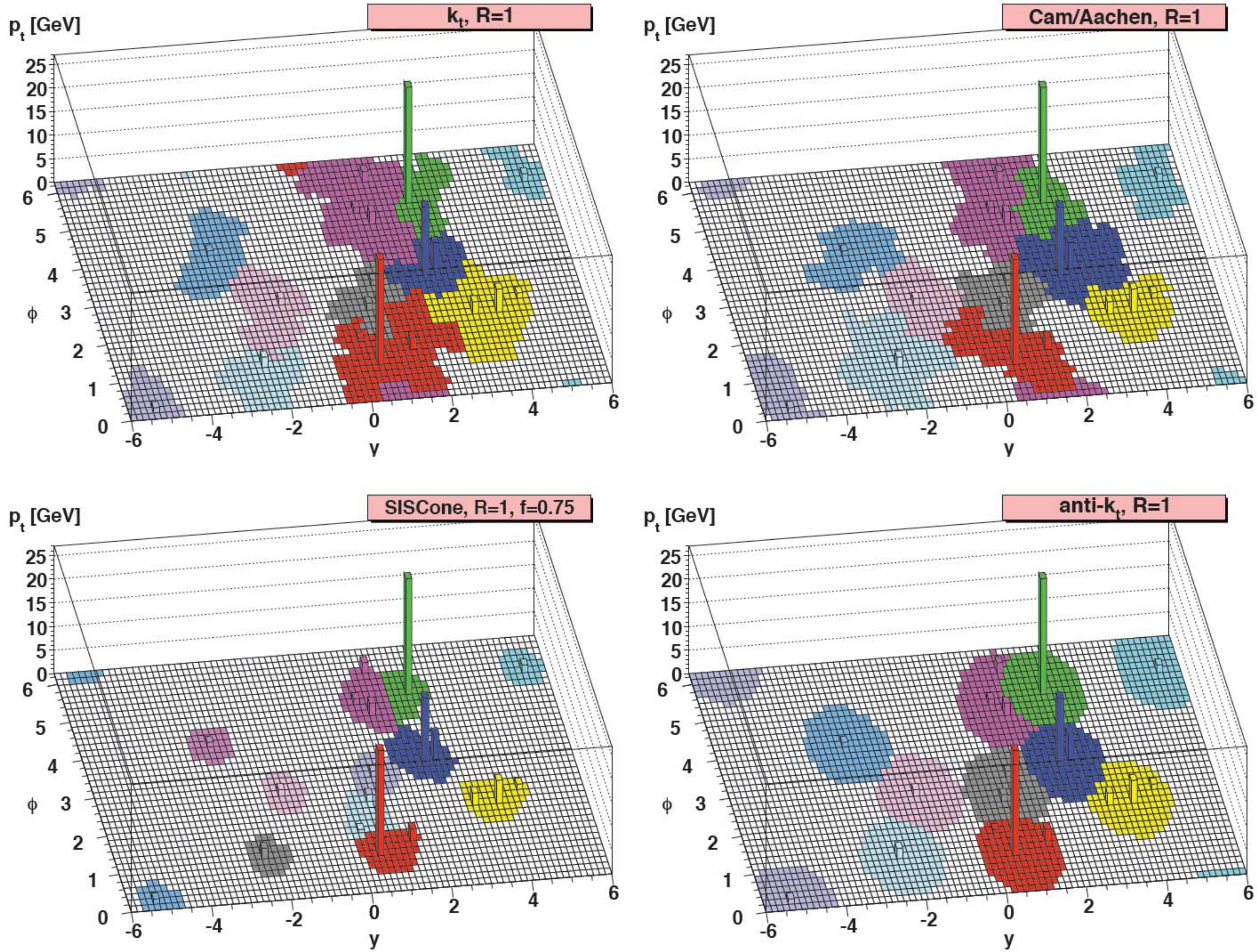
- anti-k_T algorithm   start with harder particles

$$d_{ij} = \min(p_{Ti}^{-2}, p_{Tj}^{-2})\left(\frac{\Delta R}{R_0}\right)^2, \; d_{iB} = p_{Ti}^{-2}$$

$$(\Delta R)^2 \equiv (\Delta\eta)^2 + (\Delta\phi)^2$$

$p_{TA} > p_{TB}$

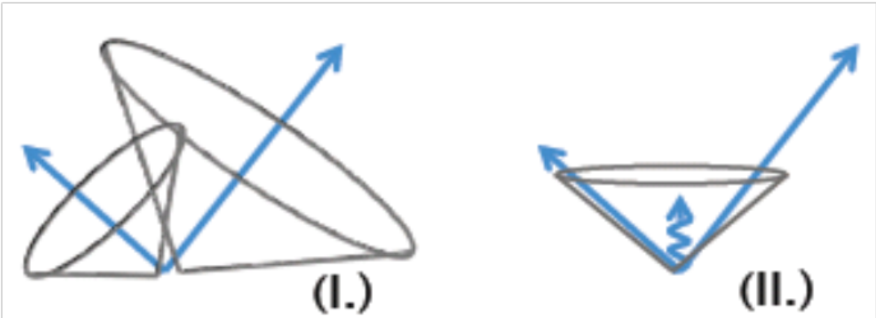




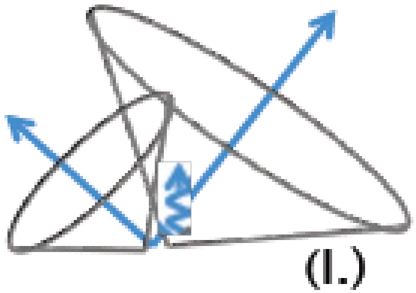
k_t, R=1
Cam/Aachen, R=1
SISCone, R=1, f=0.75
anti-k_t, R=1
p_t [GeV]
ϕ
y

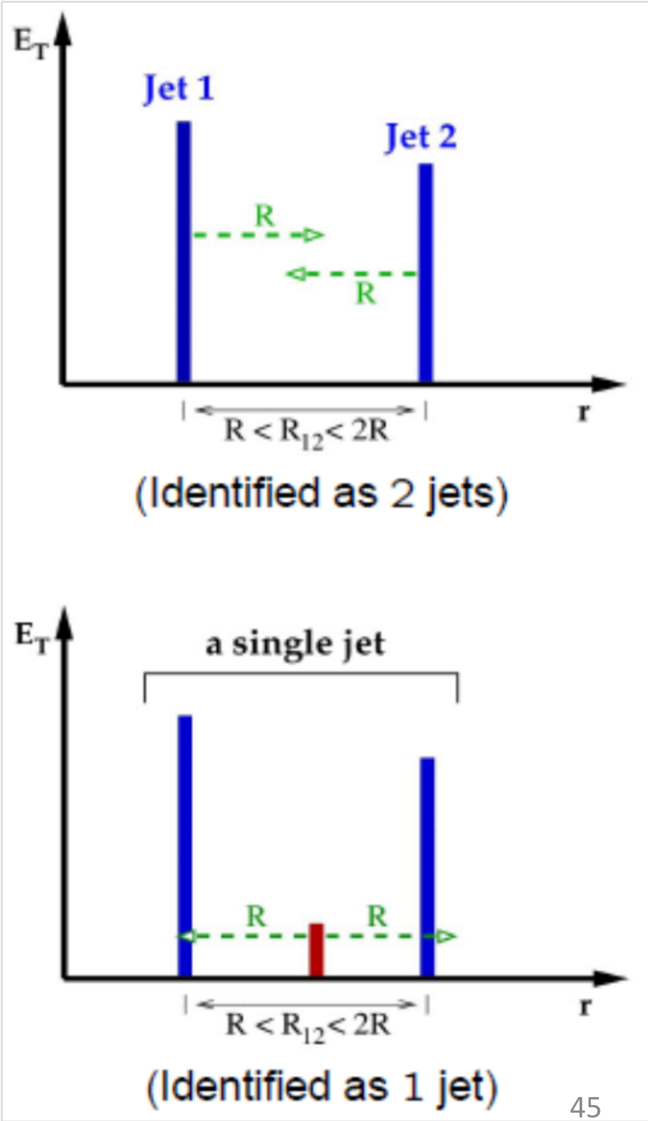
Infrared safety

- In seeded cone algorithm



- In kt algorithm, remain two jets---infrared safety





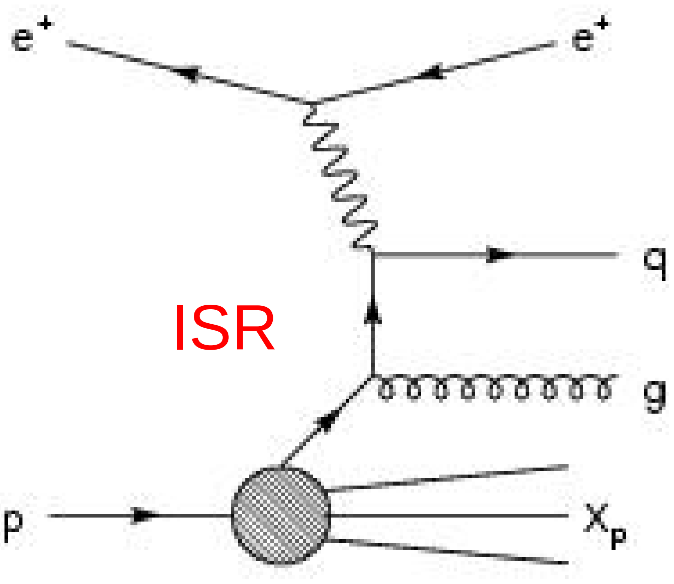
Jets in theory

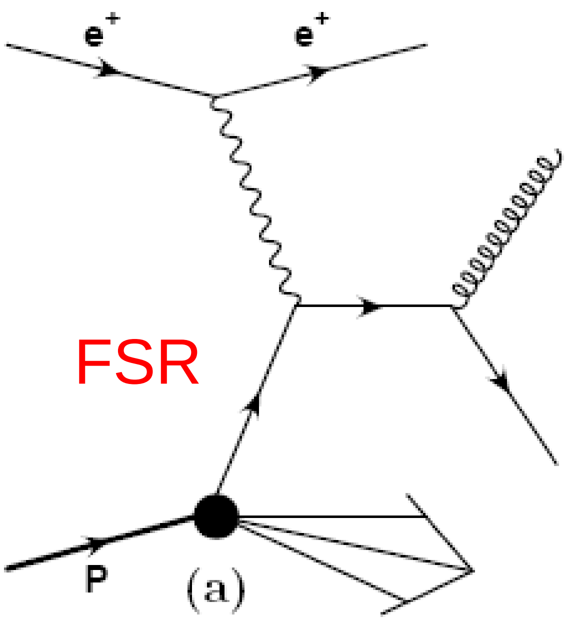
Jet production in DIS

- Restrict phase space of final-state quark and gluon in small angular separation
- Jet production enhanced by collinear dynamics

$$I_{3,\mathrm{IR}} = (2\pi) \int_0 \frac{dk}{k} \int_0 \frac{d\theta}{\theta}$$

Soper and Pumplin's lectures





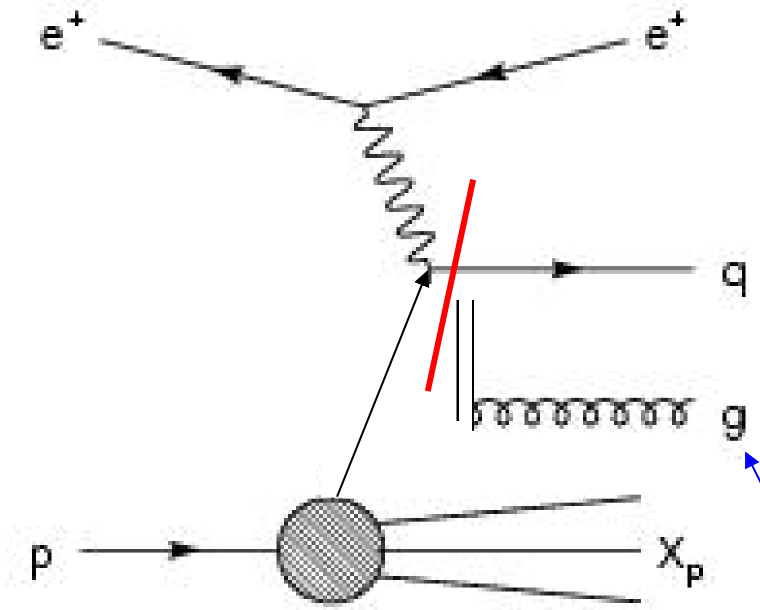
Wilson link

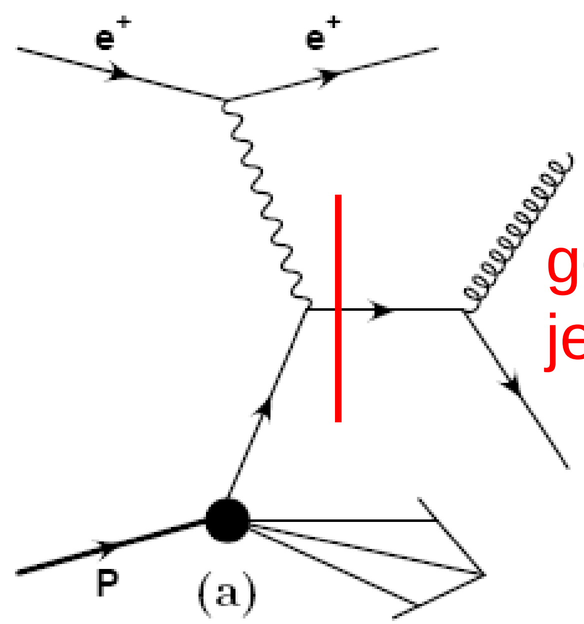
- Feynman rules are reproduced by Wilson link

$$\Phi_\xi^{(f)}(\infty,0;0) = \mathcal{P}\left\{ e^{-ig\int_0^\infty d\eta\, \xi\cdot A^{(f)}(\eta\,\xi^\mu)} \right\}$$

- Represented by double lines

direction of
initial quark



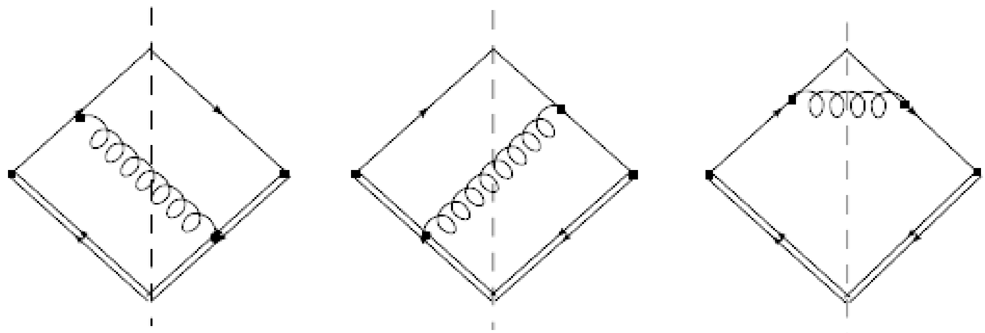


go into
jet function

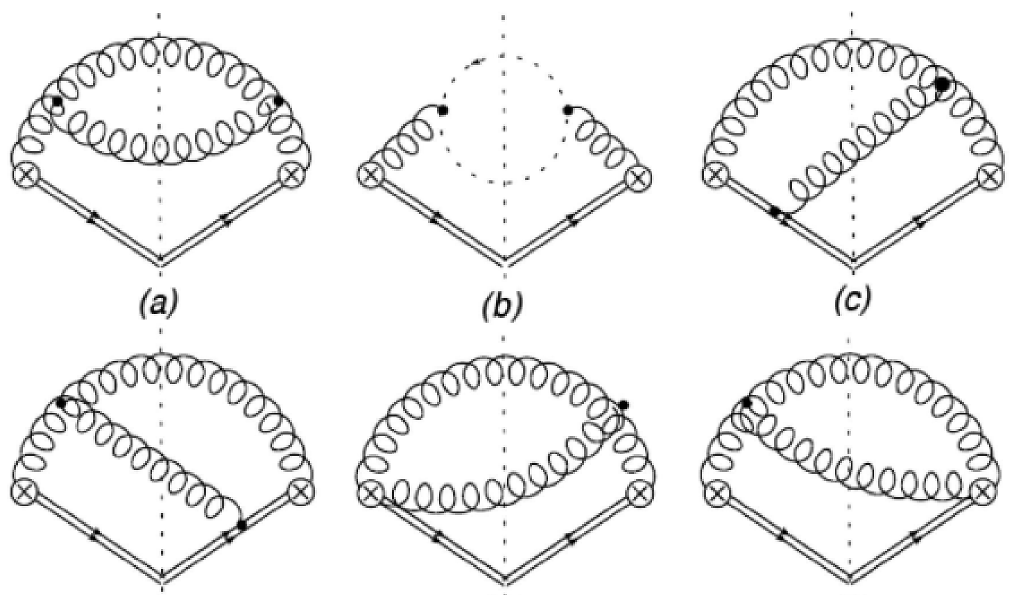
collinear gluon detached and factorized

NLO diagrams

- quark jet
- gluon jet



(a) (b) (c)



(a) (b) (c)

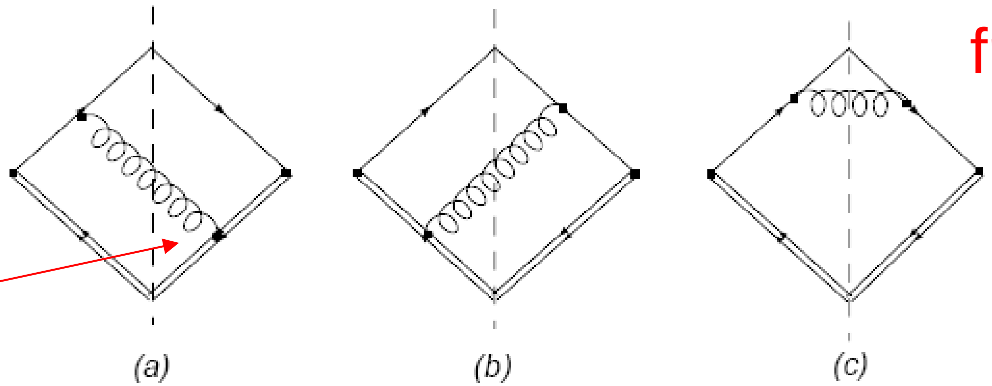
(d) (e) (f)

Power counting

- ISR, FSR are leading power, and should be included in jet (gauge invariant) definition

from FSR

from ISR



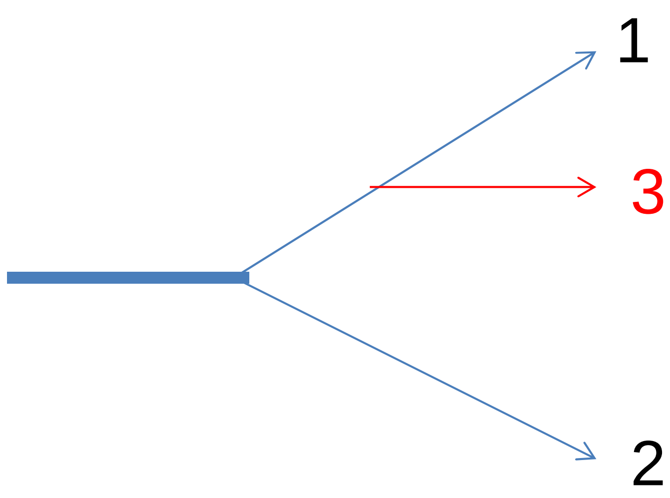
- MPI are sub-leading power: chance of involving more partons in scattering is low. They should be excluded in principle

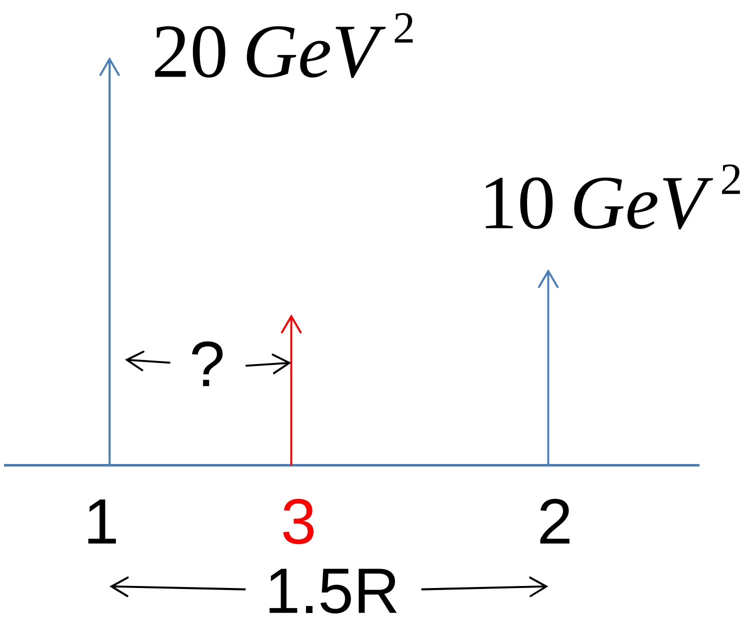
Relation to jet algorithms

- Jet definitions are formal, independent of jet algorithms
- Extraction of physics from jet definitions depends on algorithms
- Difference vanishes for extremely energetic jets
- Theoretical construction (factorization) of jets must follow some algorithm, in order to be compared with jets in experiment (data)

NLO factorization

- At NLO for gluon jet of cone radius R, need to integrate over kinematics of 3rd gluon
- What is the allowed kinematic region for combining 3rd gluon with 1st gluon ?





Algorithm-dependent kinematics

- For kT algorithm

$$d_{13} \le d_{23}, \quad \Delta R_{13} \le \Delta R_{23}$$

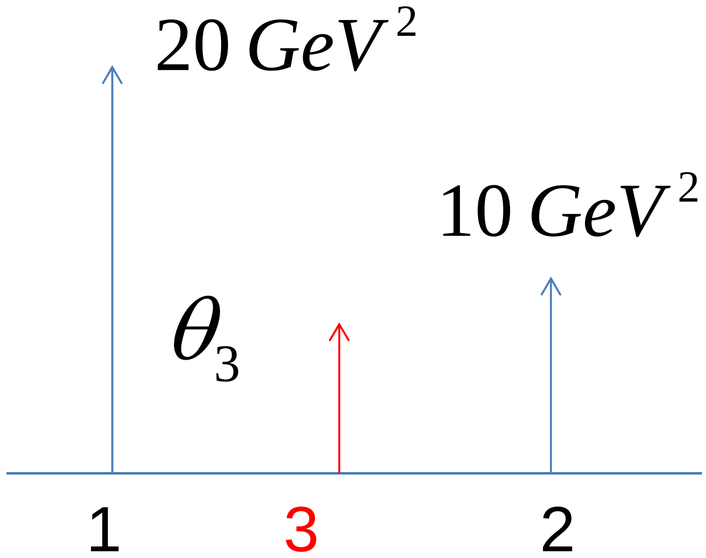
$$\theta_3 \le 0.75R$$

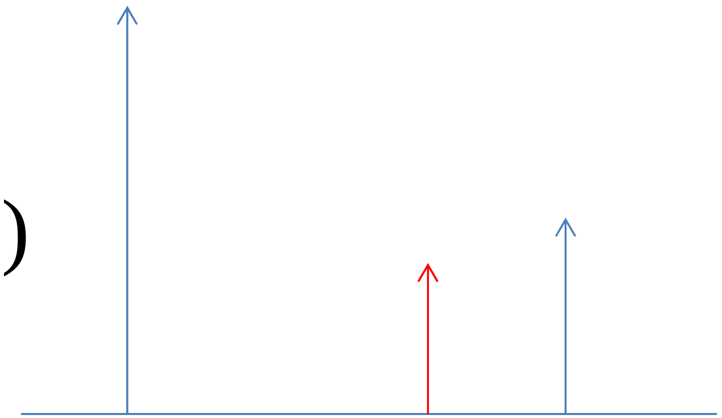
- For anti-kT algorithm

$$d_{13} \le d_{23}, \quad \Delta R_{13} \le 2\Delta R_{23}$$

$$\theta_3 \le R$$

- Angular bounds are $O(R)$
- Energetic jets become insensitive to algorithms





Summary

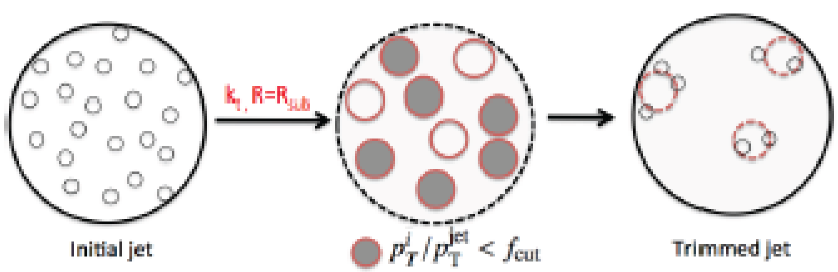
- QCD gives jets due to infrared enhancement
- Infrared safe jet definitions are needed in both experiment (sequential algorithms) and theory (ISR, FSR)
- Both experimental and theoretical constructions of jets depend on algorithms
- Must specify algorithm, before making meaningful comparison between data and predictions

Back-up slides

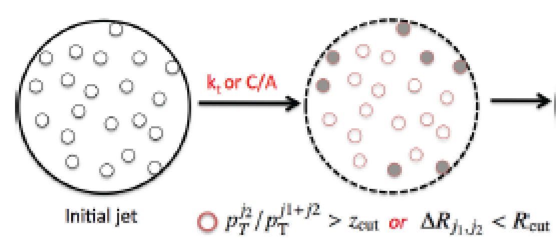
Jet grooming techniques

- **Filtering**:
  - keep only the 3 leading sub-jets

- **Trimming**:
  - remove soft sub-jets



- **Pruning**:
  - reject soft and wide-angle radiation at each clustering step



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